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Risk-Related Return Premia in the Swedish Term Structure

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Abstract

According to the expectations theory of the term structure the yield on a multi-period bond is equal to the expectation of the average of future one-period spot rates, with the possible exception of a constant term premium. In this paper holding period return premia for Swedish Treasury bills and bonds are related to the volatility in the markets for these securities. The derived yield premia and forward premia are close to zero during most of the period examined. However, the premia are substantial in a few periods of high volatility. Thus, there have been periods when the yield has increased because of higher risk in the market rather than because of a change in expectations about future one-period interest rates. The evidence of time-varying risk premia contradicts a strict interpretation of the expectations theory of the term structure.

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1. Introduction

According to the expectations theory of the term structure the yield on a multi-period bond is equal to the expectation of the average of future one-period spot rates. However, the return on a bond is risky unless you hold it until maturity and investors would therefore demand a risk premium. A constant risk premium could easily be allowed for in the expectations theory, but not a time-varying premium. One reason that there could be a time-varying risk premium is if risk varies over time.

In two recent papers, Engle, Ng, and Rotschild (1990) (hereafter ENR) and Engle and Ng (1993) examine the possibility of risk-related return premia in the term structure. They relate the excess holding period returns on individual T-bills to the conditional variance of an equally weighted bill portfolio. Thus, volatility in the T-bill market represents risk in the model. This specification leads to return premia that vary with the volatility of the T-bill market. The model seems to fit the U.S. monthly data well for the time period 1964-89. The premia are quite large in periods of high volatility.

We specify and test a version of the ENR model on Swedish data. An important extension of the analysis is to include T-bonds as well as T-bills. This also leads us to consider a multifactor model. The estimated model is a two-factor model with a bill factor and a bond factor. We derive risk premia that vary with the volatility in the T-bill market and the T-bond market. The finding of time-varying return premia contradicts the expectations theory. Yield premia and forward premia are also derived. These have been quite substantial in a few periods of high volatility during the period 1987-1994.

Looking at previous empirical studies of the Swedish term structure, the one that is closest in spirit to the present paper is Hördahl (1995). He uses the multivariate GARCH-M model of Bollerslev, Engle, and Wooldridge (1988) in which the premium on Swedish benchmark bonds is related to the conditional covariance of a market portfolio consisting of stocks and bonds. He finds premia of substantial magnitudes during periods of great uncertainty and concludes that the expectations theory is rejected. Other studies of the Swedish term structure have yielded mixed results. Hörngren (1986), Warne (1990), and Blix (1996) reject the expectations theory, while Dahlquist and Jonsson (1994) and Hördahl (1994) are not able to reject the theory. Dillén (1996) presents a regime shift model, in which investors' fears that the economy will switch to a high-inflation regime give rise to a regime shift premium for holding bonds. In addition to this, he also finds evidence of a volatility-driven time-varying term premium.

The plan of the paper is the following. In the next Section the ENR model is presented. In Section 3 we take a look at the Swedish data. ARCH models for the

T-bill and T-bond portfolios are estimated in Section 4, while one-month holding premia for the individual bills and bonds are estimated in Section 5. In Section 6 yield premia and forward premia are computed from the holding premia. Section 7 concludes.

2. The ENR Model

2.1. The Theoretical Model

Engle, Ng, and Rotchild (1990) postulate the following K-factor linear model for the one-period excess holding returns[1] of N assets,

$$\mathbf{y}_t = \mu_t + \sum_{k=1}^{K} \mathbf{g}_{kt} f_{kt} + \mathbf{v}_t, \tag{2.1}$$

where μ_t is the $N \times 1$ vector of conditional means of the excess returns, f_{kt} is the k:th factor with the $N \times 1$ vector of factor loadings $\mathbf{g}_{kt}$, and $\mathbf{v}_t$ is a $N \times 1$ vector of residuals. It is further assumed that the following moment conditions hold,

$$\begin{aligned} E_{t-1}\left[f_{kt}\right] &= 0, \forall k, t, \\ E_{t-1}\left[f_{kt} f_{jt}\right] &= 0, \forall j \neq k, t, \\ E_{t-1}\left[\mathbf{v}_t \mid f_{1t}, f_{2t}, .., f_{Kt}\right] &= \mathbf{0}, \forall t, \\ E_{t-1}\left[\mathbf{v}_t\right] &= \mathbf{0}, \forall t, \\ E_{t-1}\left[\mathbf{v}_t \mathbf{v}_t^{\top}\right] &= \mathbf{\Omega}, \forall t, \end{aligned}$$

where $\mathbf{\Omega}$ is a $N \times N$ positive semi-definite matrix. The K factors $f_1, f_2, ..., f_K$ represent sources of common risk for the assets, while $\mathbf{v}_t$ represent idiosyncratic risk. The factor loading g_{ik} gives the sensitivity of the i:th asset to a change in the k:th factor.

Let β_k, $k = 1, .., K$, be linearly independent $N \times 1$ vectors and λ_{kt} be positive random variables for all k, t. We can construct a factor model with time-varying factor loadings by assuming that $\mathbf{g}_{kt} = \beta_k \lambda_{kt}^{1/2}$ and $E_{t-1}\left[f_{kt}^2\right] = 1$ for all k, t. Alternatively, by assuming that $\mathbf{g}_{kt} = \beta_k$ and $E_{t-1}\left[f_{kt}^2\right] = \lambda_{kt}$ for all k, t, we get a factor model with constant factor loadings and factors that have time-varying conditional second moments. Either set of assumtions will lead to the following conditional covariance matrix of asset excess returns,

$$H_t \equiv E_{t-1}\left[\left(\mathbf{y}_t - \mu_t\right)\left(\mathbf{y}_t - \mu_t\right)^{\top}\right] = \sum_{k=1}^{K} \beta_k \beta_k^{\top} \lambda_{kt} + \mathbf{\Omega}. \tag{2.2}$$

[1]The one-period excess holding return is the return from holding a multi-period asset for one period compared to holding a one-period asset for that same period.

A problem with the specification above is that the factors are unobservable. We deal with this problem by considering asset portfolios that represent the factors. Let α_k be a portfolio of N assets, which is only subject to k-factor risk, i.e. $\alpha_k^\top \beta_j = 0 \; \forall j \neq k$ and $\alpha_k^\top \beta_k = 1$. The conditional variance of the excess return on such a *factor representing portfolio* will be

$$\theta_{kt} \equiv \alpha_k^\top H_t \alpha_k = \lambda_{kt} + \alpha_k^\top \mathbf{\Omega} \alpha_k \, . \tag{2.3}$$

Using Eq. 2.3 to substitute out λ_{kt} in Eq. 2.2, we get

$$H_t = \sum_{k=1}^{K} \beta_k \beta_k^\top \theta_{kt} + \mathbf{\Omega}^*, \tag{2.4}$$

where $\mathbf{\Omega}^* = \mathbf{\Omega} - \sum_{k=1}^{K} \beta_k \beta_k^\top \alpha_k^\top \mathbf{\Omega} \alpha_k$. Thus, the latent variables λ_{kt} are replaced by the conditional variances θ_{kt} of the factor representing portfolios.

Let us next see how the conditional means of the excess returns, or return premia, are determined. It is assumed that there exists a pricing kernel, which is given by[2]

$$m_t = \nu + \sum_{k=1}^{K} b_k f_{kt} + w_t, \tag{2.5}$$

with moment conditions $E_{t-1}[w_t] = 0$, $E_{t-1}[w_t | f_{1t}, .., f_{Kt}] = 0$, $E_{t-1}[w_t \mathbf{v}_t] = \mathbf{0}$, and $E_{t-1}\left[(w_t)^2\right] = \sigma_w^2$ for all t. Under the above assumptions the return premia will be given by

$$\mu_t \equiv \delta Cov_{t-1}(\mathbf{y}_t, m_t) = \sum_{k=1}^{K} \beta_k \delta b_k \lambda_{kt}, \tag{2.6}$$

where δ is a preference parameter, assumed to be constant. Premultiplying by α_k' gives us the return premium of the k:th factor representing portfolio as

$$\Pi_{kt} = \alpha_k^\top \mu_t = \delta b_k \lambda_{kt}. \tag{2.7}$$

Using Eq. 2.7 the asset return premia in Eq. 2.6 can be rewritten as linear functions of the return premia of the factor representing portfolios,

$$\mu_t = \sum_{k=1}^{K} \beta_k \Pi_{kt} = \sum_{k=1}^{K} \beta_k \left(c_k + \gamma_k \theta_{kt} \right) . \tag{2.8}$$

where $c_k = -\delta b_k \alpha_k^\top \mathbf{\Omega} \alpha_k$ and $\gamma_k = \delta b_k$ are constants. The last equality is obtained by substituting Eq. 2.3 into Eq. 2.7. Hence, the asset return premia are functions of the conditional variances of the factor representing portfolios.

[2]Typically in a CCAPM framework we would think of the pricing kernel as being the rate of change of marginal utility of consumption for a representative agent with time-separable vonNeuman-Morgenstern utility.

In Eqs. 2.8 and 2.4 the means and conditional variances of the asset excess returns are conveniently expressed as linear functions of the conditional variances of the K factor-representing portfolios. In the next subsection this model will be put into testable form.

2.2. The Econometric Specification

In what ENR call a *general portfolio representation* the excess return on any factor-representing portfolio depends on the history of the past excess returns of all K factor-representing portfolios. Consistent estimates of the conditional covariances, θ_{kt}'s, can be obtained by estimating a system of K factor-representing portfolios. Hence, for each k=1,...,K, and all t, we have

$$\begin{aligned} P_{kt} &= \Pi_{kt} + u_{kt} = c_k + \gamma_k \theta_{kt} + u_{kt} \\ \theta_{kt} &= \omega_k + \textstyle\sum_p \sum_j \left(\phi_{kj} u^2_{k,t-p} + \varphi_{kj} \theta_{k,t-p} \right), \end{aligned} \tag{2.9}$$

where P_{kt} is the excess return on portfolio k, u_{kt} is normally distributed with mean zero and conditional variance θ_{kt}, a GARCH-M specification is assumed to start with, and the summation is over all K portfolios. The estimation is further simplified if the system has a *recursive portfolio representation*, such that the excess return on the k:th portfolio depends only on its own history and the history of the first k-1 portfolios (the second summation in Eq. 2.9 goes from j=1 to k-1). In the most restrictive case the K factor-representing portfolios will have a *univariate portfolio representation*, meaning that the excess return on a portfolio depends only on its own history (the summation in Eq. 2.9 collapses to j=k).

The procedure to be followed is this. First, a univariate portfolio representation will be estimated. After testing whether it should be expanded into a recursive or general representation, the preferred portfolio models are estimated. After that it will be possible to estimate return premia for the individual assets. The model to be used for this purpose is derived from Eqs. 2.8 and 2.4. For all assets i and all time periods t, we have

$$\begin{aligned} y_{it} &= \psi_i + \textstyle\sum_k \beta_{ki} \Pi_{kt} + \varepsilon_{it} \\ h_{it} &= \sigma_{ii} + \textstyle\sum_k \beta^2_{ki} \theta_{kt} \end{aligned} \tag{2.10}$$

where ε_{it} is normally distributed with mean zero and conditional variance h_{it}. The constant ψ_i has been included in the mean equation to capture a possibly constant term premium. The constant σ_{ii} is the i:th diagonal element in the Ω^* matrix in Eq. 2.4. In estimating the model in Eq. 2.10 the Π_{kt}'s and θ_{kt}'s are predetermined variables from the factor-representing portfolio model Eq. 2.9 above. The model in Eq. 2.10 will be referred to as the Factor ARCH model.

3. The Data

The sample period is June 1987 to December 1994. This is a sample of 90 observations (since one observation will be lost computing the monthly holding returns). Data for the nine-year bond are only available starting in June 1987. Dropping the nine-year bond from the sample would not help much, since before May 1987 data are only available for a limited number of maturities. The interest rate series include T-bills (Statsskuldväxlar) with maturities of 1 to 12 months and T-bonds (Statsobligationer) with maturities of 2 to 9 years. A value-weighted stock index (Affärsvärldens Generalindex) is also considered. All series were obtained from the Swedish Central Bank's (Sveriges Riksbank) database.

For most of the period T-bills have been issued only in the middle of the month. In an effort to make the stated maturities correspond as closely as possible to the actual maturities data have been collected on the 15th of the month or on the nearest trading day after that date. This procedure has been followed for all the series to get a synchronous set of data. One exception is the starting date, which is June 22, 1987 for all series because that is the first date for which a quotation is available for the nine-year bond. In February 1990 there was a short labor market conflict in the banking sector and the data for that month are from the first trading day after the strike, which was February 19.

Figure 1 depicts the continuously compounded interest rates on 1 and 12 month T-bills along with the 5-year T-bond, while Figure 2 shows the monthly excess holding returns on the securities with the longer maturities (expressed in annual percentage terms).[3] The extremely high short-term interest rates in September 1992 reflect the attempt to defend the fixed exchange rate. The krona was eventually allowed to float on November 19 of the same year.

A visual inspection of the return series in Figure 2 indicates that return models with time-varying conditional variances could possibly yield a good characterization of these time series. The excess holding period returns are highly positively correlated across maturities. This makes it seem plausible that they could be driven by a set of common factors. In the next section we will treat these questions more rigorously.

4. Estimating Factor-Representing Portfolio Models

It is not clear how one should go about choosing the factor-representing portfolios. Using an equally-weighted bill portfolio will enable comparisons with Engle, Ng, and Rotschild (1990) and Engle and Ng (1993). On the other hand, a bond portfolio could capture some aspects of risk that are more important at the longer

[3]Holding-period returns are computed as in Shiller (1990), which also adjusts for coupons. Excess holding returns are then obtained by simply subtracting the return on the one month bill.

Table 4.1: Testing for ARCH in Portfolio Returns

Number of lags	Bill portfolio	Bond portfolio	Stock index
1	0.011	0.647	0.661
2	0.034	0.734	0.885
3	0.068	0.053	0.752
4	0.117	0.107	0.874
5	0.200	0.119	0.921
6	0.299	0.062	0.198

NOTES.- Probability values for the null hypothesis of no ARCH.

end of the maturity spectrum. The stock market could possibly provide us with additional information on the covariance structure of asset prices. Hence, we will be looking at a bill factor, a bond factor, and a stock market factor. More specifically, the following factor-representing portfolios will be considered: (1) a portfolio with equal weights on each of the bills and zero weights on all other assets, (2) a portfolio with equal weights on the bonds and zero weights on all other assets, and (3) a portfolio with a weight of one on the stock index and zero weights on all other assets.

The excess holding period returns on the portfolios are subjected to the LM-test for ARCH errors proposed by Engle (1982). The probability values from the test of no ARCH effects are presented in Table 4.1. The assumption of no ARCH effects in the T-bill returns is rejected at one and two lags at the 5 percent level of significance. ARCH in the bond returns is marginally significant (at the 5 percent level) at three lags. There is no evidence of ARCH effects in the stock market index. Hence, only the T-bill and T-bond portfolios will be used in what follows.

Based on the results in Table 5.1 a GARCH(1,1)-M and a GARCH(3,1)-M were estimated for the bill and bond portfolios respectively. Because of the leptokurtosis in the standardized residuals, $e \equiv u_t\theta_t^{-1/2}$, the robust Wald test in Bollerslev and Wooldridge (1992) was used to prune the models. After shedding superfluous variables the preferred factor-representing portfolio models are estimated as (subscript b = bill portfolio and B = bond portfolio):

$$\begin{aligned} P_{bt} &= \underset{\substack{(1.016)\\ [0.39]}}{0.002} + \underset{\substack{(4.07)\\ [0.22]}}{0.35}\ \theta_{bt} + u_{bt} \\ \theta_{bt} &= \underset{\substack{(2.99)\\ [0.46]}}{0.0003} + \underset{\substack{(4.19)\\ [0.23]}}{3.30}\ u_{b,t-1}^2 \end{aligned} \tag{4.1}$$

$$
\begin{aligned}
P_{Bt} = & \underset{\substack{(1.01)\\ [0.46]}}{-0.09} + \underset{\substack{(1.01)\\ [0.67]}}{2.252}\ \theta_{Bt} + u_{Bt} \\
\theta_{Bt} = & \underset{\substack{(4.79)\\ [1.16]}}{0.03} + \underset{\substack{(1.29)\\ [0.86]}}{0.28}\ u^2_{B,t-3}
\end{aligned}
\tag{4.2}
$$

where ordinary t-statistics are shown in parentheses under the estimated coefficients and t-statistics that are robust to departures from normality are shown in square brackets. The robust t-statistics for the bill portfolio are much smaller than the ordinary t-statistics. The reason for the imprecise estimates is the high kurtosis in the standardized residuals (the excess kurtosis measure, which is zero for the normal distribution, takes the value 4.70). The ARCH(3)-M model for the bond portfolio does not have the same problem. The normalized kurtosis is a more managable 1.8, even though it is still highly significant. In both models the standardized residuals are negatively skewed (-.88 and -.96 respectively). The assumption of normality is clearly violated and it is important to use robust statistics in evaluating the Quasi Maximum Likelihood estimates (see Bollerslev and Wooldridge (1992)). The joint significance of the two coefficients of interest in Eq. 4.2 (not the constant terms) was tested using a robust Wald test. They were found to be jointly significant at any reasobable level of significance.

Because of the exceptionally high kurtosis in the standardized residuals from the bill portfolio model we consider an alternative model, an EGARCH-M model, which may fit the bill returns better.[4] This model is estimated as

$$
\begin{aligned}
P_{bt} = & \underset{\substack{(1.18)\\ [0.48]}}{-0.003} + \underset{\substack{(2.54)\\ [1.25]}}{1.62}\ \theta_{bt} + u_{bt} \\
\ln\theta_{bt} = & \underset{\substack{(9.48)\\ [4.46]}}{-7.19} - \underset{\substack{(0.90)\\ [0.41]}}{0.10}\ \ln\theta_{b,t-1} \\
& + \underset{\substack{(7.49)\\ [2.96]}}{1.34}\left(\left|\frac{u_{b,t-1}}{\sqrt{\theta_{b,t-1}}}\right| - \sqrt{\frac{2}{\pi}} - \underset{\substack{(2.68)\\ [1.40]}}{0.37}\ \frac{u_{b,t-1}}{\sqrt{\theta_{b,t-1}}}\right).
\end{aligned}
\tag{4.3}
$$

The t-value for the GARCH-coefficient is very low, but reestimating the model without it does not change any of the other coefficients. From the coefficient within the brackets (-0.37), we can tell that a negative innovation has a greater effect on the conditional variance than a positive innovation. There is no straightforward way of comparing the models in Eqs. 4.3 and 4.1 since they are not

[4]The EGARCH specification was suggested in Nelson (1991).

nested. However the t-values in Eq. 4.3 are generally higher and the excess kurtosis has been reduced from 4.70 to 1.89. An additional benefit is that the skewness has fallen to -.2, which is insignificant. A test of any remaining ARCH effects in the standardized residuals of the models in Eqs. 4.2 and 4.3 was conducted and no such effects were found. Thus, the models seem to adequately describe the autoregressive conditional heteroskedasticity in the return series. Tests were also conducted to ascertain whether there was any influence from the volatility in one market on the volatility in the other market. No such causality in variance was detected. Hence, we stick to the simple univariate portfolio representation of Eqs. 4.2 and 4.3.

5. Estimating Individual Asset Return Premia

We estimate the return premia of the 11 bills and 8 bonds using the Factor ARCH model with two factors,

$$\begin{aligned} y_{it} &= \psi_i + \beta_{bi}\Pi_{bt} + \beta_{Bi}\Pi_{Bt} + \varepsilon_{it} \\ h_{it} &= \sigma_{ii} + \beta_{bi}^2\theta_{bt} + \beta_{Bi}^2\theta_{Bt} \end{aligned} \tag{5.1}$$

Quasi Maximum Likelihood is employed in estimating the models in Eq. 5.1 for the 19 different maturities: bills of 2 to 12 months and bonds of 2 to 9 years. The results of these estimations are presented in Table 5.1. To save space the constant terms are not reported. The ψ_i were small and insignificant for all maturities. Ordinary t-statistics are reported in parentheses after the estimated coefficients and robust t-statistics are reported in square brackets.

Even if the precision in the estimates of the bill betas are low, there is a clear pattern for the shorter maturities. The bill betas for 2 month bills to 10 month bills are monotonically increasing. A similar pattern is even more evident for the bond betas. With the exception of the 11 and 12 month bills, the bond betas are increasing for maturities of 3 months up to 7 years. Bond betas for maturities of more than 9 months are measured with satisfactory precision, with robust t-values ranging from 1.83 to 4.04.[5] We will relate these findings to previous studies on U.S. data at the end of this section.

We would like to check the assumption that the standardized residuals $e_{kt} \equiv u_{kt}\theta_{kt}^{-1/2}$ are standard normally distributed by testing for skewness and kurtosis. Under the null hypothesis of normality, the measures of skewness $(6T)^{-1/2}\sum_{t=1}^{T} e_{kt}^3$ and excess kurtosis $(24T)^{-1/2}\sum_{t=1}^{T}(e_{kt}^4 - 3)$ are both asymptotically standard normally distributed. In addition, the sum of the squares of these measures is asymptotically distributed as Chi-square with two degrees of freedom, which

[5] All 19 models were also estimated using the model in Eq. 4.1 in lieu of Eq. 4.3. The main difference was a deterioration in the precision of the QML estimates and lower bond betas for the bills. However, the other results were mainly unaltered.

Table 5.1: Restricted 2-Factor ARCH Model for T-bills and T-bonds

	β_{bi}	t	rt	β_{Bi}	t	rt	sk	ku	DH	LR1	LR2
2 m	0.24	(5.40)	[0.17]	0.08	(18.08)	[0.58]	-1.15	8.67	58.81*	5.46	23.11*
3 m	0.41	(7.20)	[0.44]	0.06	(1.32)	[0.12]	-0.91	7.40	57.46*	5.82	2.83
4 m	0.54	(7.57)	[0.51]	0.12	(1.35)	[0.12]	-0.78	6.45	52.35*	2.74	7.15*
5 m	0.70	(7.65)	[0.69]	0.12	(2.18)	[0.45]	-0.73	5.32	40.66*	1.07	6.16**
6 m	0.81	(7.34)	[0.74]	0.17	(2.47)	[0.66]	-0.72	4.47	31.37*	2.26	7.94*
7 m	0.92	(7.01)	[0.78]	0.18	(2.69)	[0.82]	-0.73	3.68	22.85*	1.87	8.51*
8 m	1.05	(7.19)	[0.78]	0.21	(2.92)	[1.00]	-0.64	3.35	21.77*	2.34	10.31*
9 m	1.31	(6.99)	[1.65]	0.34	(3.98)	[1.83]	-0.44	1.61	8.79**	7.18**	13.52*
10m	1.49	(6.86)	[1.64]	0.43	(4.62)	[2.16]	0.49	2.28	13.87*	1.87	36.47*
11m	1.06	(5.45)	[1.07]	0.30	(3.56)	[1.91]	-0.38	1.45	7.94**	2.51	12.30*
12m	1.01	(4.88)	[1.16]	0.34	(3.70)	[2.14]	-0.27	1.33	7.45**	2.16	13.38*
2 y	0.90	(2.93)	[1.18]	0.56	(4.08)	[3.34]	-0.32	0.49	2.41	1.09	11.87*
3 y	0.88	(2.09)	[0.66]	0.79	(3.76)	[3.60]	-0.53	0.95	4.99	0.92	4.68**
4 y	1.33	(2.78)	[0.76]	0.97	(3.52)	[3.30]	-0.68	1.40	7.30**	1.39	7.04*
5 y	1.05	(1.64)	[0.36]	1.18	(3.31)	[3.28]	-0.77	1.48	8.16**	0.96	2.83
6 y	0.46	(0.48)	[0.08]	1.18	(3.40)	[4.04]	-0.93	1.76	10.78*	0.91	0.51
7 y	0.60	(1.15)	[0.85]	1.26	(2.88)	[3.17]	-0.76	1.25	7.73**	0.09	1.80
8 y	0.21	(0.36)	[0.59]	1.19	(2.79)	[3.43]	-0.99	1.79	12.34*	0.03	0.12
9 y	0.38	(0.84)	[1.67]	0.99	(2.42)	[3.95]	-0.94	2.32	11.53*	0.67	0.60

NOTES.- Ordinary t-values are reported in parentheses to the right of the coefficient estimates. Robust t-values are reported in square brackets. sk is the measure of skewness (normal=0), ku is a normalized measure of kurtosis (normal=0), and DH is the Doornik-Hansen measure of normality described in the text. LR1 is a likelihood ratio test comparing the restricted models to the unrestricted models and LR2 is a likelihood ratio test comparing the restricted two-factor model with the restricted one-factor model (a bill factor for the bills and a bond factor for the bonds). ** and * indicate significance at the 5 percent and 1 percent level respectively.

yields a test of the joint hypothesis of symmetry and no excess kurtosis.[6] One problem with this test is that the kurtosis measure converges very slowly to standard normal. Doornik and Hansen (1994) suggest an alternative test statistic with better size properties in small samples, where the above measures are transformed to be closer to standard normal. This is the measure denoted DH in Table 5.1. The assumption of normality is rejected at the 5 percent level of significance for all maturities except the 2- and 3-year bonds.

LR1 is a likelihood ratio test comparing the restricted models to the unrestricted models. The nature of the restriction is that the parameters appearing in the conditional variance equation are the same as the parameters in the mean equation squared. The statistic is distributed as Chi squared with 2 degrees of freedom. The restriction is not rejected for any maturity (except the 9-month bill) at the 5 percent level of significance. This is strong evidence in favor the restrictions imposed by the theory.[7]

We have used the bill and bond portfolios as factor representing portfolios. This implies that we should have $\alpha_b^T \beta_b = 1$ and $\alpha_B^T \beta_B = 1$. Using the estimates in the table we get $\alpha_b^T \hat{\beta}_b = 0.87$ and $\alpha_B^T \hat{\beta}_B = 1.01$.

We may want to consider the possibility that the security returns would be better modeled with a 1-Factor ARCH specification. LR2 is a likelihood ratio test comparing the restricted two-factor model with the restricted one-factor model (a bill factor for the bills and a bond factor for the bonds respectively). The test statistic is distributed as Chi square with one degree of freedom. The restriction to one factor is rejected at the 5 percent level for all maturities of less than 5 years (except for the 3 month bill). For maturities of 5 years and longer a bond factor seems to be sufficient. We also have $\alpha_B^T \hat{\beta}_B = 0.98$.

The QML estimates of a one-factor model for the bonds are given in Table 5.2. The bond betas are marginally lower than in the two-factor model. The robust t-statistics are also somewhat lower. LR1 is a likelihood ratio test comparing the restricted one-factor model to the unrestricted one-factor model. The restrictions are not rejected for any maturity for any reasonable significance level. Hence, again the restrictions imposed by the theoretical model are supported by the data. LR2 is the same as in Table 5.1. The restriction to one factor is only rejected for the shorter bonds (2-4 years). The overall impression is that the bill portfolio does not contribute much to the pricing of bonds with a maturity of 5 years and more.

The results pertaining to the bill portfolio betas for the bills are very similar to t'.e results on U.S. data in Engle and Ng (1993). They price bills using a

[6]This test is sometimes referred to as the Bera-Jarque test.

[7]The estimates of the unrestricted two-factor model for the 2- and 4-month bills were obtained after considerable difficulties in getting the optimization algorithm to converge. For all other estimates reported in this paper convergence was rapid and not very sensitive to the choice of initial values.

Table 5.2: Restricted 1-Factor ARCH Model for T-bonds

	β_{Bi}	t	r-t	**LR1**	**LR2**
2 years	0.54	(3.51)	[2.03]	0.04	11.87*
3 years	0.75	(3.45)	[2.93]	0.01	4.86**
4 years	0.88	(3.19)	[1.95]	0.03	7.04*
5 years	1.11	(3.14)	[2.81]	0.01	2.83
6 years	1.16	(3.28)	[3.69]	0.00	0.51
7 years	1.24	(3.04)	[2.94]	0.00	1.80
8 years	1.18	(2.79)	[3.30]	0.02	0.12
9 years	0.98	(2.40)	[3.07]	0.33	0.60

NOTES.- t-values in parentheses and robust t-values in square brackets. LR1 is a likelihood ratio test comparing the restricted model with the unrestricted model and LR2 is the same as in the previous table. ** and * indicate significance at the 5 percent and 1 percent level respectively.

one-factor model with a bill portfolio. Their beta estimates range from a low of 0.17 for the 2-month bill and then rise monotonically to a high of 1.77 for the 11-month bill. The beta for the 12-month bill is 1.57. They advance the change in the definition of the 12-month bill by CRSP as an explanation for the 12-month beta being lower than the 11-month beta. The findings in this paper suggest an alternative explanation. Note that in Table 5.1 the bill beta is highest for the 10-month bill and then seem to decrease with maturity and becomes unimportant for the longer bonds (according to the LR2 statistic). This is because there is a second factor that gains in importance with maturity, viz. the bond factor.

The bond beta is highest for the 7-year bond (1.26) and then falls with maturity (to 1.19 for the 8-year bond and 0.99 for the 9-year bond). In analogy with our discussion earlier, this could indicate that there should be a third factor which is important for long maturities and which is missing from the model. That is, unless one is willing to believe that the 8- and 9-year bonds are less risky than the 7-year bond.

In summary, a 2-Factor ARCH with a bill factor and a bond factor seem to give a good description of the one-month excess holding returns of the T-bills and the short T-bonds, while a 1-Factor ARCH with a bond factor seem to be sufficient in pricing the long T-bonds.

6. Yield Premia and Forward Premia

The yield on an n-period bond can be divided into two parts,[8]

$$r_t^{(n)} = p_t^{(n)} + q_t^{(n)}, \tag{6.1}$$

where the first part is the expectation of average future one-period spot rates and the second part is the premium component,

$$p_t^{(n)} = E_t\left[\frac{1}{n}\sum_{j=1}^{n} r_{t+j-1}^{(1)}\right], \tag{6.2}$$

$$q_t^{(n)} = E_t\left[\frac{1}{n}\sum_{j=1}^{n} \mu_{t+j-1}^{(n-j+1)}\right]. \tag{6.3}$$

Engle and Ng (1993) refer to $q_t^{(n)}$ as the *yield premium.* According to the expectations theory of the term structure these premia should be equal to zero. In the applied literature one has allowed for a premium that is constant over time. Hence, according to the expectations theory the yield on a multiperiod bond should be equal to the expectation of average future one-period spot rates, with the possible exception of a constant term premium. The expectations theory is a maintained assumption in the growing literature on using the term structure as an indicator for monetary policy.[9] If there are time-varying term premia this indicator will not work as well as in the absence of such premia.

Based on Eqs. 5.1 and 2.8 we can write the return premium for an asset with maturity n as

$$\mu_t^{(n)} = a_n + s_{bn}\theta_{bt} + s_{Bn}\theta_{Bt}, \tag{6.4}$$

where $a_n = \psi_n + \beta_{bn}c_b + \beta_{Bn}c_B$, $s_{bn} = \beta_{bn}\gamma_b$, and $s_{Bn} = \beta_{Bn}\gamma_B$. Combining Eqs. 6.3 and 6.4 we can express the yield premium on an n-period bond as

$$q_t^{(n)} = \frac{1}{n}\sum_{j=1}^{n}\left[a_{n-j+1} + s_{b,n-j+1}E_t\left(\theta_{b,t+j-1}\right) + s_{B,n-j+1}E_t\left(\theta_{B,t+j-1}\right)\right]. \tag{6.5}$$

We can use the portfolio models in Eqs. 4.2 and 4.3 to obtain the multiperiod forecasts of future volatility that are needed in Eq. 6.5. These forecasts and the estimated coefficients plugged into Eq. 6.5 will enable us to compute the yield premia for bonds with different maturities.

In Figure 3 the yield and yield premium for a 3-month T-bill is shown (expressed in annual percentage terms). For most of the period the yield premium has been positive but small. But there are some periods when the premium has been substantial. In February 1990 it was 1.2 %, in December 1991 it was 4.2%,

[8]See Engle and Ng (1993) for the derivation.
[9]See for example Svensson (1993).

and finally towards the end of the fixed exchange rate regime during the period August to November 1992 it was 0.7%, 2.1%, 0.2%, and 0.5% in the respective months. The premium continued to be relatively high for a few months after the abandonment of the fixed exchange rate on November 19. Towards the second half of 1994 the yield premium has also been relatively high. An increase in the yield is often but not always accompanied by a higher risk premium. In early 1990 the increased yield depends to some extent on a higher yield premium, but the increased yield towards the end of 1990 seems not to have anything to do with risk as the yield premium is practically zero.[10]

It is possible to identify the causes of the uncertainty in the three periods we have identified above. At the end of 1989 the Swedish krona came under pressure. An ensuing political crisis culminated in the resignation of the government in early February 1990. This led to a great deal of uncertainty about the future direction of economic policy. On November 15, 1991 Finland devalued its currency. The Swedish krona also came under pressure. This led the Riksbank to raise the marginal overnight rate from 10.5 to 11.5 percent on November 26 and then to 17.5 percent on December 5. This action seemed to convince the market that Swedish exchange rate policy would stand firm. In the fall of 1992 the Swedish krona again came under pressure and was eventually allowed to float on November 19.

At this stage we can also easily compute the *forward premia*. We start by defining the forward rate. The one-period forward rate at time t for the period from time n-1 to time n is

$$f_t^{(n)} \equiv n r_t^{(n)} - (n-1) r_t^{(n-1)}. \tag{6.6}$$

Using Eq. 6.1 we get

$$f_t^{(n)} = E_t\left[r_{t+n-1}^{(1)}\right] + n q_t^{(n)} - (n-1) q_t^{(n-1)}. \tag{6.7}$$

The forward premium is defined as the difference between the forward rate and the expected future spot rate,

$$L_t^{(n)} \equiv f_t^{(n)} - E_t\left[r_{t+n-1}^{(1)}\right], \tag{6.8}$$

which, using Eq. 6.7, can be rewritten in terms of the yield premia,

$$L_t^{(n)} = n q_t^{(n)} - (n-1) q_t^{(n-1)}. \tag{6.9}$$

In Figure 4 the one-month interest rate, forward rate, and forward premium for the period from two to three months into the future are shown. Not surprisingly, the pattern is similar to the one for the yield premium in Figure 3. The premium is of importance only in a few periods of very high volatility.

[10]In the 1990 Annual Report of Sveriges Riksbank it is stated that "As the year drew to a close there were market expectations of currency outflows, partly in conjunction with the tightening of capital adequacy requirements for banks". Not a great deal of uncertainty seems to have been connected to these expectations according to Figure 3.

7. Conclusion

A Factor ARCH model was considered as a way of modeling the risk premia in interest-bearing securities. In this model the monthly holding returns are related to the volatility in two factors. These factors are represented by a portfolio of equally-weighted T-bills and a portfolio of equally-weighted T-bonds. The risk premia are linearly related to the conditional variances of these portfolios.

The model was fitted to the Swedish term structure for the period June 1987 to December 1994. A 2-Factor ARCH with a bill factor and a bond factor seem to fit the data for the one-month excess holding returns of the T-bills and the short T-bonds quite well, while a 1-Factor ARCH with a bond factor is sufficient for the long bonds. There is some reason to believe that the inclusion of a third factor (replacing the bill factor) would provide a better model for the longest maturities. The one-month holding premia are (generally) shown to be increasing in maturity.

A risk premium for the yield to maturity was derived from the one-month holding premia. The yield premium for the 3-month T-bill has been close to zero for most of the period. But there are shorter periods within this period when the yield premium has been quite substantial. A forward premium for the period from two to three months into the future was also derived. This premium showed a time path very similar to that of the yield premium.

It is quite clear that there have been instances when the yield has increased because of increased risk in the market rather than because of a change in expectations about future one-period interest rates. This evidence of time-varying premia contradicts a strict interpretation of the expectations theory of the term structure.

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Figure 1. Yields on selected bills and bonds

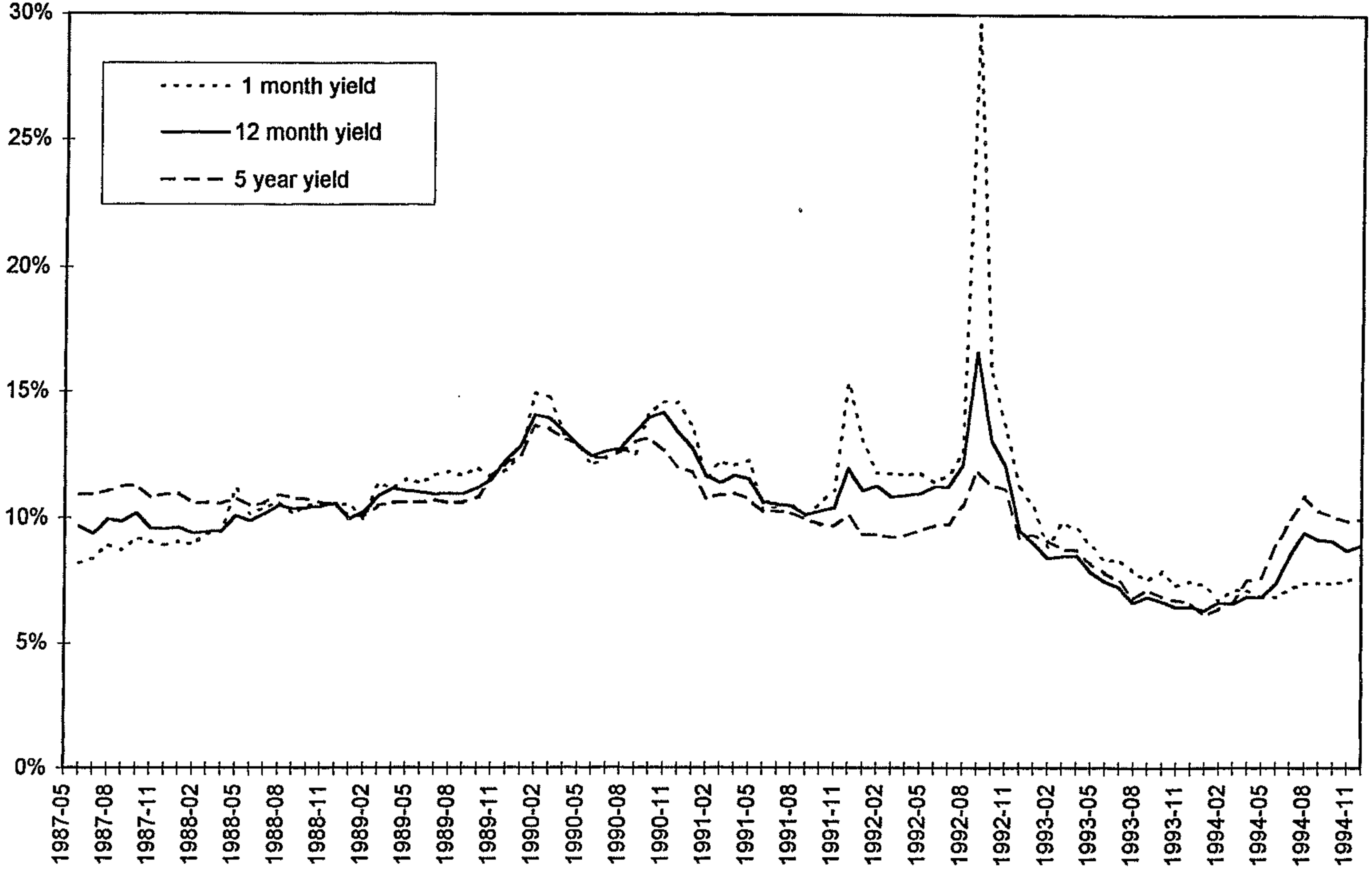

Figure 2. One month excess holding period returns

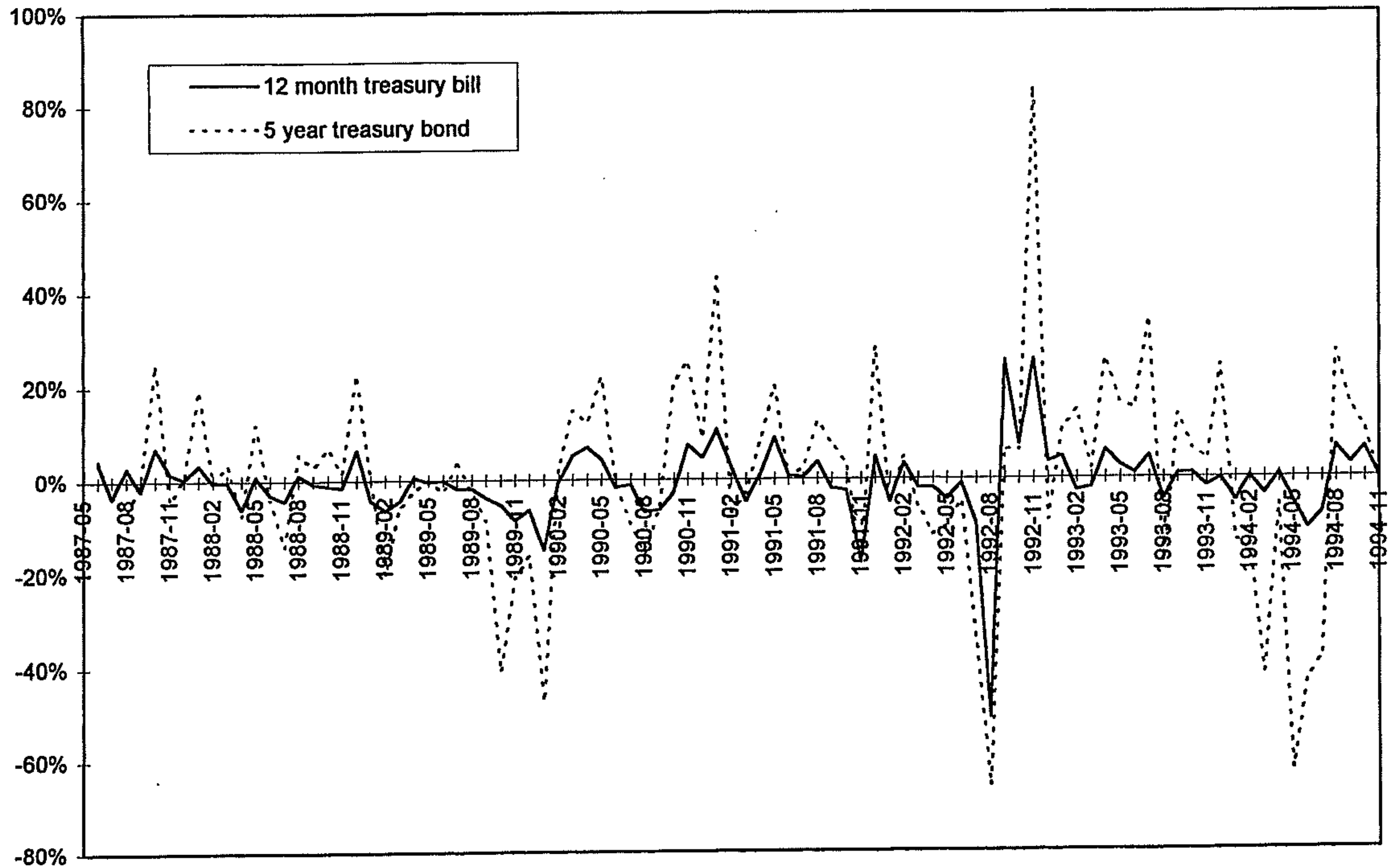

Figure 3. Yield and yield premium on 3-month T-bill

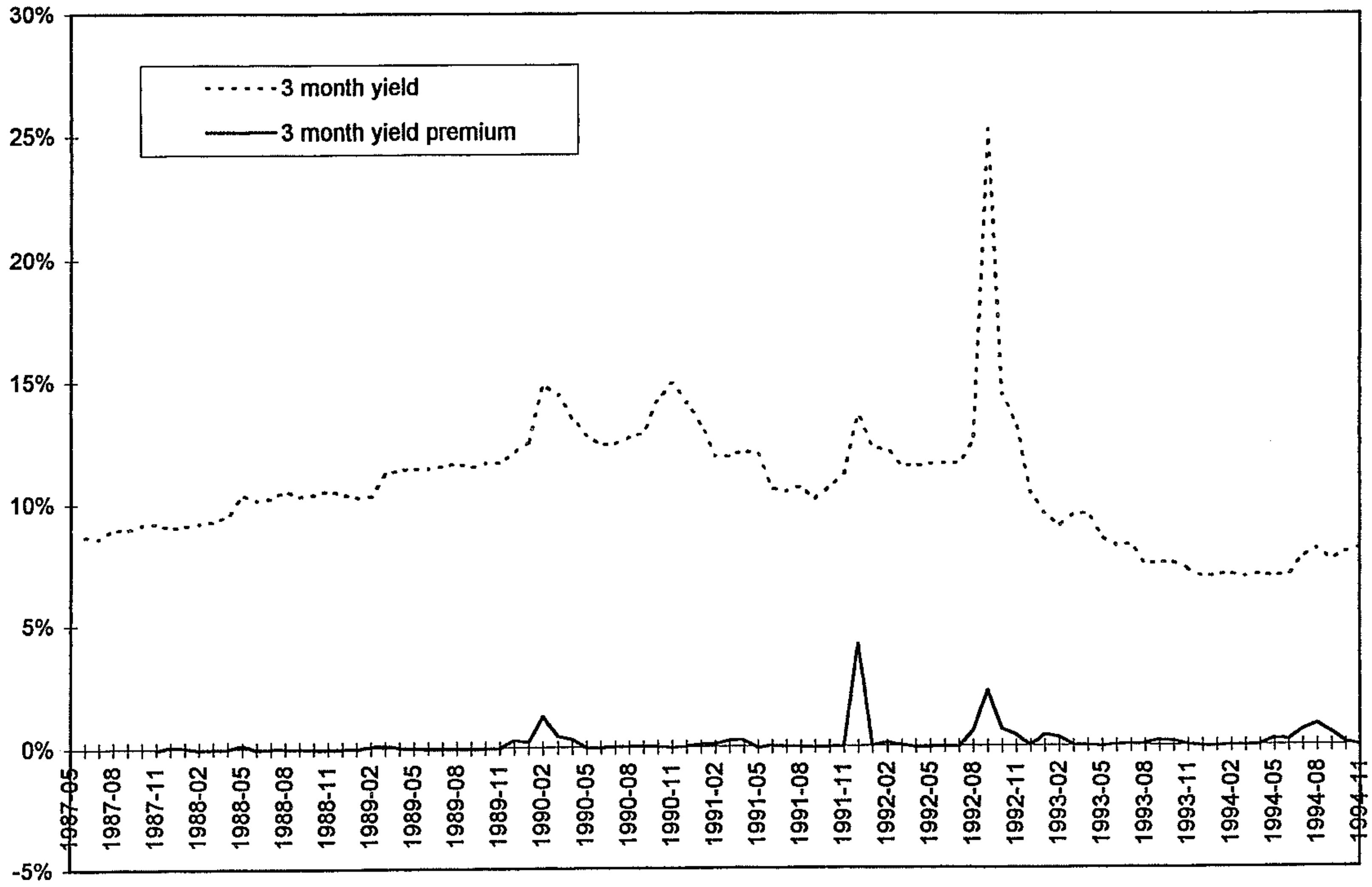

Figure 4. The one month forward rate and forward premium with a two month horizon