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Long Run Real Exchange Rates - a Cointegration Analysis

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Abstract

Long run purchasing power is tested on 16 OECD countries using data from 1960 to 1994. PPP is rejected for some countries (Canada, USA, Japan, Switzerland, Belgium, Austria, Italy and Spain) and not rejected for several other countries (Sweden, Denmark, Finland, Norway, France, Holland and the United Kingdom). For the latter countries, impulse response functions show that half of a disturbance to the equilibrium real exchange rate disappears within three years. The method used is Johansen's maximum likelihood approach to cointegration. Simulations are performed to obtain empirical critical values of the tests.

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1 Introduction

The idea of long run purchasing power parity (PPP) may be expressed as the hypothesis that there exists a stationary equilibrium real exchange rate. If PPP holds, the real exchange rate (defined as EP*/P or the foreign price level times the nominal exchange rate divided by the domestic price level) fluctuates around an equilibrium value. This implies that shocks to real exchange rate only have temporary effects. If the real exchange rate is pushed below (above) its' equilibrium level, it can be expected to rise (fall).

Empirical tests tend to reject the hypothesis that real exchange rates are stationary. However, most tests of PPP use short sample periods and focus on the bilateral real exchange rates between the USA and other major countries. In contrast, this paper investigates multilateral PPP, using a fairly long sample period and including a number of previously neglected small European countries. PPP also often is said to lack solid theoretical underpinnings. Still, it is frequently used as a building block of models in international macroeconomics. Its popularity could perhaps be attributed to the absence of obvious alternative assumptions about how the real exchange rate is determined. However, I will try to argue that the case for PPP is considerably better than its reputation, both when considering theoretical underpinnings and the results of empirical tests.

The simplest theoretical motivation of PPP is that if money is neutral in the long run, the real exchange rate should not be influenced by monetary disturbances. If the shocks are monetary in nature, PPP holds in any model that is characterized by long run neutrality of money. Furthermore, PPP always follows as a special case in models of equilibrium real exchange rates. For instance, in models where the real exchange rate is determined by country differences in productivity or relative productivity in the tradable and non-tradable sectors, PPP holds if countries have the same productivity development.[1] If the real exchange rate is driven by the terms of trade, it will be stationary if the terms of trade are stationary.[2] These two approaches to the equilibrium real exchange rates have been combined in a single model by Gregorio and Wolf (1994). Another concept has been to derive a long run equilibrium rate from the condition that the current account has to balance in the long run.[3]

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1 The most popular models where the equilibrium real exchange rate is determined by relative productivity developments or relative productivity developments in the tradable and non-tradable sectors are Balassa (1964) and Samuelsson (1964).

2 Models where the terms of trade or relative price movements within the tradable sector determines the real exchange rate include Frenkel and Razin (1992), Ostry (1988) and Greenwood (1984).

3 Examples of models built on current account balance are Kimbrough (1982), Dornbush and Fisher (1980), Frenkel and Rodriguez (1975) and Kouri (1976).

Models where the development of real variables determine the real exchange rate results in PPP in two cases: If the real variables in question are unchanged or if the real shocks affect the countries in a "symmetric" way that leaves the equilibrium real exchange rate between them unchanged.

PPP has been tested empirically numerous times and rejected more often than not. There is widespread agreement that it does not hold in the short run. The disagreement concerns whether PPP holds in the long run and how long the long run is. In section two of this paper, it will be argued that whether PPP is rejected or not depends partly on the choice of countries, the length of the sample period and the econometric techniques used.

Available evidence suggests that if there is mean reversion in real exchange rates, it is slow. Estimates of the autoregressive parameters indicate that half of the effect of a disturbance disappears within two to four years. When PPP is tested on data covering only 10 to 15 years, mean reversion or stationarity may not be discovered. In their overview of empirical evidence of PPP, Froot and Rogoff (1994) construct an example with a true AR coefficient of 0.981. In monthly data, this corresponds to a half life of disturbances to the equilibrium relationship of three years. They show that it would be necessary to have a data set covering 72 years to be able to reject a unit root at the 95% significance level, using the Dickey-Fuller test. Thus, either very long time series or more powerful tests would be needed to confirm that the real exchange rate does not contain a unit root. It is definitively relevant to be able to distinguish between a unit root and an AR-coefficient of 0.98 in monthly data. In the latter case, most of the disturbance to the equilibrium real exchange rate can be expected to disappear within an average business cycle, while shocks have permanent effects if there is a unit root.

Most tests of PPP have focused on bilateral real exchange rates between major industrial nations like USA, Japan and Germany. A case can be made that since these countries have rather different economic structures, the real exchange rates between them are less likely to be stationary than the real exchange rates between more homogenous European countries. For instance, asymmetric real shocks like different productivity developments and variable terms of trade may be more important when considering the real exchange rate between Japan and the United States than for the real exchange rate between Germany and Belgium. As will be discussed in the next section, there are rather few studies of whether the real exchange rates between European countries are stationary.

If the mechanism driving PPP has to do with international competitiveness, it may be more relevant to study multilateral PPP than bilateral. For a single country, a real appreciation against one partner country may offset a real depreciation against another. However, as shown by Nessén (1994), multilateral PPP cannot hold simultaneously for all countries involved unless bilateral PPP holds between all countries. Multilateral or effective real exchange rates are simply linear combinations of bilateral real exchange rates. It is not possible to form a full set of stationary linear combinations of bilateral real exchange rates unless all of them are stationary.[4] Thus, unless bilateral PPP holds between all the countries, at least one multilateral real exchange rate has to be non-stationary.

In this paper, multilateral PPP is tested on monthly data for 16 OECD countries from 1960 to 1994. The sample period, 35 years, is longer than in most studies of PPP except those using data since the turn of the century. The method used to test for cointegration is Johansen's (1988) maximum likelihood approach, which incorporates tests of linear restrictions on the cointegrating vectors. Since it has been shown that the empirical distributions of the test statistics may differ considerably from the asymptotic distributions even with several hundred observations, Monte Carlo tests and parametric bootstrapping provide some additional information about how the test results should be interpreted. [5] Empirical distributions and corresponding critical values given normally distributed residuals are generated. Since the residuals are not normally distributed but display excess kurtosis and skewness, bootstrapping is used to shed some light on how the tests work given the present distribution.

The paper is organized as follows: Section two is an overview of empirical tests of PPP. The data are described in section three and section four discusses the statistical method. The empirical results and the simulations of the tests are presented in section five, followed by the conclusions in section six.

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4 This is similar to the more familiar result that there cannot be k cointegrating vectors in a system of k I(1) variables unless all variables are stationary.

5 Simulations of the Johansen procedure have been made by Jacobson, Vredin and Warne (1994), among others.

2 An overview of the empirical literature on PPP

Before the virtual explosion of PPP tests that followed the introduction of econometric techniques designed to handle non-stationary data, it had become more or less a stylized fact that PPP was rejected in empirical tests. Standard textbooks in international economics like Krugman and Obstfeld (1986) concluded that PPP does not hold. Evidence presented by Adler and Lehman (1983) supported this view also for extremely long data sets (1900–1972). In contrast, it was shown by Frenkel (1978, 1981) that PPP could not be rejected for a number of hyperinflationary economies and for the US versus Germany, Britain and France in the 1920:s. These papers were exceptions to the rule.

Since the concepts of integration and cointegration became common knowledge, empirical tests of PPP have proliferated. The results are mixed and no final verdict has been reached. Studies covering less than 15 years of data almost always reject PPP, while those covering the entire century usually do not. Furthermore, rejections are much more frequent for the United States and Canada than for the European countries.

One approach has been to investigate whether the real exchange rates contain a unit root, which is incompatible with PPP. As discussed in the introduction, the power of unit root tests to distinguish between AR-coefficients close to one and unit roots is questionable in small samples. There is a handful of studies that use very long sample periods. Kim (1990) and Abuaf and Jorion (1990) apply unit root tests to a data set that covers 1900–1972 but do reject a unit root in the real exchange rate.[6] Froot and Rogoff (1994) discuss whether the fact that PPP seems to hold in the very long run is due to what they call a survivor bias: Data from the beginning of the century is only available for a handful of countries. Could it be that PPP holds better for these early industrialisers than for other countries for which data is unavailable?

Other tests of unit roots in real exchange rates include Bleaney (1992), who rejects a unit root only for the real exchange rate between Germany and France when investigating all bilateral rates between Germany, France, Italy, Belgium and Holland between 1979 and 1988. Edison and Fischer (1991) study the same countries but use data from 1973 to 1989. They reject a unit root (allowing for a structural break in the relationship in 1979) for six of 15 bilateral exchange rates. Mark (1990) studies five intra-European real exchange rates between 1973 and 1989 and concludes that they all contain a unit root. The Dickey-Fuller statistics exceed 2.0 in three cases, however.

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[6] This is the same data set as Adler and Lehmann (1983) used.

Given the low power of the test, it is unlikely that a true AR coefficient of, say, 0.95 would have been discovered by the ADF test.

A second approach has been to investigate whether nominal exchange rates and price levels are cointegrated. Studies using cointegration techniques have quite often found cointegration among nominal exchange rates and price levels. A problem with many of these studies is that the existence of a stationary linear combination of exchange rates and prices does not necessarily mean that PPP holds. According to PPP, it is the real exchange rate that should be stationary. This implies certain restrictions on the cointegrating vector(s). Unfortunately, it is difficult to test hypothesis on the cointegrating vectors in several of the most commonly used cointegration procedures. Examples of studies that claim to report evidence supportive of PPP without testing whether the cointegrating vectors are compatible with PPP are Canarella, Pollard and Lai (1990) and Enders (1988), Heir and Theurillat (1990) and Kim (1990). The last two papers report point estimates of the cointegrating vectors close to those required by PPP.

A few papers do incorporate tests of restrictions on the cointegrating vectors. In a study that is often referred to as presenting evidence favourable to PPP, Fisher and Park (1991) reject the null hypothesis of no cointegration for all G-10 countries except the US and Canada on data from 1973 to 1988. However, the hypothesis that the real exchange rate is stationary is rejected in 43 of 55 cases using CPI and 51 of 55 using wholesale prices. Nessén (1994) finds four cointegration vectors in the seven variable system of dollar exchange rates and price levels for the US, Britain, Germany and Japan. Tests of whether the real exchange rates are cointegrating vectors led to rejections of the hypothesis, however. Data cover 1970-1990. Cheung and Lai (1993) also find cointegration among exchange rates and price levels of the US versus Britain, France, Germany, Switzerland and Canada for the sample period 1974-1989 but reject the restrictions imposed by PPP.

Appendix A compiles the results from a large number of tests of PPP. Three observations emerge. Investigations of real exchange rates using US data have dominated the literature. The small European countries have largely been ignored, especially the Nordic countries. Second, PPP has fared worse in tests focusing on the United States. Rejections are fewer when studying the real exchange rates between other countries.

Third, bilateral PPP has been tested much more often than multilateral PPP. Possible reasons for this are that the choice of weights is rather arbitrary and that the hypothesis of stationary effective real exchange rates is testable within multivariate systems of price levels and bilateral exchange rates. However, as the number of partner countries grows, the dimension of the multivariate system turns prohibitively large. Many of the European countries have diversified trade patterns and it would be unfortunate to restrict the number of partner countries so drastically that one multivariate system could be used. In this paper, as in Johansen and Juselius (1992), PPP is therefore tested within three variable systems of domestic prices, weighted averages of foreign prices and weighted averages of nominal exchange rates.

3 Data

Monthly data from 1960 to 1994 on exchange rates and consumer price indices have been collected from OECD:s Main Economic Indicators. Other conceivable choices of price indices would be industrial prices and wholesale prices that contain a larger share of tradeables. Logarithms of level data are used throughout.

Multilateral real exchange rates have been constructed using the OECD overall weights. This weighting system considers not only the share of trade between any pair of countries but also the extent to which they compete on third markets. For instance, the direct trade between Japan and Sweden is rather small, but Swedish and Japanese products compete on third markets. This gives Japan a larger weight in the Swedish effective exchange rate than what is motivated by the trade volume between the countries. An alternative would be to use the similarly constructed MERM weights used by IMF.

Appendix B shows the real effective exchange rates for the 16 countries studied in this paper. Simply looking at the figures, the real exchange rates of Belgium, Finland, France, Italy, Sweden, United Kingdom and possibly the US and Germany appear to be stationary. The real exchange rates of Austria, Canada, Denmark, Holland, Japan, Norway, Spain and Switzerland appear to be non-stationary. As section five will show, this coincides only partly with the test results.

Before proceeding to cointegration analysis, it is useful to know the time series properties of the individual series. There are three possibilities to consider when performing unit root tests on nominal variables like price levels: Data may be stationary around a deterministic time trend or have one or two unit roots. A second unit root means that the first difference of a variable – the inflation rate, for instance – is non-stationary, while the second difference is stationary.

A variable x_t is said to be integrated of order d if it has a stationary, invertible, non-deterministic ARMA representation after differencing d times. If a variable is integrated of order d, it has d unit roots. The number of unit roots in the time series on price levels and nominal exchange rates is investigated using the Dickey-Fuller test:

(1) $$\Delta x_t = \rho x_{t-1} + \sum_{i=1}^{i=p} \Delta x_{t-i} + \mu + \varepsilon_t$$

p is the number of lags in the AR-process (the number of lags needed to get rid of higher order residual autocorrelation in the last section). A unit root implies that ρ is equal to one. This hypothesis is tested against the alternative that ρ is less than one. Two unit roots in the level of x_t is equivalent to one unit root in the first difference of x_t, which is tested using (1) on difference data. Table 1 reports the results for nominal exchange rates and price levels. A linear trend is included in the tests.

When the presence of two unit roots is suspected, Dickey and Pantula (1987) show that the hypothesis of a double unit root should be tested first. If it is rejected, the presence of a single unit root may be investigated as usual. The Dickey-Pantula test indicates a single unit root in all variables, as does the Dickey-Fuller test without a deterministic trend.

Table 1. Unit roots in price levels and nominal exchange rates, t-statistics from the ADF-test

	CPI, levels	CPI, differences	Exchange rates, levels	Exchange rates, differences
Canada	-0.556	-2.640*	-0.913	-5.984**
US	-0.152	-2.846*	-1.537	-6.254**
Japan	-2.247	-3.862**	-0.594	-6.204**
Austria	-0.937	-7.935**	-0.124	-6.892**
Belgium	-0.785	-3.403**	-1.047	-6.254**
Denmark	-1.598	-4.655**	-1.860	-6.578**
Finland	-0.877	-3.940**	-0.732	-6.645**
France	-0.723	-2.643*	-1.349	-6.558**
Germany	-0.730	-6.180**	-0.463	-6.925**
Holland	-1.829	-5.382**	-0.166	-6.748**
Italy	-0.429	-2.711*	-0.714	-6.089**
Norway	-0.334	-4.491**	-0.062	-6.748**
Spain	-0.437	-3.513**	-1.860	-5.282**
Sweden	-0.718	-4.844**	-0.361	-6.312**
Switzerland	-0.792	-5.429**	-0.334	-6.881**
United Kingdom	-0.537	-4.110**	-1.349	-7.032**

Critical values are taken from Fuller (1976)
* significant at 5%
** significant at 1%

All series have a first unit root but none of them contains a second unit root. Thus, they are all I(1), integrated of order one. Some of the inflation rates are close to non-stationary: Canada, France, Italy and USA.

4 The statistical model

Cointegration tests look for linear combinations of I(1) time series that are stationary (or, more generally, linear combinations of I(d) time series that are integrated of an order lower than d). Since price levels and nominal exchange rates are integrated of order one, they are cointegrated if a linear combination of them is stationary.

The first generation of cointegration tests were based on stationarity analysis of the residuals from a cointegrating regression. When more than two variables are involved, Johansen's (1988) multivariate procedure is more suitable. It focuses on the rank of the Π-matrix in equation (2).

(2) $$\Delta x_t = \mu + \Gamma_1 \Delta x_{t-1} + \ldots + \Gamma_{t-p+1} \Delta x_{t-p} + \Pi x_{t-1} + \varepsilon_t ,$$

When the variables are integrated of order one, their first difference is stationary. Thus, the right hand side of equation (2) must be stationary as well. If there is no cointegration between the variables, Π must be a zero matrix. If the Π-matrix has reduced rank, implying that $\Pi = \alpha\beta'$, the variables are cointegrated, with β as the cointegrating vector. If the variables themselves are stationary, Π would have full rank.

The maximum likelihood estimation of β boils down to solving an eigenvalue problem.[7] The rank of the system is determined by investigating how many of the eigenvalues that are non-zero. A non-zero eigenvalue can loosely be interpreted as a positive correlation between the corresponding linear combination of x_t and (the stationary) Δx_t. Since the asymptotic correlation between a stationary and a non-stationary process is zero, a positive correlation means that this linear combination of x_t is stationary. With three variables, there will be three eigenvalues with corresponding eigenvectors. If the true rank is r, there will be r non-zero eigenvalues. Johansen (1988) has developed two tests of the number of non-zero eigenvalues.

First, the eigenvalues are ordered from the largest to the smallest. The maximal eigenvalue statistic tests whether eigenvalue number s+1 is non-zero. It is computed as:

(3) $$\lambda_{max} = -T \ln(1 - \lambda_{s+1})$$

The trace test investigates whether all eigenvalues from number s+1 to r are zero:

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7 See Johansen (1988) for a full discussion of the procedure.

(4) $$trace = -T\sum_{i=s+1}^{r}\ln(1-\lambda_i)$$

Asymptotic distributions of the two cointegration rank tests are derived in Johansen (1988, 1991 a and b). The distributions are not invariant to various restrictions due to the treatment of constants. Critical values that take this into consideration have been simulated by Osterwald-Lenum (1992).

The Johansen procedure identifies the vector space spanned by the cointegrating vectors. Tests of linear restrictions on the cointegrating vectors investigate whether a certain vector belong to this space. If one coefficient in each vector is normalised to one, the other two can be determined. Linear restrictions on β are tested by comparing the likelihood of the restricted and unrestricted β. Asymptotically, the likelihood ratio test statistic is χ^2 distributed. It is computed as:

(5) $$-2\ln Q = T\sum_{i=1}^{r}\left[\ln(1-\lambda_i^R)-\ln(1-\lambda_i)\right],$$

where λ_i are the estimated eigenvalues from the unrestricted model and λ_i^R are the eigenvalues when the restrictions are imposed. When only r1 of the r cointegrating vectors are restricted, a slightly different test is used:

(6) $$-2\ln Q = T\left[\sum_{i=1}^{r1}\ln(1-\lambda_i^R)+\sum_{i=r1}^{r-r1}\ln(1-\lambda_i^E)-\sum_{i=1}^{r}\ln(1-\lambda_i)\right],$$

where λ_i^E are the estimated eigenvalues of the (r-r1) unrestricted cointegration vectors, given the restriction on the r1 first vectors.

A cointegrated VAR-model as in equation (2) can be interpreted as an error correction representation where the α −parameters are the error correction parameters. It can also be rewritten as a moving average representation as in equation (7):

(7) $$\Delta x_t = C(1)\mu + C(L)\varepsilon_t$$

Impulse response functions show how a variable reacts over time to a shock. They are obtained from the Wold moving average representation (7) of the cointegrated VAR-system in (2). The impulse response functions simply use the estimates of C(L). The vector of shocks is set to one standard error in period t and zero in all other periods. There will be an immediate effect on

Δx_t in period t, determined by the estimated coefficients of C(0), a one period lagged effect, determined by the coefficients of C(1) and so on.

The moving average representation in (7) can be written as a common trends model. If there are K variables and r cointegrating vectors, there will be K-r stochastic trends that drive the system. Innovations to the stochastic trends have permanent effects on the variables. The error terms in equation (7) are linear combinations of the permanent and transitory shocks in the common trends representation. Given a sufficient number of identifying assumptions, the structural shocks can be distinguished. The identifying assumptions are usually obtained from economic theory. Given the lack of theoretical foundations of PPP, no attempt to identify the shocks will be made here.

5 Empirical results

After a few model specification tests, the cointegration rank, the cointegrating vectors and the dynamic adjustment mechanism will be investigated. Since the critical values from the asymptotic distribution are not necessarily appropriate for a small sample, simulations will be used to generate empirical critical values.

In the Monte Carlo test, 10 000 synthetic data sets are generated using the estimates of the parameters in the data generating process (2) and normally distributed disturbance terms. Tests for cointegrating rank and restrictions on the cointegrating vectors are applied to the data sets and the test statistics are collected. The resulting distributions of the test statistics under the true null hypothesis are used to derive empirical critical values. The power of the tests can be investigated by testing a false hypothesis and see how often it is rejected. Parametric bootstrapping is used in a similar fashion except that resampled residuals are used instead of generated normally distributed errors.

5.1 Model specification tests

The matrix x_t in the VAR-model consists of 411 observations on the domestic price level, a weighted average of foreign price levels and the nominal effective exchange rate for each country. First, the number of lags to be included has to be determined. Information criteria like AIC, BIC and LIL minimise a function in which the increase in explanatory power from additional lags is balanced by a punishment for the additional parameters that have to be estimated. In addition, autocorrelation or non-normality of the residuals may indicate that the specified number of lags is incorrect.

The information criteria are computed as:

(9)
$$AIC = \ln|\Sigma(p)| + 2pK^2 / T,$$
$$BIC = \ln|\Sigma(p)| + 2\ln TpK^2 / T, \text{ and}$$
$$LIL = \ln|\Sigma(p)| + 2\ln\ln TpK^2 / T,$$

where $\sum(p) = E[\varepsilon_t(p)\varepsilon_t(p)']$ or the residual variance matrix when p lags are used, K is the number of variables and T is the number of observations.

Results from the model specification tests are presented in Table 2. In order to reduce the amount of statistical information, the multivariate LM test for first order autocorrelation and the Box-Ljung test for higher order autocorrelation are reported for the preferred lag length only. For fewer lags, at least one of the tests indicate serial correlation. In many cases, more lags are needed to remove higher order serial correlation than to remove first order serial correlation. Priority is then given to higher order autocorrelation. The number in parentheses after the p-values of the LM-test are the number of lags needed for the test statistics to become insignificant.

Table 2. Specification tests

	Preferred lag length: AIC	LIL	SBC	LM(1), p-val	Box-Ljung, p-val	Bera-Jarque preferred lag	Bera-Jarque preferred w/ dummies	Pre-ferred lag length
Canada	2	2	2	0.38	0.06	318.7	26.82 (0.00)	6
US	5	2	2	0.59 (3)	0.05	94.4	42.53 (0.00)	9
Japan	2	2	2	0.06	0.05	93.3	12.34 (0.05)	7
Austria	5	5	2	0.33 (3)	0.07	370.7	17.33 (0.01)	7
Belgium	7	3	2	0.36	0.06	328.1	15.23 (0.02)	7
Denmark	3	2	2	0.17	0.07	322.6	28.86 (0.00)	7
Finland	4	2	2	0.24	0.08	1136.7	38.19 (0.00)	5
France	5	2	2	0.05	0.08	371.7	36.18 (0.00)	4
Germany	5	2	2	0.28 (3)	0.07	306.5	32.50 (0.00)	7
Italy	7	2	2	0.94	0.445	384.0	73.45 (0.00)	7
Holland	6	2	2	0.08	0.06	508.4	29.94 (0.00)	7
Norway	6	2	1	0.15 (3)	0.06	426.5	44.45 (0.00)	8
Spain	5	2	2	0.25	0.05	254.4	28.16 (0.00)	7
Sweden	7	2	2	0.01 (3)	0.24	287.8	23.70 (0.00)	6
Switzerland	5	4	2	0.03 (7)	0.12	138.5	11.16 (0.08)	5
United Kingdom	7	2	2	0.56 (3)	0.05	179.6	10.31 (0.11)	10

The Bera-Jarque test for normality of the residuals (excess kurtosis and skewness) indicates non-normality for all countries and all lag lengths. This is not surprising given the data on fixed exchange rates with occasional realignments. A number of spikes in the nominal exchange rates coincide with devaluations. There are also spikes in the domestic price levels that are due to changes of indirect taxes Adding dummy variables to removes these large residuals reduces non-normality very much for almost all countries. Except in the case of Italy, the Bera-Jarque statistics are reduced with about 90 percent. Although a notable improvement, significant non-normality remains in most cases. An indication of how the cointegration tests behave with non-normal residuals may be found among the simulation exercises later in this section. The Bera-Jarque statistics are reported for the preferred lag length only.

Enough lags to remove residual autocorrelation have to be included. The last column shows the preferred number of lags. Seven lags are chosen for half of the countries. In the cases of Norway, the United States and United Kingdom, more lags are necessary for the residuals to pass the Box-Ljung test for higher order autocorrelation. For Finland, France, Sweden and Switzerland, fewer than seven lags are suffcient. The results in section 5 are fairly robust to the choice of lag length.

5.2 Cointegration rank

The testing procedure starts by investigating the null hypothesis that all the eigenvalues are zero, implying a zero Π-matrix and no cointegration. If r=0 is rejected, the largest eigenvalue must be non-zero. Next, the hypothesis r=1 or at least two non-zero eigenvalues is tested. For instance, the largest eigenvalue in the three variable system of Canada's effective exchange rate, the domestic price level and the foreign price level is 0.0394. The $\lambda-\max$ statistic tests whether this eigenvalue is zero and the trace statistic tests whether all three eigenvalues are zero. Both tests reject the null. The second largest eigenvalue is 0.0166. The $\lambda-\max$ statistic does not reject that λ_2 is zero and the trace statistic does not reject that λ_2 and λ_3 are zero. Thus, the cointegration rank for this system is 1. Table 3 shows the results from the cointegration rank tests.

The last column shows the number of cointegrating vectors. Three different cases emerge from Table 3. For Austria, Japan, Italy, Britain and Holland, there are two cointegration vectors. Since the number of common trends is the number of variables minus the number of cointegration vectors, the presence of two cointegration vectors imply that there is only one stochastic trend in the system. In the US three variable system of the effective nominal exchange rate and domestic and foreign price levels, the tests indicate no cointegration, especially not if empirical critical values are used. For the remaining ten countries, one cointegration vector is found, implying two common trends. A common interpretation of this would be that the system is driven by a domestic monetary policy trend and a foreign monetary policy trend.

Table 3. Cointegrating rank

	λ_1	λ_2	λ_3	Trace			λ-max			Rank
				r=0	r=1	r=2	r=0	r=1	r=2	
Critical values				21.63	10.47	2.86	15.59	9.52	2.86	
Canada	0.0945	0.0115	0.0002	44.93	4.74	0.07	40.19	4.66	0.07	1
US	0.0226	0.0204	0.0023	18.37	9.19	0.92	9.18	8.26	0.92	0
Japan	0.1033	0.0700	0.0001	73.37	29.34	0.04	44.04	29.30	0.04	2
Austria	0.0925	0.0472	0.0071	61.57	22.36	2.88	38.21	19.49	2.88	2
Belgium	0.0637	0.0120	0.001	32.09	5.50	0.63	26.59	4.87	0.63	1
Denmark	0.0417	0.0063	0.0026	20.84	3.64	1.07	17.20	2.57	1.07	1
Finland	0.0397	0.0191	0.0003	23.68	7.71	0.10	15.97	7.61	0.10	1
France	0.0381	0.0101	0.0046	21.84	6.04	1.89	15.80	4.15	1.89	1
Germany	0.0655	0.0195	0.0001	35.32	7.97	0.03	27.35	7.94	0.03	1
Italy	0.1261	0.0388	0.0034	72.54	17.54	1.40	55.00	16.14	1.40	2
Holland	0.1479	0.0424	0.0026	83.24	18.57	1.06	64.66	17.51	1.06	2
Norway	0.0574	0.0150	0.0044	31.68	7.88	1.79	23.80	6.08	1.79	1
Spain	0.0403	0.0131	0.0002	22.03	5.41	0.07	16.62	5.34	0.07	1
Sweden	0.0524	0.0120	0.0039	28.19	6.48	1.59	21.81	4.89	1.59	1
Switzer-land	0.108	0.0137	0.0053	51.14	7.81	2.16	43.33	5.64	2.16	1
United Kingdom	0.0531	0.0270	0.0062	35.32	13.45	2.49	21.87	10.96	2.49	2

The empirical distributions of the test statistics under various circumstances can be investigated by running the tests on a large number of synthetic data sets that contains a known number of cointegrating vectors. The data sets are created using the data genererating process (2). The parameters are estimated conditional on the rank of Π. First, normally distributed errors with the estimated variance-covaraince matrix are used. Since inclusion of dummy variables is known to change the critical values, data sets with dummy variables and normally distributed errors are generated as well. In order to investigate what happens under non-normality, bootstrapping with resampled residuals is used, both with the original highly non-normal residuals and with dummy variables and the slightly non-normal residuals that are actually present in the data sets.

The results for Sweden are shown in Table 4. The results for the cointegration rank tests differ only slightly between countries and the remaining critical values appear in Appendix C.

Table 4. Critical values from the empirical distributions: The case of Sweden

	95% percentiles for the empirical distributions							
	The normal distribution		Actual distribution (bootstrapping)		Normal distribution		Actual distribution, w/ dummy variables (bootstrapping)	
	trace	λ-max	trace	λ-max	trace	λ-max	trace	λ-max
r>0	30.43	21.49	52.68	41.47	29.36	20.12	60.27	51.39
r>1	13.66	11.71	20.03	17.07	11.72	10.47	12.31	10.63
r>2	5.92	5.92	4.93	4.93	3.17	3.17	4.53	4.53

Since the critical values from the empirical distribution are considerably higher than those from the asymptotic distribution, a true hypothesis is rejected too often using the latter. There are differences between countries, but they are rather small. The results are well in line with those of Jacobson, Vredin and Warne (1994) in a similar simulation exercise with about 100 observations in each sample. They also found empirical critical values that are about 20 percent higher than the asymptotic. Thus, the tests are oversized and too many cointegrating vectors tend to be found when asymptotic critical values are used.

The cointegration rank test statistics from the simulations can also be used to investigate how often the tests find the correct number of cointegrating vectors. For instance, when the true rank of Π is one, the λ-max statistics indicates zero cointegration vectors in 74,1 percent of the cases, one in 29,3 percent and two in 9,65 percent. Table 5 shows the results for Sweden, using normally distributed residuals and no dummy variables. The results for different countries differ very little.

Table 5. Frequencies of preferred rank, using asymptotic critical values

	Test result:r=0		Test result:r=1		Test result:r=2		Test result: r=3	
True rank	Trace	λ-max	Trace	λ-max	Trace	λ-max	Trace	λ-max
r=0	**66.13**	**74.13**	24.85	22.21	5.65	3.10	3.37	0.01
r=1	48.41	60.88	**33.92**	**29.30**	11.73	8.17	5.94	1.65
r=2	43.31	57.17	34.92	30.55	**13.64**	**9.65**	8.13	2.63

Table 5 shows that the power of the tests is mediocre in the present setting. They tend to indicate that there are no cointegrating vectors far too often and pick the correct number of cointegrating vectors in a minority of the cases except when the true rank is zero. For instance, when the true rank is one, both the tests indicate that the rank is zero about half of the time. Even when the true rank is equal to two, the tests very often result in zero cointegrating vectors.

As evident from Table 6, the situation deteriorates further if the empirical critical values are used instead of the asymptotic. The trace test performs better than the λ-max test. Since the power to distringuish between the hypotheses is so low when critical values from the empirical distributions are used, it does not seem appropriate to apply these critical values.

Table 6. Frequencies of preferred rank, using empirical critical values

True rank	Test result:r=0		Test result:r=1		Test result:r=2		Test result: r=3	
	Trace	λ-max	Trace	λ-max	Trace	λ-max	Trace	λ-max
r=0	**87.19**	**90.00**	6.73	8.72	2.46	1.18	0.01	0.00
r=1	78.93	82.71	**12.93**	**13.47**	6.17	3.44	1.97	0.00
r=2	74.20	79.31	14.98	15.47	**7.75**	**4.54**	3.06	0.01

When critical values from the asymptotic distribution are used, one cointegrating vector is found in most cases. The tests indicate two cointegrating vectors in five cases and zero in one (the United States). The simulations show that the asymptotic critical values are too large. Reconsidering the test results in table 3 with this in mind, one might suspect that the number of cointegrating vectors is lower than what the test indicate. If the critical values from the empirical distributions given normally distributed errors are used, no cointegration is found for the US, Denmark, Finland, France and Spain. Finally, using empirical critical values from the boothstrapping exercise, cointegration is not rejected only for Japan, Italy and Holland.

The simulations also indicate that the power to reject a false null hypothesis of no cointegration is unaccaptably low when empirical critical values are used. The test tends to indicate zero cointegrating vectors most of the time even when the true rank is two. Thus, the results from the cointegration rank tests using critical values from the empirical distributions are not reliable either. I will return to this issue in the concluding section.

5.3 The cointegrating vectors

PPP implies not only that home prices, foreign prices and nominal exchange rates should be cointegrated but also that the cointegrating vector should be [-1, 1, 1]. The latter condition means that it is the real exchange rate that is constant in the long run, not some other linear combination of exchange rates and price levels.

As with the trace and λ-max statistics above, the empirical distribution for particular samples and parameter values can be simulated. The empirical distributions are located further to the right than the asymptotic distributions, which means that the critical values are higher than for the χ^2 distribution. Thus, a true hypothesis will be rejected too often when critical values from the asymptotic distribution are used. Critical values and p-values from the empirical distributions appear in Table 7 along with the estimated cointegrating vectors and the likelihood ratio test statistics.

Table 7. The cointegrating vectors

	Cointegration vector(s)	χ^2 and p-value for [-1, 1, 1] $\in \beta$	Empirical critical value	Size
Austria	[-1.00, 0.162, 0.065] [-1.00, 0.849, 0.213]	12.12 (0.00)	10.386 (0.02)	17.31
Belgium	[-1.00, 0.827, -0.506]	20.25 (0.00)	14.745 (0.01)	41.65
Canada	[-1.00, 1.017, 0.267]	30.95 (0.00)	20.746 (0.01)	51.01
Denmark	[-1.00, 0.902, 0.260]	3.38 (0.18)	12.690 (0.48)	25.83
Finland	[-1.00, 1.445, 0.395]	4.26 (0.12)	14.974 (0.45)	31.60
France	[-1.00, 1.106, 0.163]	6.04 (0.05)	14.939 (0.33)	33.48
Germany	[-1.00, 0.764, 0.435]	6.36 (0.04)	9.957 (0.15)	16.70
Holland	[-1.00, -1.194, -1.025], [-1.00, 1.063, 0.914]	0.81 (0.37)	6.39 (0.68)	9.15
Italy	[1.00, 01.46, 00.33], [1.00, 01.76, 00.55]	14.74 (0.00)	10.14 (0.03)	14.28
Japan	[-1.00, 1.125, 0.392] [-1.00, 1.621, -0.446]	13.28 (0.00)	9.25 (0.02)	11,25
Norway	[-1.00, 1.124, 0.462]	5.97 (0.05)	13.123 (0.27)	27.12
Spain	[-1.00, 1.925, 0.193]	9.87 (0.01)	13.436 (0.11)	27.41
Sweden	[-1.00, 1.117, 0.439]	5.15 (0.08)	13.18 (0.31)	26.25
Switzer-land	[-1.00, 0.87, 0.33]	33.92 (0.00)	16.099 (0.00)	38.01
United Kingdom	[1.00, 1.1883, -5.452, [1.00, -1.549, 0.005]	1.46 (0.23)	12.76 (0.56)	15.04

The coefficients of the estimated cointegrating vectors usually have the expected signs. However, few of them are very close to the [-1, 1, 1] required by PPP. Above all, the coefficient on the nominal exchange rate is less than 0.5 in all cases where there is only one cointegrating vector.

Using the asymptotic critical values at the 5 percent significance level, the hypothesis of a constant real exchange rate is not rejected for Sweden, Denmark, Norway, Finland, Holland, United Kingdom and France. The marginal significance of the test statistic for Germany is 4 percent. Relying instead on critical values from the empirical distributions, PPP may hold also for Germany. The hypothesis of a stationary real exchange rate is solidly rejected for Austria, Belgium, Italy, Japan, Switzerland and Canada. Since the cointegration rank for the US was found to be zero in the previous section, it is not meaningful to test whether [-1, 1, 1] is a cointegrating vector.

If the test results are compared to the visual impression of real exchange rates in Appendix A, a number of cases seem rather puzzling. For instance, the figures for Austria, Denmark and Holland look similar, but the test results differ very much. PPP is rejected for Austria while Holland is the most clear cut non-rejection. The real exchange rates of Italy, the US and possibly Belgium look more stationary than the real exchange rates of Norway, Holland and Denmark. Still, the tests reject PPP for the former but not for the latter. The point estimate of the cointegrating vector for Denmark is [-1, 0.902, 0.260], which certainly differs considerably from [-1 , 1, 1] .

Simulations may shed some light on the size and power of the likelihood ratio test. There are more pronounced differences between the countries when the likelihood ratio test is considered than the case was with the cointegration rank tests. Again, the test is oversized in all cases. As Table 8 shows, the highest empirical critical values are about three to four times as high as the corresponding values from the χ^2 distribution. The lowest empirical critical values are only slightly higher than the asymptotic ones. Jacobson, Vredin and Warne (1994) also came to the conclusion that empirical distributions for the likelihood ratio test are located to the right of the χ^2 distribution. Their empirical critical values are about twice the asymptotic ones. It seems to matter whether there are one or two cointegrating vectors. Tests of whether the real exchange rate belongs to the space spanned by two cointegrating vectors may be less oversized than tests of whether a single vector is equal to [-1, 1, 1]. Whether this is the case is not a testable hypothesis within this framework.

The four first columns of Table 8 contain the empirical critical values at various significance levels. The final column shows the size of the likelihood ratio test when the asymptotic 5 percent critical value is used. The size varies between 17 and 51 percent. A true cointegrating vector of [-1, 1, 1] is rejected far too often when critical values from the asymptotic distribution are used.

To conclude, PPP is rejected for some countries (Canada, the US, Japan, Switzerland, Belgium, Austria, Italy and Spain) and not rejected for several other countries (Sweden, Denmark, Finland, Norway, France, Holland and the United Kingdom. Germany is a borderline case where the marginal significance of the test statistics is 0.04 using the asymptotic distribution and 9.96 using the empirical.

Table 8. Empirical distributions of the likelihood ratio test

	99%	95%	90%	50%	Rejection percentage using the 95% critical value
Asymptotic	9.21	5.99	4.61	1.39	5.00
Austria	15.56	10.38	7.88	2.37	17.31
Belgium	22.62	15.74	12.90	4.94	41.65
Canada	28.92	20.75	16.49	6.14	51.01
Denmark	18.61	12.69	9.90	3.22	25.83
Finland	22.65	14.97	11.47	3.68	31.60
France	21.63	14.94	11.77	3.88	33.48
Germany	15.25	9.96	7.67	2.32	16.70
Holland	12.31	9.84	6.16	2.73	15.43
Italy	10.14	8.12	5.34	3.07	14.28
Japan	9.25	7.25	5.01	2.49	11.25
Norway	20.04	13.12	10.23	3.31	27.12
Spain	20.71	13.44	10.43	3.22	27.41
Sweden	20.50	13.18	10.26	3.08	26.25
Switzerland	22.60	16.10	12.64	4.51	38.01
United Kingdom	12.76	9.52	7.38	3.62	15.04

5.4 Impulse response functions

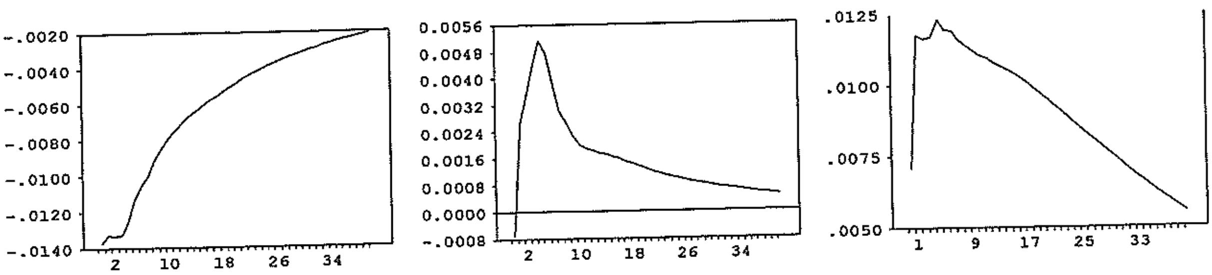
The dynamic adjustment towards PPP after a disturbance can be illuminated using impulse response functions. The polynomial C(L) in the moving average representation in equation (7) contains an infinite number of lags. Here, impulse responses are calculated up to 40 lags or almost three and a half years.

We are primarily interested in how the real exchange rate adjusts to disturbances. Equation (7) is easily transformed to levels of the variables. Levels of prices and exchange rates are in turn easily transformed into levels of real exchange rates. Figure 1 shows the impulse response functions for the Swedish real exchange rate. The corresponding figures for the other countries are presented in Appendix D.

With one cointegration vector, there are two independent stochastic trends that drive the system. In this system of domestic and foreign price level and the nominal exchange rate, it seems natural to identify the trends with domestic and foreign monetary policy. Innovations to the stochastic trends have permanent effects on the price levels and the nominal exchange rate. In addition, there is a transitory shock that has transitory effects on the variables. One identifying assumption is needed to distinguish the two permanent shocks. The only obvious alternative is the small country assumption that the foreign price level is unaffected by the domestic price level. Only a few of the countries could be characterised as small. In addition, the error correction coefficients on the foreign price levels are significant in most cases, even for the small countries. A small country assumption does not seem appropriate. However, our interest is primarily in speed of the adjustment process. It can be investigated without identifying the shocks.

Figure 1. Impulse response functions for the real exchange rate: Sweden

Time profile of the response to a transitory shock	*Time profile of the response to the first type of permanent shock*	*Time profile of the response to the second type of permanent shock*



The main conclusion from an examination of the impulse response functions is that the bulk of the adjustment is completed within 40 months or a little more than three years. There is an adjustment back to PPP in almost every case, even for the countries for which PPP was rejected. The cointegrating vector corresponding to PPP is imposed as a long run restriction when estimating the impulse response functions, but this does not force the short run parameters to imply a rapid adjustment to PPP. Indeed, there are a few cases where the impulse response functions do not show an adjustment back to PPP within 40 months: The real exchange rates of Belgium and United Kingdom do not seem to adjust to the second type of permanent shocks.

6 Conclusions

Before the virtual explosion of PPP tests that followed the introduction of econometric techniques designed to handle non-stationary data, it had become more or less a stylised fact that PPP was rejected in empirical tests. One conclusion from the review of empirical evidence in section two is that PPP has indeed fared badly when American data are used. However, focusing instead on the more homogenous European countries or even Japan, the hypothesis of a stationary real exchange rate is rejected less often. Excluding tests on short sample periods decreases the proportion of rejections further. There are few studies that include small European countries like Sweden, Norway, Denmark and Finland and use a long sample period. Also, almost all studies have focused on bilateral rather than multilateral real exchange rates.

In this paper, multilateral PPP has been tested on data from 1960 to 1994 on 16 OECD countries, using Johansen's maximum likelihood approach that incorporates tests of linear restrictions on the cointegrating vectors. The hypothesis of a stationary real exchange rate is rejected for Austria, Switzerland, Belgium, Canada and Japan. However, it is not rejected for the Nordic countries, Holland, France and United Kingdom even using the asymptotic critical values that have been shown to reject a true hypothesis too often.

A few of the test results are surprising in light of the figures on the real exchange rates. The real exchange rates of Belgium, Italy and the US look rather stationary, but the tests reject stationarity. The real exchange rates of Denmark, Holland and Norway look non-stationary, but the tests do not reject stationarity. Perhaps the main lesson from the simulation exercises is to encourage caution when interpreting the test results.

Empirical distributions and critical values are simulated for the λ-max and trace rank tests as well as the likelihood ratio test for linear restrictions on the cointegrating vector. They show that there may be considerable discrepancies between asymptotic and empirical distributions of the test statistics. All the tests are oversized. The power of the cointegration rank tests is unacceptably low when critical values from the empirical distributions are used.

Appendix A: Empirical tests of PPP

Table 10: The number of rejections of PPP and the number of times a pair of countries was tested

	Aut	Bel	Fra	Ger	Hol	Swe	Swi	Ita	Fin	Nor	Den	UK	US	Jap	Can
Aut	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Bel		-	0/1	0/1	0/1	1/2	0/1	0/1	-	-	-	2/2	2/2/**0/1**	1/2	-
Fra			-	2/3	1/2	1/2	0/1	0/1/**1/2**	-		-	1/1/**0/2**	5/5/**1/7**	1/2	1/2/**0/2**
Ger				-	0/1	0/1	1/2	3/3	-	-	-	1/1	4/4/**3/3**	1/2	3/3
Hol					-	0/1	1/2	1/1	-	-	-	1/1	1/1/**1/3**	1/2	1/1
Swe						-	1/2	-	-	-	-	-	-	1/2	1/1
Swi							-	2/2	-	-	-	-	3/3/2/2	2/3	-
Ita								-	-	-	-	**1/2**	**2/7**	**1/2**	-
Fin									-	-	-	-	-	-	-
Nor										-	-	-	**0/1**	-	-
Den											-	-	-	-	-
UK												1/1	3/4/5/9	1/1	1/2/**0/2**
US													**6/8**	2/2/6/8	4/5/**7/11**
Jap														-	-
Can															-
>20	0/0	0/1	3/13	3/3	1/3	0/0	2/2	5/13	0/0	0/1	0/0	6/15	35/60	6/8	8/17
excl*	0/0	0/0	1/6	0/0	0/0	0/0	0/0	3/6	0/0	0/0	0/0	1/6	0	0/0	0
>15	0/0	6/13	12/20	15/21	7/13	5/11	10/16	6/8	0/0	0/0	0/0	8/10	24/26	10/16	9/12
excl*	0/0	4/11	6/13	8/14	5/11	4/10	7/13	6/8	0/0	0/0	0/0	6/6	0	8/14	0

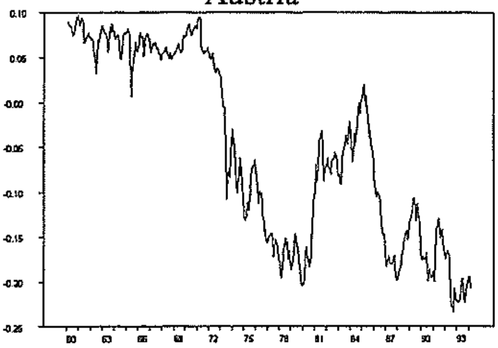
Only tests using sample periods longer than 15 years are included. Fat numbers denote studies on data covering more than 20 years. Tests of multilateral PPP are placed on the diagonal of the matrix. Searching through major data bases of journal articles and the references of the papers discussed above, all papers covering sufficiently long sample periods are included in table 10. Among studies covering more than 20 years of data, this amounts to the following papers: Adler and Lehmann (1990), Kim (1990), Abuaf and Jorion (1990), McNown and Wallace (1990), Ardeni and Lubian (1991), Layton and Stark (1990) and Glen (1992). With sample periods of 15-20 years, the studies by Cheung and Lai (1993), Glen (1992), Nelson (1990), Fisher and Park (1991) and Johansen and Juselius (1992) are included.

A few observations can be made:
- 60 of 72 tests focuses on the US real exchange rate.
- Of the test using sample periods longer than 20 years, 35 of 60 tests on the US real exchange rate reject PPP. Among tests on other real exchange rates, only 3 of 13 rejects it.
- Of tests using 15 to 20 years of data, 87 percent of the tests on the North American dollars reject PPP, compared to 51 percent of the tests on other currencies.

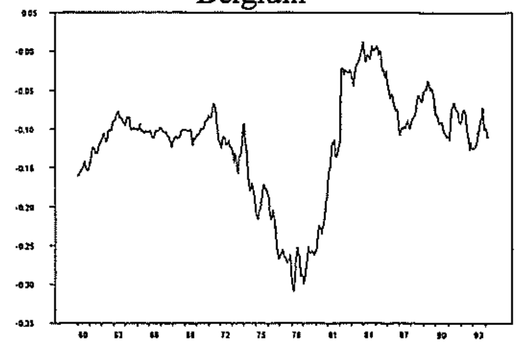
* Tests on real exchange rates against the United States and Canada have been excluded.

Appendix B: Figures on the real exchange rates

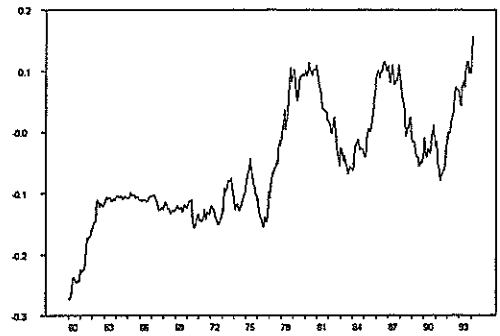
Austria



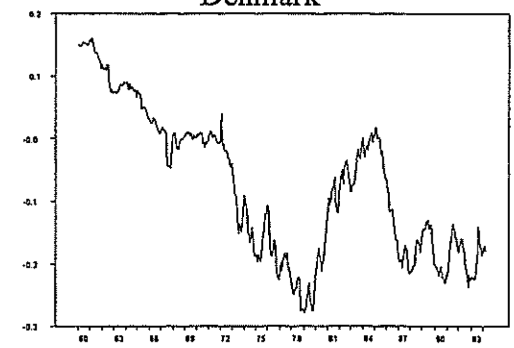
Belgium



Canada

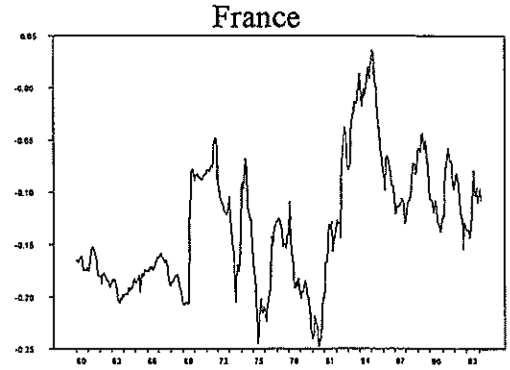


Denmark

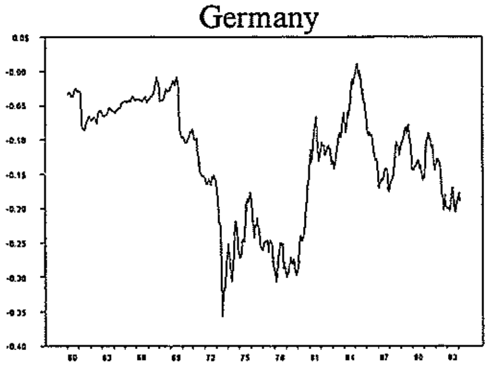


Finland

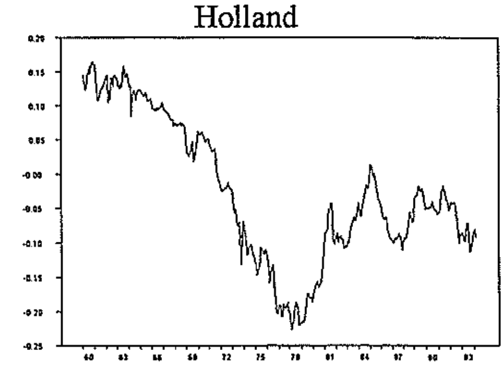
France



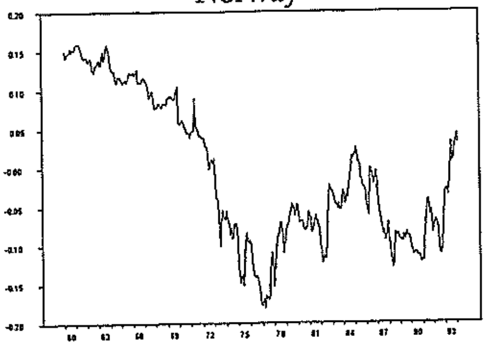
Germany



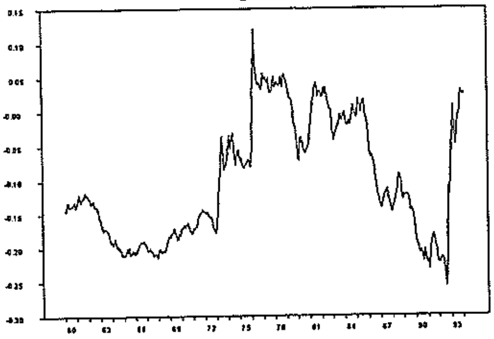
Holland



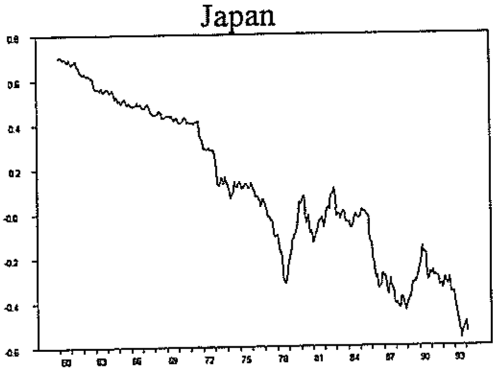
Norway



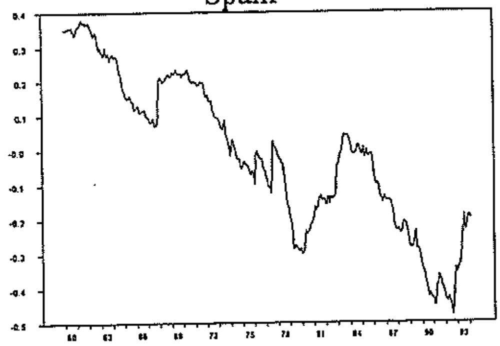
Italy



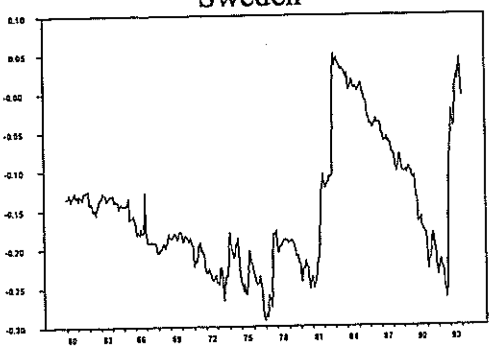
Japan



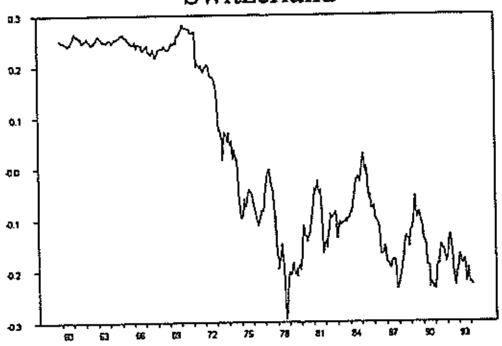
Spain



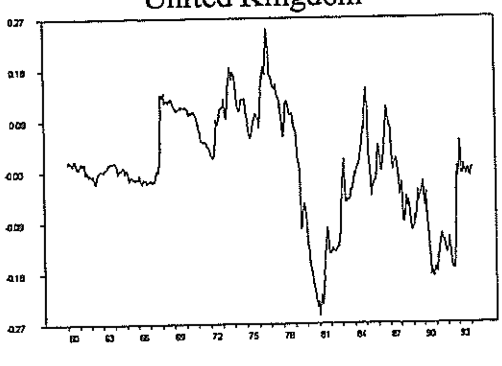
Sweden



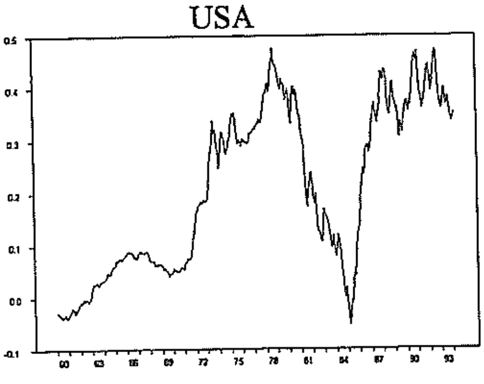
Switzerland



United Kingdom



USA



Appendix C: Critical values from the empirical distributions of the cointegration rank tests

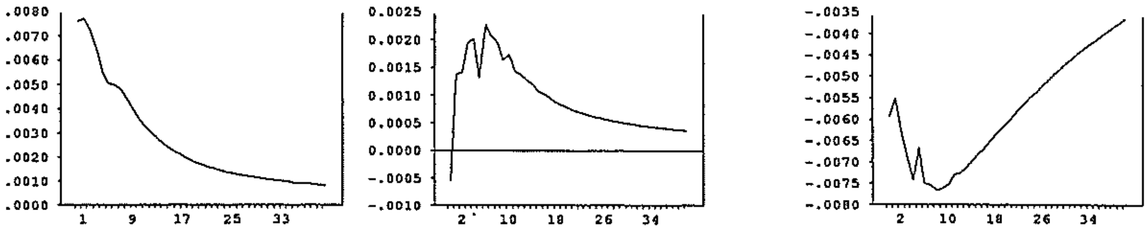
	95% percentiles for the empirical distributions							
	Normal distribution		Actual distribution (bootstrapping)		Normal distribution		Actual distribution w dummy variables (bootstrapping)	
	Trace	λ-max	Trace	λ-max	Trace	λ-max	Trace	λ-max
Canada								
r>0	30.97	21.44	53.28	42.01	30.05	21.77	48.35	37.55
r>1	13.29	11.29	13.29	11.40	11.63	9.82	11.57	9.85
r>2	4.11	4.11	3.94	3.94	3.71	3.71	3.44	3.44
USA								
r>0	31.20	21.97	40.81	29.69	30.52	21.56	37.68	27.10
r>1	15.70	13.29	15.23	13.13	15.32	12.83	15.28	13.00
r>2	5.80	5.80	5.58	5.58	7.26	7.26	6.20	6.20
Japan								
r>0	31.11	21.84	43.48	29.43	31.75	22.44	40.65	29.66
r>1	15.96	13.80	15.19	13.59	16.60	14.02	16.98	14.60
r>2	7.56	7.56	7.05	7.05	9.14	9.14	8.49	8.49
Austria								
r>0	30.38	21.38	78.14	66.80	30.91	21.08	66.49	55.71
r>1	14.35	12.31	14.46	12.55	17.62	14.18	13.79	12.13
r>2	4.81	4.81	4.72	4.72	5.25	5.25	6.70	6.70
Belgium								
r>0	31.10	21.78	68.58	54.51	29.97	21.10	56.73	44.29
r>1	13.01	11.00	13.16	11.13	12.56	10.53	12.86	10.56
r>2	4.81	4.81	4.87	4.87	4.33	4.33	4.37	4.37
Denmark								
r>0	30.70	21.68	52.68	41.47	31.07	21.03	52.27	41.76
r>1	15.62	13.53	15.91	13.84	15.06	12.50	15.12	12.77
r>2	5.30	5.30	4.93	4.93	5.73	5.73	5.85	5.85
Finland								
r>0	31.09	21.51	48.19	36.88	29.85	20.51	46.36	36.54
r>1	13.37	11.19	13.19	11.37	11.70	10.53	12.08	10.75
r>2	5.56	5.56	5.27	5.27	4.54	4.54	4.99	4.99
France								
r>0	31.00	21.40	58.59	47.93	30.27	21.12	59.28	48.67
r>1	12.73	10.93	12.75	11.09	11.87	10.36	11.94	10.59
r>2	4.13	4.13			3.49	3.49	3.10	3.10

	95% percentiles for the empirical distributions							
	Normal distribution		Actual distribution (bootstrapping)		Normal distribution		Actual distribution w dummy variables (bootstrapping)	
	Trace	λ-max	Trace	λ-max	Trace	λ-max	Trace	λ-max
Germany								
r>0	31.04	21.61	55.18	42.39	32.61	22.77	51.39	37.48
r>1	14.45	12.29	14.66	12.48	13.93	12.28	13.78	12.20
r>2	4.84	4.84	4.78	4.78	4.68	4.68	4.21	4.21
Italy								
r>0	30.69	21.66	46.94	35.56	32.45	23.10	39.35	29.44
r>1	15.88	13.74	16.15	14.24	13.57	11.34	14.38	12.33
r>2	4.53	4.53	4.18	4.18	3.22	3.22	3.57	3.57
Holland								
r>0	30.68	21.52	67.58	54.65	34.87	26.36	75.36	62.34
r>1	16.37	14.67	15.74	13.88	14.67	12.94	15.60	13.53
r>2	4.71	4.71	4.96	4.96	4.87	4.87	4.97	4.97
Norway								
r>0	30.87	21.67	50.35	38.36	30.57	21.66	49.60	36.59
r>1	13.30	11.28	13.14	11.37	13.87	11.49	13.28	12.16
r>2	4.80	4.80	4.98	4.98	4.97	4.97	4.89	4.89
Spain								
r>0	31.22	21.91	48.27	36.25	31.02	21.68	45.59	34.83
r>1	12.73	10.94	12.61	11.07	13.18	11.13	11.39	10.35
r>2	3.90	3.90	3.67	3.67	4.02	4.02	2.36	2.36
Switzerland								
r>0	30.93	21.85	41.66	29.59	33.86	22.74	37.90	27.07
r>1	13.36	11.44	13.76	11.70	14.30	12.14	14.14	12.93
r>2	4.64	4.64	4.68	4.68	3.06	3.06	3.04	3.04
United Kingdom								
r>0	30.65	21.65	48.10	36.83	29.08	19.91	49.08	38.34
r>1	12.99	10.93	13.38	11.29	12.42	10.52	13.02	10.66
r>2	4.97	4.97	3.66	3.66	4.94	4.94	5.42	5.42

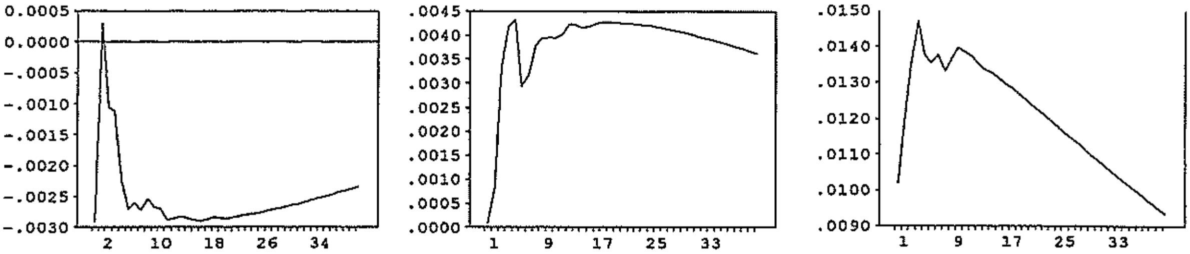
Appendix D: Impulse response functions for the real exchange rates

Time profile of response to a transitory shock | *Time profile of response to the first type of permanent shock* | *Time profile of response to the second type of permanent shock*

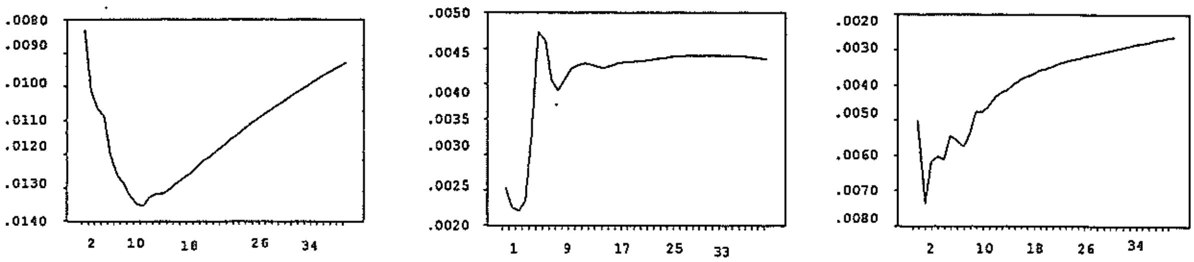
Austria



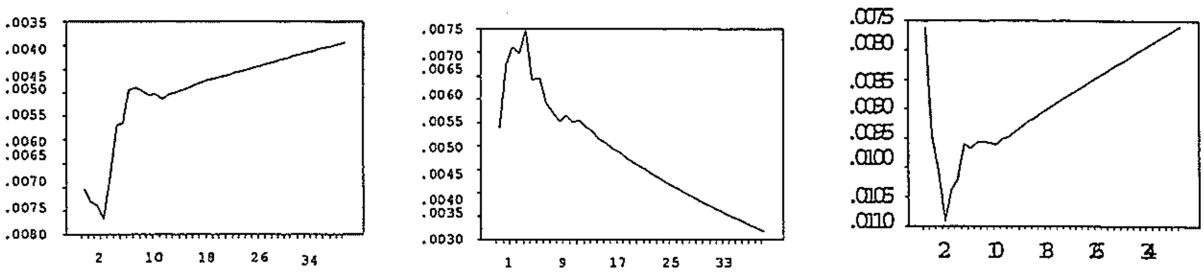
Belgium



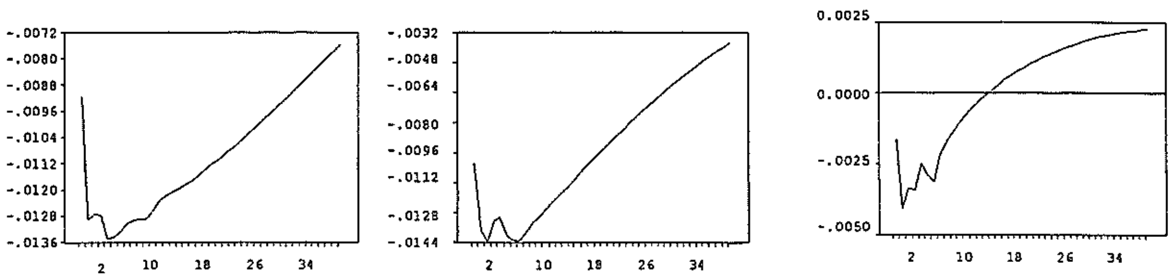
Canada



Denmark

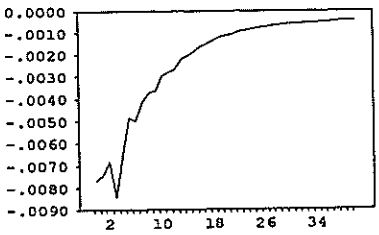


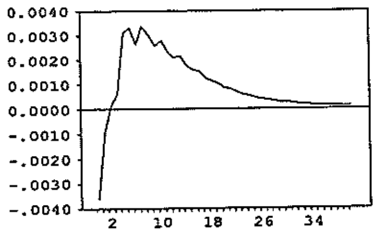
Finland

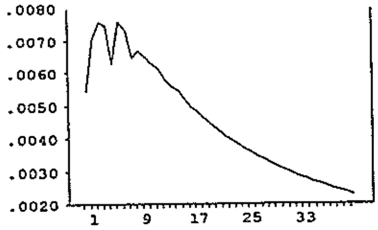


Time profile of response to a transitory shock	*Time profile of response to the first type of permanent shock*	*Time profile of response to the second type of permanent shock*

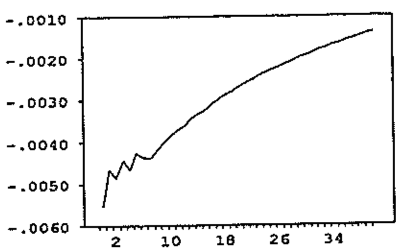
France

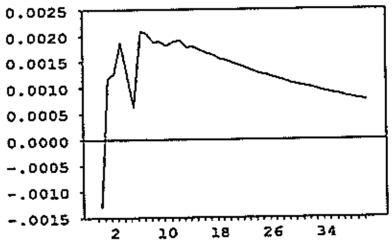


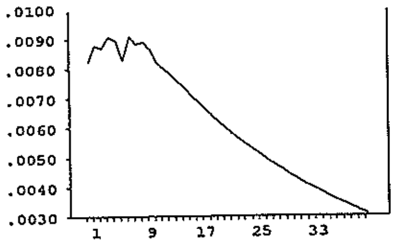




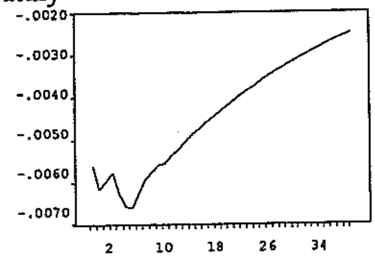
Germany

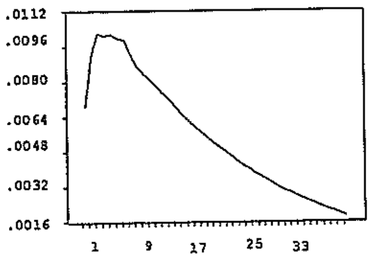


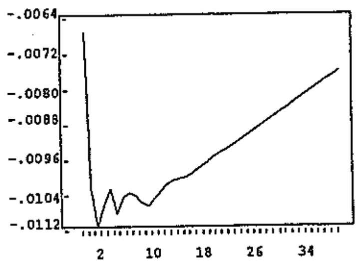




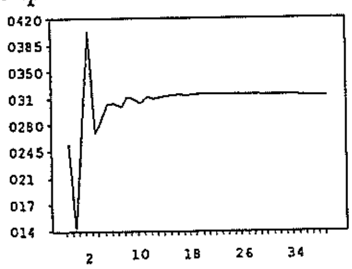
Italy

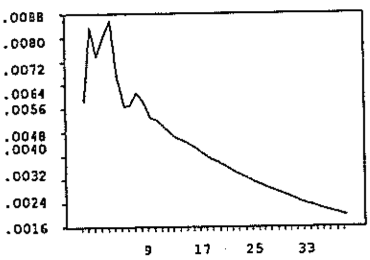


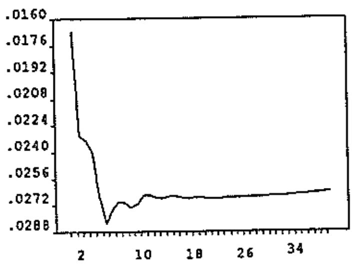




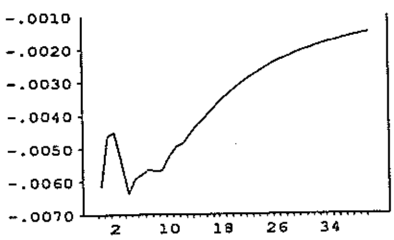
Japan

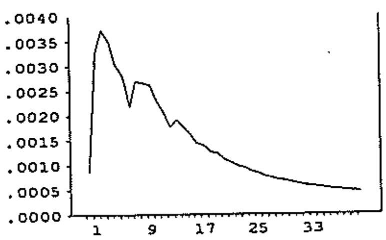


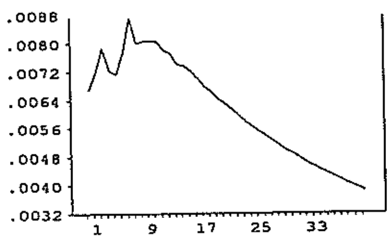




Netherlands

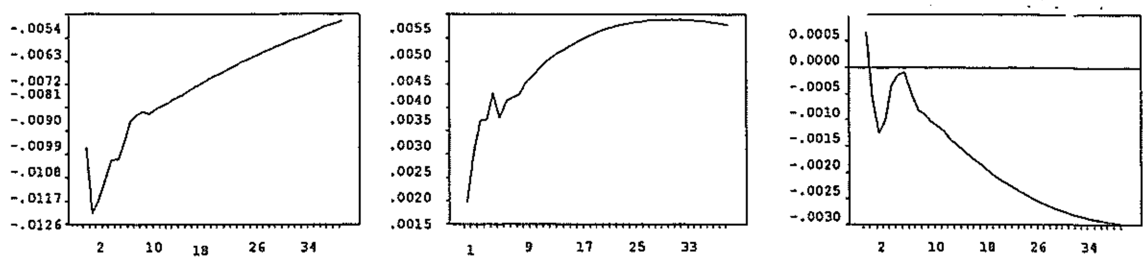




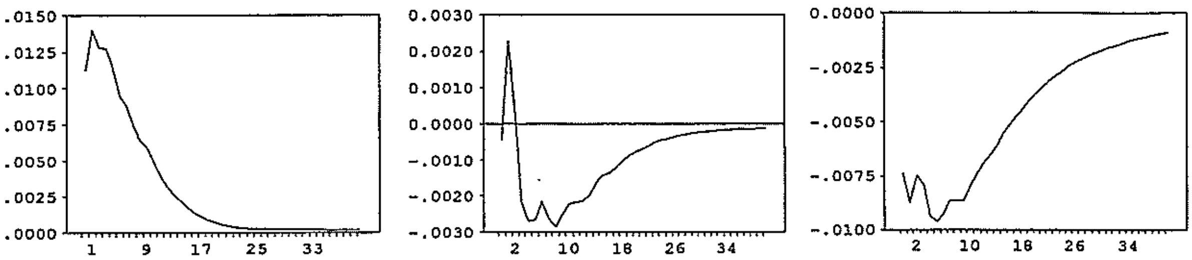


Time profile of response to a transitory shock | *Time profile of response to the first type of permanent shock* | *Time profile of response to the second type of permanent shock*

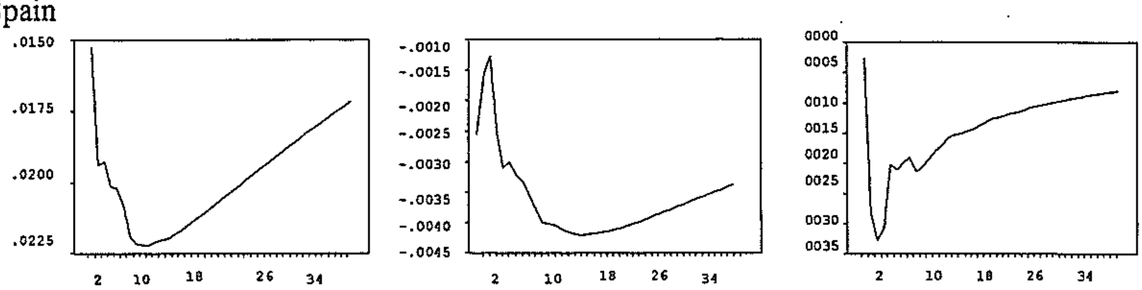
Norway



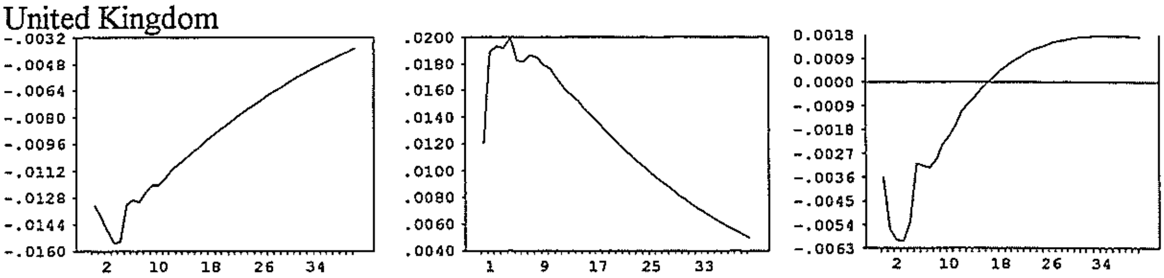
Switzerland



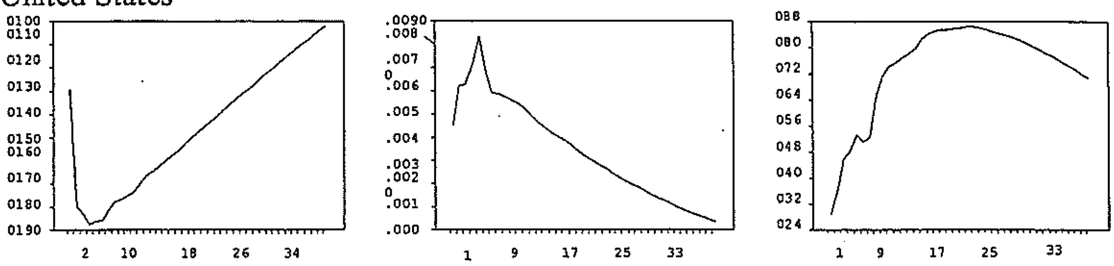
Spain



United Kingdom



United States



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