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Regime Shift Premia in the Swedish Term Structure: Theory and Evidence

by

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Abstract: This paper presents a regime shift model, in which investors' fears that the economy will switch to a high-inflation regime give rise to a regime shift premium for holding bonds. In this way the term structure effects of a not fully credible inflation target can be explicitly modelled. A nice feature of the model is that it provides an explanation for the volatility of long term interest rates relative to the short interest rate. An empirical investigation on Swedish data yields evidence that supports the main features of the model. In addition, a time varying term premium, driven by holding return volatility of ARCH type, seems to exist, although its significance is not so large. The presence of regime shift and term premia has important implications for the use of spot and forward interest rates as indicators of expectations of future short term interest rates and inflation rates.

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1. Introduction

The abandonment of a fixed exchange rate as an intermediate monetary policy target in Sweden (and elsewhere in Europe) has increased the need for indicators of monetary policy. Recently attention has been paid to the use of forward interest rates as a monetary policy indicator. As demonstrated by Svensson (1994) forward interest rates are indicative of market expectations of the evolution of future short term interest rates, inflation rates and currency depreciation rates. The dramatic movements of spot and forward interest rates in recent years do, however, give rise to interpretative problems. The problem is essentially that the fluctuations in long term rates have been very volatile and it is difficult to relate this interest volatility to fluctuations in economic factors that usually are believed to affect interest rates. Indeed the fact that long term interest rates have been more volatile than short term interest rates is hard to reconcile with fundamental economic theory, in particular the expectation hypothesis.[1]

According to the vast majority of commentators, the high Swedish interest rates to a large extent reflect the worrisome development of the national debt - higher budget deficits lead to higher interest rates. These commentators are, however, not very precise in pointing out the mechanisms behind such a relationship. Dillén (1995) discusses the possibility that investors have fears that an increased real burden of the national debt may lead to a situation where the current low inflation policy is abandoned and the inflation rate becomes very high. It is reasonable to believe that these fears are reinforced by Sweden's poor inflation record. Moreover, it is shown in Dillén (1995) that a high/low inflation switching regime model proposed by Dillén (1994) is quite promising in explaining recent movements in the Swedish term structure. This model can explain non-trivial phenomena such as volatile long term interest rates and a positive correlation between the exchange rate and the interest rate differential.

The purpose of this paper is to present a variant of the high/low inflation switching regime model mentioned above and evaluate the model empirically on Swedish data. This exercise will result in an estimate of a regime shift premium, showing the effect of investors' expectations of a switch to a high inflation regime (i.e. the imperfect credibility of the low inflation policy). Moreover, the regime shift adjusted forward rate (the forward rate minus the regime shift premium) should be indicative of market expectations of the evolution of future short term interest rates, inflation rates and currency depreciation rates contingent on the non-occurrence of a regime shift. Thus, by adjusting for credibility effects in this manner, the precision of forward rates as a monetary policy indicator can be improved.

The regime shift premium is a first moment factor since it reflects expectations. In the financial literature, however, attention often has been paid to second moment factors capturing the volatility of asset prices as a determinant of various kinds of risk premia. Therefore, also a time varying term premium, reflecting the compensation for risk that volatile bond prices give rise to, will be considered. The term premium will essentially be estimated by means of the ARCH-M model suggested by Engle, Lilien and Roberts (1987), though a variant of this estimation technique will also be used.[2] On Swedish data this type of model has been examined by Hördahl (1995), while Sellin (1996) estimates term premia by a factor ARCH model. The use of forward rates as monetary policy

[1] See Shiller (1979) or Singleton (1980) for an analysis of the expectation hypothesis and its implications for the volatility of long term interest rates. A review of the expectation hypothesis with a focus on the recent development on the bond markets is provided by Campbell (1995).

[2] ARCH stands for Auto Regressive Conditional Heteroscedicity and ARCH-M stands for ARCH in mean.

indicators can be further improved by making a proper adjustment for a time varying term premium. The inclusion of a time varying term premium also makes it possible to compare the significance of regime shift premia (first moment effects) with that of time varying term premia (second moment effects). Moreover, if regime shift premia turn out to be important, then the exclusion of this factor can be expected to lead to spurious ARCH effects. Thus it is important to estimate regime shift and term premia simultaneously. Regime shift effects in the term structure were originally analyzed by Hamilton (1988) but in a somewhat different context. Regime shift effects have also been examined by Blix (1995) on Swedish (and UK) data, but those regime shifts were designed to model a time varying term premium rather than a regime shift premium that a not fully credible inflation target gives rise to. This study is instead more related to the kind of inflation regimes considered by Evans and Wachtel (1993) and Evans and Lewis (1995).

The outline of the paper is as follows. Section 2 introduces the notion of forward rates and excess returns and discusses how these concepts can be interpreted economically. Special attention is paid to the problem of decomposing systematic fluctuations in the excess return into a forecast error term, that expectations of future regime shifts give rise to, and a time varying term premium. A simple regime shift model is presented in section 3 in order to illustrate some general features of such models with emphasis on the implications for the forward rate curve. On the basis of this model different econometric models are developed in section 4. The econometric models are estimated in section 5 and the results are evaluated partly from the view of monetary policy. Summary, conclusions and suggestions for future research are given in section 6.

2. Forward rates, excess returns, regime shifts and term premia

Let $i(t)$ denote the short term interest rate at time t. We may think of this as the central bank's monetary instrument, though, as discussed below, that does not mean that the short term interest rate also reflects other economic factors. Since the expected future short term interest rate will play an important role in the analysis, the nature of the stochastic process that the short term interest rate follows deserves special consideration. In continuous time models it is usually assumed that the short term interest rate follows a diffusion process, whereas empirical studies often are carried out within the ARCH framework. The common feature of all these models is the implication that no drastic changes (jumps) occur in the short term interest rate. We will deviate from this modelling device and assume that there is a probability of a *regime shift* taking place. The kind of regime shift to be considered in this paper is typically a drastic economic change that leads to a situation with high inflation and interest rates. A model with these features will be presented in the next section. Formally, the expectation of a future regime shift of the kind discussed above can be written as

$$
\begin{aligned}
E_t[i(t+\tau)] = (1-p(t,\tau))E_t[i(t+\tau) \mid \text{no regime shift}] \\
+ p(t,\tau)E_t[i(t+\tau) \mid \text{regime shift}]
\end{aligned}
\tag{1}
$$

where $p(t, \tau)$ is the probability that a regime shift will take place during $[t, t+\tau]$, E_t is the expectation operator contingent on information known at time t. If Δi is the expected increase in the short rate implied by a regime shift relative to the future short rate level that would prevail in the absence of a regime shift, then (1) can be rewritten as

$$E_t[i(t+\tau)] = E_t[i_0(t+\tau)] + f_{RS}(t,\tau), \qquad f_{RS}(t,\tau) = p(t,\tau)\Delta i \tag{1'}$$

where we have introduced the notation that a subscript zero means that a regime shift has not occurred. Thus, given that forward rates are unbiased estimates of future short rates, i.e. the term premium is zero, we have

$$f(t,\tau) = E_t[i(t+\tau)] = f_0(t,\tau) + f_{RS}(t,\tau) \tag{2}$$

where the forward rate, $f(t,\tau)$, is the interest rate agreed at time t for a short term loan at time $t+\tau$ and $f_0(t,\tau) = E_t[i_0(t+\tau)]$.[3] Thus, the forward rate is decomposed into a *regime shift adjusted component*, $f_0(t,\tau)$, representing the expected future short rate if a regime shift does not occur during $[t, t+\tau]$ and a *regime shift premium*, $f_{RS}(t,\tau)$ quantifying the effect on forward rates that regime shift expectations give rise to.

The regime shift adjusted forward rate should reflect expectations not only of fundamental economic factors such as international interest rates and normal changes in the exchange and inflation rate, but also monetary policy. It is beyond the scope of this paper to develop a theory of how the economic factors interact with monetary policy. A reasonable conjecture, however, is that expectations of monetary policy dominate in the short run, whereas in the long run $f_0(t,\tau)$ mainly reflects expectations of changes in economic factors. Thus, it is of potential interest to extract the regime shift adjusted forward rate since it is likely to contain information about investors' expectations of future economic conditions within the current regime. Contingent on the non-occurrence of a regime shift, the regime shift adjusted forward rate is related to the future short rate according to

$$f_0(t,\tau) = i_0(t+\tau) + e(t,\tau), \qquad E_t[e(t,\tau)] = 0 \tag{3}$$

where $e(t,\tau)$ is a forecast error term. Combining (2) and (3) we see that the excess forward return, $\eta(t,\tau) \equiv f(t,\tau) - i(t+\tau)$, in periods with no regime shift is of the form

$$\eta_0(t,\tau) = f_{RS}(t,\tau) + e(t,\tau) \tag{4}$$

where $\eta_0(t,\tau) \equiv f(t,\tau) - i_0(t+\tau)$. We can generalize the set-up above by introducing a non-zero (forward) term premium, $\rho_f(t,\tau)$, defined as $\rho_f(t,\tau) \equiv f(t,\tau) - E_t[i(t+\tau)]$ implying that (4) will change to

$$\eta_0(t,\tau) = f_{RS}(t,\tau) + \rho_f(t,\tau) + e(t,\tau) \tag{5}$$

Sometimes it is more convenient to analyze regime shifts effects and term premia in terms of excess holding returns, $\nu(t,\tau;s)$, defined as $h(t,\tau;s) - i(t)$, where $h(t,\tau;s)$ is the holding return during the period $[t,t+s]$ of a zero coupon bond that matures at $t+\tau$ $(\tau > s)$. The excess holding return (contingent on no regime shifts) can then be written in a manner analogous to (5) as

$$\nu_0(t,\tau;s) = \nu_{RS}(t,\tau;s) + \rho_h(t,\tau;s) + u(t,\tau;s) \tag{6}$$

[3] Continuously compounded spot and forward interest rates are related to each other by the relation: $y(t,\tau) = \tau^{-1}\int_t^{t+\tau} f(t,s)ds$, where $y(t,\tau)$ is the spot interest rate (or yield) at time t on a bond maturing at $t+\tau$, and $f(t,s)$ is the forward interest rate at t on a short (instantaneous) loan with settlement date $t+s$.

where $\nu_{rs}(t,\tau;s)$ is the (holding) regime shift premium, $\rho_h(t,\tau;s)$ is the (holding) term premium, and where $u(t,\tau;s)$ is an error term. In empirical investigations, systematic fluctuations in the excess forward/holding returns often have been interpreted as a time varying term premium.[4] However, equations (5) and (6) suggest that fluctuations which *seem* to be systematic also can be attributed to fluctuating expectations of future regime shifts. If regime shifts are rare events, peso-type problems make it very difficult (or even impossible) to distinguish statistically between these two competing explanations. In other words, the reader is free to consider what the paper calls a regime shift premium as an additional component of the term premium.

3. Inflation rate regimes and the forward rate curve

Some empirical and theoretical considerations

A suitable specification of a regime shift model of the Swedish economy should be able to explain some of the observed characteristics of the Swedish yield and forward rate curves. One such characteristic is that long term interest rates have been more volatile than short term interest rates. This contrasts to the vast majority of term structure models, which imply instead that the volatility of interest rates declines towards zero as time to maturity tends to infinity. This is an implication of the expectation hypothesis, assuming the existence of a stationary level for the short term rate. Thus, to explain the volatility of Swedish long term interest rates one has to assume expectation formation of a non-standard type and maybe relax the assumption of a stationary level of the short term rate. The regime shift model to be presented shortly exhibits these non-standard features.

A high/low inflation regime shift model for Sweden

In this section a model that quantifies the regime shift (or credibility) effects on forward rates is presented. The rate of inflation is assumed to fluctuate around certain levels, π_i, which follow a continuous time Markov chain $\{S(t)\}$, see figure 1. We will assume that the Swedish economy is in the low inflation state of low credibility (state 2), representing a critical stage in which the economy will switch either to a bad state (state 1) characterized by high inflation or to a good state of low inflation with high credibility (state 3). The switching (or jump) intensities to state 1 and state 3 are α and β, respectively. There is also some probability that, once it has reached state 1 or state 3, the economy will switch back to state 2 and we assume for simplicity that the switching intensity back to state 2 equals γ for both state 1 and 3. In the following we will often discuss features of this model under the presumption that state 1 and state 3 are highly persistent, i.e. γ is small. In the extreme case $\gamma = 0$, the economy eventually will be absorbed in state 1 or 3. The inflation level, π_0, associated with the low inflation states 2 and 3 can be seen as the Riksbank's declared inflation target of 2 per cent.[5]

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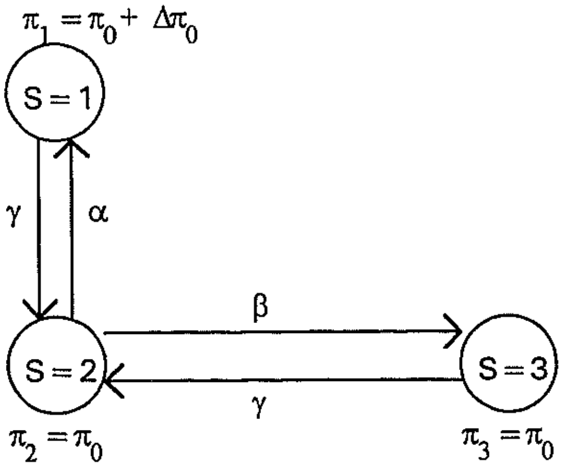
[4] See e.g. Fama and Bliss (1987) or Engle, Lilien and Robins (1987) for empirical investigations on US data and Hördahl (1995), Dahlquist and Jonsson (1995) or Sellin (1995) for investigations on Swedish data.

[5] Besides the inflation levels, π_i i= 1,2 or 3, there is also a component representing the deviation from these levels. Forecast errors of future deviations will show up in the forecast error term, $e(t,\tau)$, in (3).

Figure 1. A high/low inflation regime shift model



Let $P(\tau) = \text{Prob}[S(t+\tau) = 1 \mid S(t) = 2]$ denote the probability that the economy is in the high inflation state τ years ahead given that the economy today is in the low inflation state of low credibility. It is shown in the appendix that this probability is given by

$$P(\tau) = \frac{\alpha}{\lambda}[1\text{-}e^{-\lambda\tau}], \quad \lambda = \alpha + \beta + \gamma \tag{7}$$

Notice that $P(\tau)$ is increasing in τ towards

$$P \equiv \lim_{\tau\to\infty} P(\tau) = \alpha/\lambda \tag{8}$$

where P can be interpreted as the stationary probability of being in the high inflation state.[6] Since the high inflation state is associated with an increase in the inflation level by $\Delta\pi_0$ relative to the current inflation target, it is reasonable according to the Fisher hypothesis that the regime shift effect (or premium) on the forward rate curve, $f_{rs}(t, \tau)$, is the expected increase in the future inflation level, i.e.

$$f_{rs}(t, \tau) = P(\tau)\Delta\pi_0 = \varphi(\tau) f_{rs}^{\infty}, \qquad \varphi(\tau) = \left[1 - e^{-\lambda\tau}\right] \tag{9}$$

where $f_{rs}^{\infty} = \lim_{\tau\to\infty} f_{rs}(t, \tau) = P\Delta\pi_0$ can be seen as the asymptotic credibility effect. Thus, the regime shift premium is the product of a pure *credibility factor*, f_{rs}^{∞}, and a *credibility sensitivity factor*, $\varphi(\tau)$, showing the credibility effect on the forward rate curve for different forecast horizons. We will in the next section present an exogenous measure of the credibility factor f_{rs}^{∞} and a main concern of the rest of the paper is to estimate the credibility sensitivity $\varphi(\tau)$ for different τ and check if the estimates are consistent with $\varphi(\tau)$ given in (9). Finally, we notice that the regime shift model above also implies a regime shift premium for spot interest rates, $y_{rs}(t, \tau)$, of the form

$$y_{rs}(t, \tau) = \phi(\tau) f_{rs}^{\infty}, \quad \phi(\tau) = \left[1 - \left(1 - e^{-\lambda\tau}\right)/\lambda\tau\right] \tag{10}$$

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[6] If $\gamma = 0$ there does not exist a stationary distribution and P is then the probability that the economy will be absorbed in the high inflation state. In this case the inflation (and hence the interest rate) is nonstationary.

Furthermore, it is straightforward to show that the regime shift component of the excess holding return is given by

$$v_{rs}(t,\tau) = [y_{rs}(t,\tau)\tau - y_{rs}(t+s,\tau-s)(\tau-s)]/s = \psi(\tau,s) f^{\infty}_{rs} \tag{11}$$

where

$$\psi(\tau,s) = \left[1 - e^{-\lambda\tau}\left(e^{\lambda s} - 1\right)/\lambda s\right]$$

Notice that the length of the holding period, s, usually is short, implying that $\psi(\tau,s) \approx \varphi(\tau)$.

Implications and generalizations

The regime model above is in some respects primitive and it can be extended to be more realistic at the cost of increased complexity. The model does, however, serve the purpose of illustrating some quite general implications that regime shift models of this kind normally exhibit. Especially, two testable implications will be investigated:

(i) *The credibility sensitivity,* $\varphi(\tau)$, *(and hence the regime shift premium) is increasing in the forecast horizon,* τ.

(ii) *The credibility sensitivity,* $\varphi(\tau)$, *(and hence the regime shift premium) is concave in the forecast horizon,* τ.

A third implication that we will not investigate is that

(iii) *There is no regime shift premium incorporated in the spot short term interest rate i.e.,* $\varphi(0) = 0$.

This is a result of an implicit assumption that the expectations of a regime shift only affect inflation expectations and not the real interest rate. In a more realistic model with price rigidities it is likely that there is a regime shift effect on the short term interest rate. The reason for this is that even if prices react slowly to a change in the regime, the effect on the exchange rate presumably will be drastic since financial markets react rapidly to new information. Thus, a shift to a high inflation regime will probably lead to a nominal as well as a real drastic depreciation of the domestic currency, i.e. the exchange rate will jump as in the overshooting model of Dornbush (1976). It is beyond the scope of this paper to develop a model with exchange rate effects of the kind discussed above. Furthermore, the presence of exchange rate effects does not affect the methodology, but it will change the interpretation of the results. For instance, both the short term interest rate and the regime shift adjusted forward rate, $f_0(t,\tau)$, partly reflect expectations of a drastic future depreciation that regime shift expectations give rise to. In other words, the regime shift premium according to (9) does not necessarily represent the total effect that regime shift expectations cause.

In the model above the credibility (and thus also the regime shift premium) is constant over time. It is, however, possible to extend the set-up above and obtain models in which the credibility varies stochastically over time. One possibility is to introduce more low inflation states and fluctuations; credibility can then be seen as switches between different low inflation regimes. Another possibility is to let the switching intensities be positive stochastic processes. Since there are strong reasons to suspect that the regime shift premium does vary over time, in the following we will extend the set-up above by allowing for a time dependent credibility factor, i.e., $f_{\mathrm{rs}}^{\infty} = f_{\mathrm{rs}}^{\infty}(t)$. The introduction of a fluctuating credibility factor then provides an explanation of the non-trivial phenomenon that long term interest rates often appear to be more volatile than short term rates, since long term rates are more sensitive to shocks in the credibility factor, i.e., $\varphi(\tau)$ is increasing in τ.[7]

The important implication of regime shift models of the kind discussed above is that one should incorporate factors that reflect expectations of future regime shifts. Thus, according to this view, the traditional way of characterizing expectations of future short term interest rates, by estimating a process for the short rate that fits historical observations well, is likely to be an insufficient approach since it fails to incorporate regime shift expectations.

4. Econometric implementation of the model

In this section an econometric implementation of the model is presented. First we have to find a measure for the credibility effect f_{rs}^{∞}, which can be interpreted as the long run expected increase in the inflation rate. Economic theory implies that the difference in long term forward rates between two countries mainly reflects the expected difference in the long run inflation rate. If we assume that Germany has a credible inflation target of 2 percent (the same as the Swedish inflation target), then the spread in the long term forward rate between Sweden and Germany should be an approximation of the credibility factor, f_{rs}^{∞}, i.e.

$$f_{\mathrm{rs}}^{\infty}(t) \approx \delta_L(t) \qquad (12)$$

Theoretically it is desirable to use the asymptotic forward rate spread between Sweden and Germany, but in practice it may be better to use the spread between long, but finite, forward rates, since it can be difficult to estimate asymptotic forward rates. Moreover, we assume that the forward and holding term premia can be written as

$$\rho_i(t,\tau) = \rho_{0i}(\tau) + \theta_i(\tau)\omega(t,\tau), \quad i = f,h \qquad (13)$$

where $\rho_{0i}(\tau)$ is a constant (with respect to time t) and $\omega(t,\tau)$ is the variance of the error term in the excess holding return equation (6). We further assume that the variance $\omega(t,\tau)$ is of GARCH(1,1) type, i.e.

$$\omega(t,\tau) \equiv \mathrm{Var}_{t\text{-}1}[(u(t,\tau)] = \alpha_0(\tau) + \alpha_1(\tau)u^2(t\text{-}1,\tau) + \beta_0(\tau)\omega(t\text{-}1,\tau) \qquad (14)$$

In (14) it is understood that the length of the holding period, s, is set to one week (1/52 years) and s is henceforth suppressed in the notation. Consequently, $t\text{-}1$ refers to the date one week before date t. Notice that $\omega(t,\tau)$ is assumed to be the relevant measure of risk also for the forward term premium. Substituting (12) and (13) into (5) and (6) yields the following expressions for the excess forward return and excess holding return equations:

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[7] Two clarifications should be made about this statement: First, in more elaborated version of the regime shift model, in which the switching intensities fluctuate, the variability will eventually decline with the term even though the decline will show up for very large τ if γ is close to zero. However, the regime shift model still provides an explanation why volatility increases (initially) with the term. Second, the variability discussed above is conditioned on that no regime shift occurs. The unconditional variability, which includes the variability that regime shifts cause, is decreasing with the term, as the expectation hypothesis implies.

$$\eta_0(t,\tau) = \rho_{0f}(\tau) + \theta_f(\tau)\omega(t,\tau) + \varphi(\tau)\delta_L(t) + e(t,\tau) \quad (15)$$

$$\nu_0(t,\tau) = \rho_{0h}(\tau) + \theta_h(\tau)\omega(t,\tau) + \psi(\tau)\delta_L(t) + u(t,\tau) \quad (16)$$

Econometric problems

The estimation of equation (15) is troublesome for several reasons. First, we have the problem that since $e(t,\tau)$ reflects shocks induced by information that arrives during $[t,t+\tau]$, adjacent error terms are affected by almost the same information, implying that the sequence $\{e(t,\tau)\}$ exhibits a strong serial correlation. As this severely reduces the efficiency of the parameter estimates, equation (15) is estimated in difference form, i.e.,

$$\Delta\eta_0(t,\tau) = \mu + \theta_f(\tau)\Delta\omega(t,\tau) + \varphi(\tau)\Delta\delta_L(t) + \varepsilon(t,\tau) \quad (17)$$

where Δ is the difference operator ($\Delta x(t) = x(t) - x(t-1)$) and where $\varepsilon(t,\tau) = \Delta e(t,\tau)$. The parameter μ is expected to be close to zero since ρ_{0f} is constant. A second, more practical problem is that the use of excess returns in the regression implies that only forward rates at time $T-\tau$ and earlier can be used in a sample where the last observation (for the short rate) is at time T. This means that if the forecast horizon is six months then the sample size is reduced by six months.

Despite these problems the excess forward return equation (17) generates considerably more efficient estimates than the holding return equation (16), since excess holding returns contain much noise. Besides serving as an alternative econometric specification, the excess holding return equation generates estimates of the variance, $\omega(t,\tau)$, which is assumed to be a determinant for the forward term premium also in the excess forward equation. Finally, it is possible that the long forward rate differential to some extent incorporates time varying term premia in Swedish and German long term forward rates. Moreover, it is reasonable to believe that the German term premium is quite stable and that the Swedish term premium mainly is driven by fluctuating fears of a shift to a high inflation regime. If this is the case then the long forward rate differential still serves as a - somewhat different - measure of the credibility of the current inflation target in the long run.[8]

Data

A recurrent problem when testing interest rate theories is that the theory deals with the yield of zero coupon bonds whereas the yields observed in existing bond markets are those of coupon bearing bonds (unless time to maturity is short). Another problem is that estimation of equations (16) and (17) requires regular observations of bonds of fixed maturity (fixed τ) which rarely is the case in reality. We circumvent these problems by estimating the whole term structure of interest rates at each point of time, using the methodology suggested by Svensson (1994). To conduct the empirical analysis we used the estimated spot and forward rates of three and six month settlement. The short term rate is represented by the marginal/repo rate. Even though the asymptotic forward interest rate can be estimated by the method mentioned above, the 10-year forward rate is used

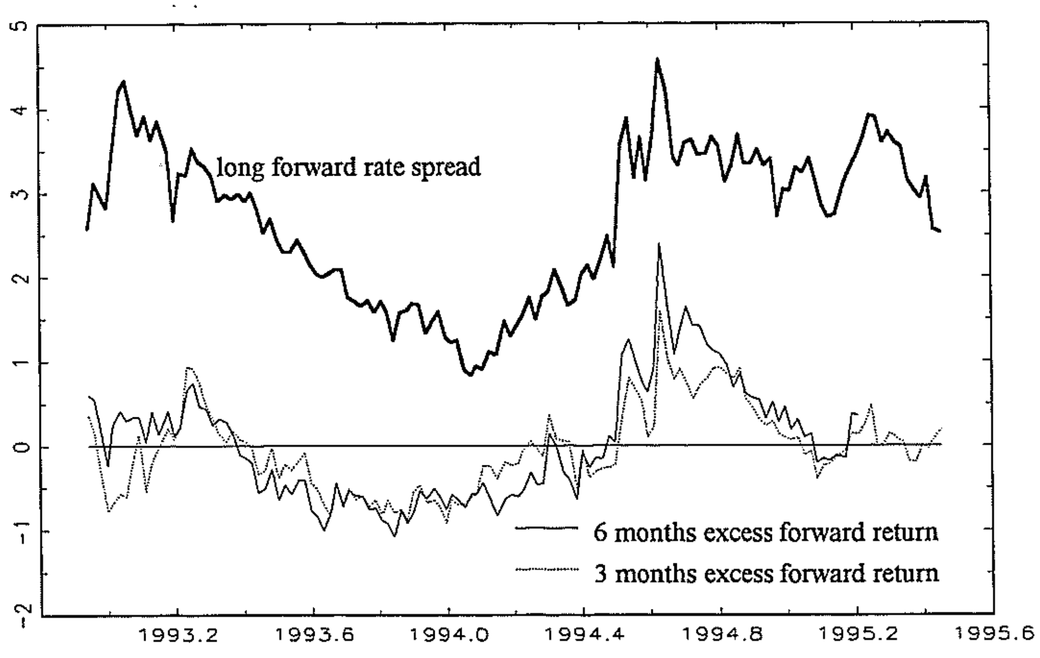
[8] Another hypothesis is that the long forward rate differential is not related to regime shifts expectation, but *only* reflects compensation the risk (i.e. variability) associated with holding long Swedish bonds. However, Sellin (1996) found that such risk will not have a significant impact on excess holding returns of short maturity bonds, when the variability of short interest rates (more precisely the variability of a portfolio consisting T-bills) is included as a risk factor. So if the hypothesis is true then we do not expect a significant impact from the long forward rate differential on excess forward/holding returns.

instead.[9] Unless the parameter λ in expression (9) is very small, the quantitative consequence of using a 10-year forward rate instead of the asymptotic forward rate is negligible.

The sample consists of weekly observations of Swedish and German interest rates (obtained from Sveriges Riksbank) from which forward rates have been calculated according to the device described above. The sample covers the period December 9, 1992 to September 12, 1995. The reason for choosing this rather small sample is that before December 1992 Sweden had a fixed exchange rate regime and the regime shift model assumes a floating exchange rate regime, in which devaluation expectations are not present. On the other hand the used sample covers a period when the long forward rate spread did exhibit substantial variation, so the efficiency of the regression will still be quite satisfactory.

The investor's ability to predict future movements in the short term interest rate can be appreciated from Figure 2, which depicts the excess forward returns as well as the long forward rate spread between Sweden and Germany. In the absence of a term premium, the excess return can be interpreted as the forecast error of the short term interest rate. As clearly seen from figure 2, the excess forward return is positively correlated with the long forward rate spread. Moreover, the six-month excess forward return seems to be more sensitive to fluctuations in the long forward rate spread. These findings are broadly consistent with the regime shift model presented in Section 3.

Figure 2. The long forward rate spread relative to Germany and excess forward returns, percent (continuously compounded)



[9] The difference between the estimated values of 10 years forward rate and the asymptotic rate is normally negligible. At some occasions, however, the difference is quite substantial, which is due to the fact that unreasonable estimates of the asymptotic forward rate are not punished when a term structure of finite maturities is observed. By using the 10 years forward rate we can check that the estimated long forward rate is in accordance with observed data.

5. Empirical examination

The primary purpose of this empirical examination is to get a rough picture of the kinds of factors that are important determinants of Swedish term structure movements. Since it is of potential interest to investigate how regime shift and term premia interact, it is natural first to examine models with only one of these premia present and then to compare with extended models in which both premia are included.

ARCH-M estimation of the restricted excess holding return equation
First we estimate holding term premia using the ARCH-M model of Engle, Lilien and Roberts (1987), i.e. we estimate equation (16) with $\psi(\tau)$ restricted to be zero and with residual variance governed by equation (14). The estimated parameters are presented in table 1. As seen from table 1 a significant time varying term premium is detectable only for $\tau = 6$ months. The large negative estimate of the intercept term, $\hat{\rho}_{0h}$, arouses suspicions since it implies that the holding term premium is around - 8 percent when in periods of low volatility (small $\omega(t,\tau)$). Another strange feature is that the dynamics of the residual variance (implied by the estimates $\hat{\alpha}_0$, $\hat{\alpha}_1$ and $\hat{\beta}_0$) differ greatly between the 3-month and the 6-month excess holding return equations. Thus, it is very probable that the restricted excess holding return equation is misspecified, implying that the estimates are not reliable.

Table 1. Estimation of restricted excess holding return equation

$$v_0(t,\tau) = \rho_{0h}(\tau) + \theta_h(\tau)\omega(t,\tau) + u(t,\tau)$$

$\tau = 3$ months	$\hat{\rho}_{0h}$	$\hat{\theta}_h$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_0$	Log L
	-0.5935	0.1405	0.2934	0.1193	0.8174	-176.7
	(-0.960)	(0.930)	(2.898)	(3.242)	(13.836)	
$\tau = 6$ months	$\hat{\rho}_{0h}$	$\hat{\theta}_h$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_0$	Log L
	-8.0908	0.3357	16.034	0.0460	0.3025	-303.3
	(-4.555)	(4.001)	(0.794)	(15.053)	(5.344)	

Note: t-values in parentheses; Log L is the maximized log likelihood value.

Estimation of the restricted excess forward return equation.
Next we consider the restricted excess forward return equation in which we only allow for a constant term premium and set the parameter $\theta_f(\tau)$ equal to zero. We see from table 2 that the estimates of the credibility sensitivity, $\hat{\varphi}$, are positive and significant for both forecast horizons. Moreover, the credibility sensitivity increases with the length of the forecast horizon and the estimates are consistent with specification (9) in the sense that we cannot reject the hypothesis that the 3- and 6-month estimates of the credibility sensitivity imply the same value of λ. Another notable observation is that the explanatory power is much higher when the forecast horizon is 6 months, as indicated by a higher R^2. The large and negative estimates of the constant term premium, $\hat{\rho}_{0f}$, should be interpreted with caution since these estimates are inefficient due to the strong autocorrelation that excess forward returns exhibit.[10]

[10] The robustness of the estimates has been investigated by Monte Carlo simulations, which show that the estimates of ρ_{0f} are very inefficient, whereas the robustness of the other estimates are broadly in accordance with the reported t-values.

Table 2. **Estimation of the restricted excess forward return equation**

$$\Delta\eta_0(t,\tau) = \mu + \varphi(\tau)\Delta\delta_L(t) + \varepsilon(t,\tau)$$

	$\hat{\mu}$	$\hat{\varphi}$	$\hat{\rho}_{0f}$	$\hat{\lambda}$	R^2
τ = 3 months	-0.0011	0.2863	-0.8063	1.3494	0.122
	(-0.051)	(4.228)		[0.667,2.173]	
	$\hat{\mu}$	$\hat{\varphi}$	$\hat{\rho}_{0f}$	$\hat{\lambda}$	R^2
τ = 6 months	-0.0057	0.4949	-1.2620	1.3658	0.310
	(-0.259)	(7.219)		[0.894,1.984]	

Note: $\hat{\rho}_{0f}$ is the implied value of ρ_{0f} appearing in equation (15) and calculated as the mean of $\eta_0(t,\tau)$ - $\hat{\varphi}\,\delta_L(t)$; $\hat{\lambda}$ is the value of λ in expression (9) that the estimate $\hat{\varphi}$ implies and the 95 percent confidence interval is given within brackets; t-values within parentheses.

ARCH-M estimation of the unrestricted excess holding return equation.
The estimate of the unrestricted excess holding return equation (16) is shown in table 3. We see that the estimates of the (holding) credibility sensitivity are positive and concavely increasing in τ, findings that broadly support a regime shift model. The regime shift effects are, however, larger in this case compared to the effects reported in table 2, but the large standard errors of the estimates call for caution in interpreting this difference. Moreover, a likelihood ratio test shows that the hypothesis of no regime shift premium (Ψ = 0) only can be rejected when τ equals 3 months. There are greater similarities between the implied dynamics of the residual variance, $\omega(t,\tau)$, in the unrestricted equation, which reinforces the impression that the restricted excess holding equation is misspecified. There is still no strong, reliable evidence of a time varying (holding) term premium, but the constant component of the term premium seems to be negative. In general, one should interpret the estimates in table 3 with caution since they are likely to be inefficient. Calculations of robust t-statistics would probably confirm this statement. Instead of characterizing the inefficiency of the estimates in table 3, we will to try to improve the efficiency by using the excess forward equation.

Table 3. **Estimation of unrestricted excess holding return equation**

$$v_0(t,\tau) = \rho_{0h}(\tau) + \theta_h(\tau)\omega(t,\tau) + \psi(\tau)\delta_L(t) + u(t,\tau)$$

	$\hat{\rho}_{0h}$	$\hat{\theta}_h$	$\hat{\psi}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_0$	Log L
τ = 3 months	-1.6088	0.0516	0.5401	0.3000	0.2023	0.7529	-172.3
	(-3.008)	(1.096)	(2.682)	(2.768)	(4.313)	(12.538)	
	$\hat{\rho}_{0h}$	$\hat{\theta}_h$	$\hat{\psi}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_0$	Log L
τ = 6 months	-2.8591	0.0310	0.8716	1.7609	0.1311	0.8148	-301.4
	(-2.238)	(0.761)	(1.655)	(0.838)	(2.862)	(18.809)	

Note: t-values in parentheses; Log L is the maximized log likelihood value.

Estimation of the unrestricted excess forward return equation

Finally, we examine the unrestricted excess forward return equation (15) using the estimated variances from the unrestricted excess holding equation as a regressor. The estimated parameters are reported in table 4. The most striking observation is the clear evidence of time varying (forward) term premia. A comparison with table 3 indicates that this observation is a result of the fact that estimations based on excess forward returns generate far more efficient estimates. Another sign that a time varying term premium has a role to play is the substantial rise in R^2 (relative to the restricted excess forward return equation). The high values of R^2 are remarkable, considering that the excess forward return is a forecast error under the expectation hypothesis.[11]

Table 4. Estimation of the unrestricted excess forward return equation

$$\Delta\eta_0(t,\tau) = \mu + \theta_f(\tau)\Delta\omega(t,\tau) + \varphi(\tau)\Delta\delta_L(t) + \varepsilon(t,\tau)$$

τ = 3 months	$\hat{\mu}$	$\hat{\theta}_f$	$\hat{\varphi}$	$\hat{\rho}_{0f}$	$\hat{\lambda}$	R^2
	-0.0005	-0.0379	0.2063	-0.7941	0.9242	0.280
	(-0.027)	(5.301)	(3.254)		[0.342,1.605]	
τ = 6 months	$\hat{\mu}$	$\hat{\theta}_f$	$\hat{\varphi}$	$\hat{\rho}_{0f}$	$\hat{\lambda}$	R^2
	-0.0050	-0.0309	0.4231	-2.006	1.1003	0.405
	(-0.245)	(4.290)	(6.403)		[0.695,1.689]	

Note: $\hat{\rho}_{0f}$ is the implied value of ρ_{0f} appearing in equation (15) and calculated as the mean of $\eta_0(t,\tau)-\hat{\theta}_f\,\omega(t,\tau)-\hat{\varphi}\,\delta_L(t)$; $\hat{\lambda}$ is the value of λ in expression (9) that the estimate $\hat{\varphi}$ implies and the 95 percent confidence interval is given within brackets; t-values within parentheses.

The negative estimates of the constant component of the term premia remain, which is somewhat strange, although it should be born in mind that the standard errors of these estimates are large. Furthermore, there are some downward adjustments of the estimates of the credibility sensitivities and hence of the implied λ values. The reduction of the implied λ is larger when the forecast horizon is three months, but still we cannot reject the hypothesis that the implied λ values are equal.

The relative impact of regime shift premia and term premia and the use of forward interest rates as monetary policy indicators

The 3- and 6-month forward rates, as well as the estimated regime shift and forward term premia based on the estimates reported in table 4 (excluding the large negative constant component), are displayed in figures 3a and 3b. The regime shift premia seem to be somewhat larger than the time varying component of the forward term premium, particularly in 1995. On some occasions, e.g. the summer and fall of 1994, the term premia are substantial, especially the 6-month term premium. It is notable that the episodes when

[11] The high values of R^2 do not imply that excess forward returns are unconditionally predictable, but rather that they are predictable conditional on no regime shift.

the forward term premium is substantial to a large extent coincide with corresponding episodes detected by Sellin (1996).

It is worth emphasizing that, due to the uncertainty about the constant component of the term premium, adjusting the forward rate for the regime and term premia, in order to obtain an unbiased measure of investors' expectations (in the current low inflation regime) of the future short term interest rate, is not a trivial matter. Moreover, the estimated 6-month term premium in August 1994, seems to be too large, indicating some kind of misspecification of the shocks that affect forward rates.[12] A closer inspection reveals that the drastic rise of the term premium coincides with the tightening of monetary policy. Thus, in a more realistic model one should explicitly incorporate interest dynamics that reflect the conduct of monetary policy.[13] Another problem has to do with the probable presence of exchange rates effects, as discussed in section 3.

Even though there are misspecification problems, and hence estimation problems, the significance of the regime shift premium is evident. The size of the 6-month regime shift premium has been in the range of 1 to 2 percentage points according to figure 3b, and it constitutes about 42 percent of the total variation in the 6-month forward rate.[14] Thus, the regime shift model is able to explain a substantial part of the fluctuations that may give rise to a time varying term premium. A weakness with models of time varying term premia is their tendency to be silent about the origin of the shocks.

From a monetary policy perspective, the message of the empirical examination is that in periods of large long term forward rate spreads and volatile interest rates one should be careful when interpreting short term forward interest rates as expectations of future short term interest rates. It is also possible to adjust medium term (say 2-year) forward rates for the regime shift premium and use them as indicators of inflation expectations within the current low inflation regime. Unless one is willing to use the functional form (9) for the regime shift premia and assume a reasonable value for λ, this kind of adjustment requires estimates of regime shift premia incorporated in medium term forward rates. An illustration of how regime shift adjusted interest rates can be used as indicators of inflation expectations is provided by Andersson and Berg (1995).

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[12] In August 1994 an excess holding return of - 29 percent (in annual rates) for the 6 months bond was observed. This observation is an extreme outlier that is not likely to be generated by shocks of GARCH(1,1) type. If we control for this observation by dummy variable in the excess holding return equation the resulting term premium becomes considerably smaller (estimated in the 6 months excess forward return equation), whereas the regime shift premium becomes somewhat larger (close to the estimate in table 2). Also an extreme observation, but of lower magnitude (-11 percent) was found in August 1994 in the sample of 3 months excess holding returns. Similar effects were found in this case when dummy adjusted variances were used in the excess forward equation, but the downward adjustment of the term premium was not so large.

[13] Expectations of monetary regime shifts will in principal give rise to regime shift effects of the kind discussed in section 2. Models with monetary regime shift features have recently been developed by Babbs and Webber (1995) and Lindberg, Orszag and Perraudin (1995).

[14] The 3 months regime shift premium explains about 25 percent of total variation in the 3 months forward rate.

Figure 3a. 3-month forward rates, regime shift premia and forward term premia (continuously compounded)

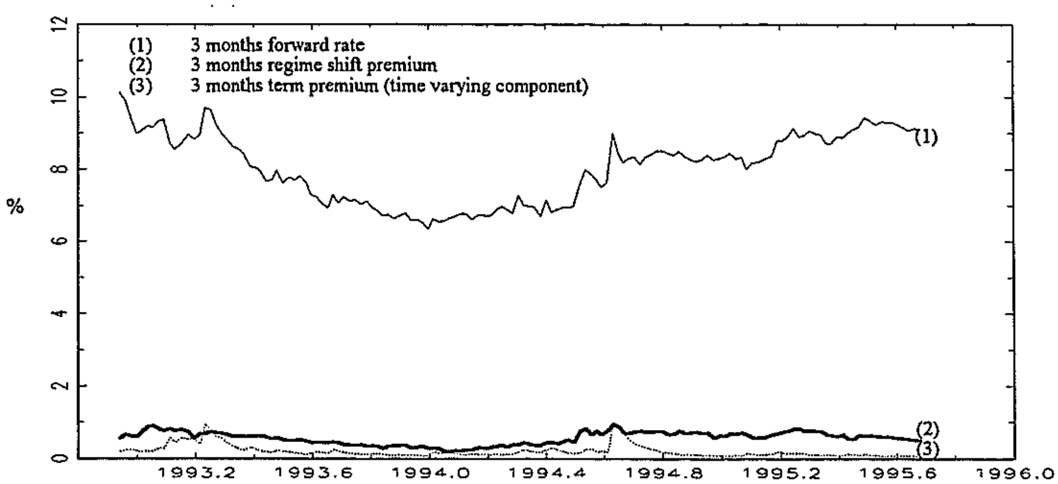
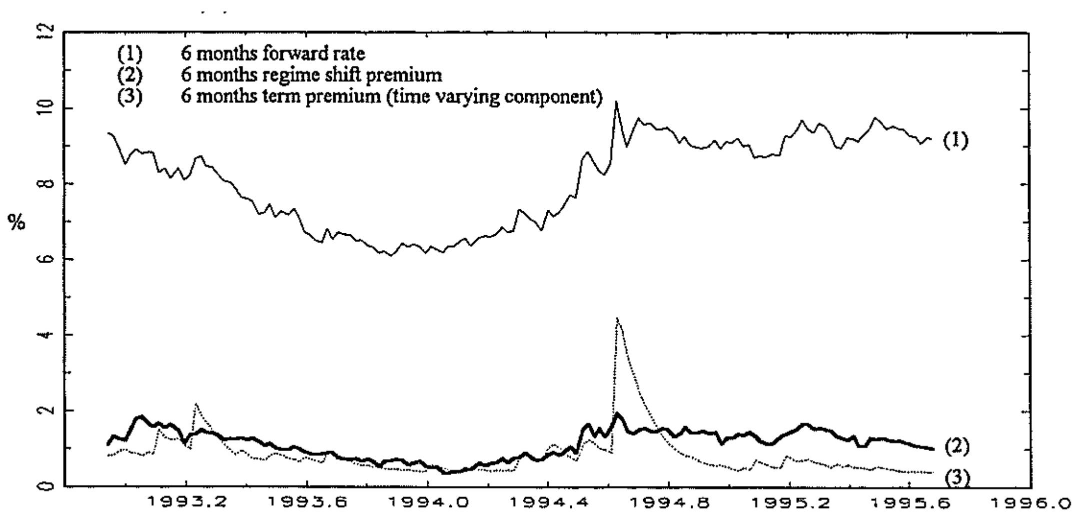


Figure 3b. 6-month forward rates, regime shift premia and forward term premia (continuously compounded)



6. Summary, conclusions and suggestions for future research

This study has established to my knowledge a new empirical relationship, namely that excess forward returns (the forward interest rate minus the future short term interest rate) are positively correlated to the long forward rate differential with Germany. This finding is broadly consistent with a proposed regime shift model in which fears that the current low inflation policy will be abandoned and the inflation rate will become high give rise to a regime shift premium. Thus, the model enables us to examine the quantitative effects of an imperfect inflation target in an explicit way. However, due to peso-type problems it is not possible to test statistically whether or not this explanation is correct. One may argue that the regime shift premium really is a component of a time varying term premium even though such an explanation is difficult to justify from economic theory. The message of this study is, however, that one should focus more on the formation of expectations and the role of regime shifts when assessing the validity of the expectation hypothesis. In particular, the presence of regime shift premia implies that estimating a process for the short interest rate on historical data is not an adequate way of obtaining accurate expectations of future short interest rates, since this approach does not take expectations of regime shifts into account.

In addition, evidence was found of time varying term premia (driven by the residual variance in an ARCH-M regression for excess holding returns). An interesting observation from a methodological point of view is that the term premia turned out to be statistically significant only when the estimated volatilities (obtained from the regression for excess holding returns) were used in the regression for excess forward returns. The reason for this is that estimates based on excess forward returns are more efficient than estimates based on excess holding returns. The time varying premium is normally of a lower magnitude than the regime shift premium, but on certain occasions it becomes substantial. It should, however, be mentioned that to a large extent the significance of a time varying term premium seems to be the result of a monetary policy shock in August 1994.

The existence of regime shift and term premia should be taken into account when using forward interest rates as monetary policy indicators, although some of the findings in this study indicate that further refinements of the set-up should be made in future research. For instance, the estimate of the constant component of the term premium is negative, implying that also the term premium is negative in low risk periods. Even though this conclusion is uncertain, due to the large standard error of this estimate, the finding does call for a reconsideration of the notion of risk. It is not obvious that the term premium is monotonically increasing with the term and that holding return volatility of ARCH type is an appropriate measure of risk. Furthermore, the long term forward rate spread relative to Germany is only a rough measure of an inflation target's lack of credibility and more attention should be paid to how to model and estimate imperfect credibility of inflation targets. Another interesting direction of research is to develop models that explicitly incorporate expectations of the conduct of monetary policy. Finally, it is highly desirable to extend the model presented in this study to incorporate sticky prices and exchange rate effects along the lines of Dornbush (1976).

Appendix: Derivation of expression (7).

Let $Q_i(t)$, i =1, 2 or 3, denote Prob[$S(t+t_0) = 1 \mid S(t_0) = i$]. It follows from the theory of finite state continuous time Markov Chains (see e.g. Karlin and Taylor (1975) p.150-152) that $Q = (Q_1, Q_2, Q_3)'$ satisfies

$$\frac{dQ}{dt} = \Lambda Q, \quad \Lambda = \begin{pmatrix} -\gamma & \gamma & 0 \\ \alpha & -(\alpha+\beta) & \beta \\ 0 & \gamma & -\gamma \end{pmatrix}, \quad Q(t_0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \text{(A1)}$$

It is straightforward to verify that the eigenvalues of Λ are 0, $-\gamma$ and $-(\alpha+\beta+\gamma)$, with corresponding eigenvectors (proportional to) $v_1 = (1,1,1)'$, $v_2 = (\beta,0,-\alpha)'$ and $v_3 = (-\gamma/(\alpha+\beta), 1, -\gamma/(\alpha+\beta))'$, implying that the solution to the system (A1) is of the form

$$Q(t) = a v_1 + b v_2 e^{-\gamma(t-t_0)} + c v_3 e^{-\lambda(t-t_0)}, \quad \lambda = \alpha+\beta+\gamma \qquad \text{(A2)}$$

The constants a, b, and c can be determined from the initial condition $Q(t_0) = (1,0,0)'$ to be

$$a = \alpha/\lambda, \quad b = 1/(\alpha+\beta), \quad c = -\alpha/\lambda \qquad \text{(A3)}$$

In particular we have $P(\tau) = \text{Prob}[S(t_0+\tau) = 1 \mid S(t_0) = 2] = Q_2(t_0+\tau) = \frac{\alpha}{\lambda}[1-e^{-\lambda\tau}]$, which was to be shown.

References

Andersson, K., C. Berg (1995), "The Swedish Inflation Target", in , *Inflation Targeting*, ed. A. Haldane, Bank of England 1995.

Babbs, S. H., and N.J. Webber (1994), "A Theory of the Term Structure with an Official Short Rate", mimeo.

Blix, M., (1995), "Term Premia under Switching Regime", mimeo, Institute for International Economic Studies.

Campbell, J., (1995), "Some Lessons From the Yield Curve", *Journal of Economic Perspective* 9, No. 3, 129-152.

Dahlquist, M., G. Jonsson (1995), "The Information in Swedish Short-Maturity Forward Rates ", *European Economic Review*, Vol 39, No 6, 1115-1131.

Dillén, H., (1994), "A Model of the Term Structure of Interest Rates in an Open Economy with Regime Shifts", in *Asset Prices in Open Monetary Economies*, PhD dissertation, Uppsala University.

Dillén, H., (1995), " The formation of Interest Rates in Theory and in Sweden", in *Financial Structure and the Monetary Transmission Mechanism*, Basle March 1995, Bank for International Settlements, 494-525.

Dornbush, R., (1976), " "Expectations and Exchange Rate Dynamics", *Journal of Political Economy* 84, 1161-1176.

Engle, R., D. Lilien and R. Robins (1987), "Estimating Time Varying Term Premia in the Term Structure: the ARCH-M Model ", *Econometrica*, 55 No 2, 391-407.

Evans, M. and K. Lewis (1995), "Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?", *Journal of Finance*, 6, No 1, 225-253.

Evans, M. and P. Wachtel (1993), "Inflation Regimes and the Sources of Inflation Uncertainty", *Journal of Money, Credit, and Banking*, Vol 25, N0 3, 475-520.

Fama, E., and R. Bliss, (1987), "The Information in Long-maturity Forward Rates", *American Economic Review* 77, 680-692.

Hamilton, J.D., (1988), "Rational Expectations Econometric Analysis of Changes in Regime: An Investigation of the Term Structure of Interest Rates", *Journal of Economic Dynamics and Control* 12, 385-423.

Hördahl, P., (1995), "Time Varying Risk Premia in Swedish Treasury Bonds", mimeo Department of Economics, Lund University.

Karlin, S., and H. Taylor, (1975), *A First Course in Stochastic Processes*, second edition, Academic Press.

Lindberg, H., M. Orszag, and W. Perraudin, (1995), "Yield Curves with Jump Short Rates", mimeo Sveriges Riksbank and Birkbeck College, London.

Sellin, P. (1996), "Risk Related Return Premia in the Swedish Term Structure" forthcoming working paper, Sveriges Riksbank.

Shiller, R., (1979), "The Volatility of Long-term Interest Rates and Expectations Models of the Term Structure", *Journal of Political Economy* 87, 1190-1219.

Shiller, R., (1990), "The Term Structure of Interest Rates", in *Handbook of Monetary Economics* vol 1, F. Hahn and B. Friedman eds., Amsterdam; North-Holland.

Singleton, K., (1980), "Expectations Models of the Term Structure and Implied Variance Bounds", *Journal of Political Economy* 88, 1159-1176.

Svensson, L. E. O., (1994), "Estimating and Interpreting Forward Interest Rates", Stockholm: Institute for International Economic Studies, Seminar Paper No. 548.