Arbetsrapport

Nr 18

Augusti 1994

Fixed Exchange Rate Collapses with Stochastic Process Switching and Bayesian Updating

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Abstract

This paper focuses on the role uncertainty about future fundamentals may play in the collapse of a fixed exchange rate regime. A model is presented in wich th eprocess that drives fundamentals switches between two states that differ with respect to their short-run compatibility with the fixed exchange rate regime. Market participants do not observe tha state shifts directly, but are able to draw probalistic inference based on the observed behavior of the data. The shifts in states and the probalistic inference influence the timing of the collapse. They also induce more variability in interest rate differentials and foreign exchange reserves, giving a more realistic collapse scenario than basic models.

I thank seminar participants at IIES and Sveriges Riksbank for comments on a previous version. In particularly I would like to express my to John Hassler, Lars Hörngren, Andrew K. Rose and Lars E.O. Svensson for help and stimulating discussions. This paper was written while I visited IIES. I thank IIES for its hospitality. The views in this paper are those of the author and do not necessarily reflect those of Sveriges Riksbank.

Contents

1 Introduction

The seminal paper by Krugman (1979) provides a simple example of a collapse of a fixed exchange rate regime.[1] Krugman sets out a model in which the currency peg must be abandoned at a point where the central bank runs out of foreign exchange reserves, the reason for the destined collapse being a deterministic trend in domestic credit. Krugman also shows that speculators with foresight inevitably will attack the currency peg before reserves are fully exhausted, according to the simplest principles of currency arbitrage.

The dramatic crises of the Nordic currencies and the ERM that culminated in August 1993 have been analysed thoroughly and induced further refinements of the basic collapse model. Eichengreen and Wyplosz (1993) and Portes (1993) argue that the crises, in part, were driven by self-fulfilling in accordance with models by Flood and Garber (1984a) and Obstfeld (1986). It is also clear that other factors than reserve availability played a crucial role for the response of governments and central banks to the crises. In the Nordic countries, it seems obvious that the effects of high interest rates and growing unemployment were influential for the decisions to float the currencies. The theoretical analysis is extended in that direction by Ozkan and Sutherland (1994) and Obstfeld (1994), among others.

The 1992-1993 crises were also marked by speculator uncertainty about future government policies and fundamentals of the economy. Krugman (1979) discusses the case when there is uncertainty about how much of its reserves the central bank is willing to use to defend the exchange rate and demonstrates that there can be a series of crises in which capital flows out of the country, and then returns, before the issue is finally settled.

This paper focuses on another source of uncertainty. The stochastic collapse model presented by Flood and Garber (1984b) is modified and extended to a case where speculators have incomplete knowledge about the processes that drive fundamentals and use Bayesian updating to assign probabilities to two states that differ with respect to their short-run compatibility with the fixed exchange rate regime. The rest of the paper is organized as follows. The model is outlined in section 2. Some simulation results are presented in section 3. Section 4 summarizes and concludes.

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[1] Krugman's model was inspired by Salant and Henderson's (1978) model of government price-fixing schemes in exhaustible resource markets.

2 The model

The *asset market view* of exchange rates emphasizes that exchange rates display many similarities to other asset prices determined in well-developed markets; they are strongly influenced not only by current events but also by market participants' expectations of future events. This gives expectations about future exchange rates a key role in determining current rates. A standard way of formalizing this notion is

$$(1) \qquad s_t = f_t + \alpha(\mathrm{E}_t s_{t+1} - s_t),$$

where s_t is the logarithm of the exchange rate at time t, f_t a fundamental determinant of the value of the currency, α a positive parameter that can be interpreted as the semi-elasticity of money demand and E_t the time t expectations operator. The saddle-path solution to (1) can be written as

$$(2) \qquad s_t = \frac{1}{1+\alpha}\mathrm{E}_t \sum_{j=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^j f_{t+j},$$

showing that s_t is the expected present value of future fundamentals. The fundamental determinant of the exchange rate, f_t, is defined as

$$(3) \qquad f_t = m_t + v_t,$$

where m_t is the logarithm of the money stock and v_t the logarithm of the income velocity of money and other macro variables exogenous to the exchange rate.[2] For notational convenience, the latter will henceforth be called just velocity.

Domestic money, m_t, is held by domestic residents only and consists entirely of base money at the central bank. Let D_t be the domestic credit and R_t the book value of foreign exchange reserves. Suppose that $\bar{s}$ is the fixed exchange rate and that the central bank is willing to defend the fixed exchange rate to the point where foreign exchange reserves are exhausted, that is, $R \geq 0$.[3] Assuming that $\ln D(t) = \ln \overline{D} = 0$ enable us to express the restriction $R \geq 0$ as

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2 Equation (1) can be derived from a flexible price monetary model that consists of a money demand function, $m_t - p_t = \psi y_t - \alpha i_t$, a definition of the real exchange rate, $q_t = s_t + p_t^* - p_t$, and an equilibrium condition in asset markets (the definition of the exchange risk premium) $\rho_t \equiv i_t - i_t^* - (\mathrm{E}_t s_{t+1} - s_t)$. Hence, $v_t \equiv q_t + \alpha\rho_t + \alpha i_t^* - \psi y_t$.

3 In reality R is not restricted to be non-negative since the central bank may borrow reserves or intervene in the forward exchange market. However, the restriction as such seems appropriate since

(4) $$m_t = \ln(\overline{D}+R_t) \equiv r_t \geq 0.$$

Assume that velocity follows the stochastic process;

(5) $$v_t = v_{t-1} + \mu_0 + z_t\mu_1 + \varepsilon_t,$$

where μ_0 and μ_1 are positive drift parameters. The state variable z_t, which is associated with μ_1, takes the values $z_t = 0$ or 1. This implies that the drift in velocity is μ_0 in state zero $(z_t = 0)$ and $\mu_0 + \mu_1$ in state one $(z_t = 1)$. The term ε_t represents a random disturbance with zero mean which obeys

(6) $$\varepsilon_t = -\frac{\beta}{\lambda} + \omega_t,$$

where ω_t is gamma distributed, $\omega_t \sim \Gamma(\beta,\lambda)$, with the (unconditional) density function

(7) $$f_\omega(\omega_t) = \begin{cases} \frac{\lambda^\beta}{\Gamma(\beta)} \exp(-\lambda\omega_t)\omega_t^{\beta-1} & \omega_t \geq 0 \\ 0 & \omega_t < 0. \end{cases}$$

Hence, ω_t has the mean β/λ and the variance β/λ^2. To ensure a non-negative growth in v_t, which is necessary in order to obtain a well defined solution of the model, it is assumed that $\mu_0 \geq \beta/\lambda$.[4]

At this stage, it might be fruitful to spell out the differences to the collapse model by Flood and Garber (1984b). First, the model outlined above is written in logarithms, not in levels. Second, the drift in fundamentals occurs in the variable labelled velocity, not in domestic credit. This is, however, merely for convenience since the expressions become a bit messier when the drift occur in domestic credit. I leave it to the reader to interpret the drift in fundamentals as a drift in domestic credit when fiscal policy is at odds with the fixed exchange rate regime or as a drift in the variables that determine the demand for the currency, for instance, the income velocity or the real exchange rate. Third, the random disturbance in the model is gamma distributed, not exponentially distributed. However, the exponential distribution is just a special case of the gamma distribution with $\beta = 1$. The

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the central bank will make losses on forward contracts and other foreign debts when the currency is floated.

4 Flood and Garber (1984b) argue that it would be permissible to allow for negative realizations in the process that drives fundamentals in a log-linear model. However, negative realizations in Δv_t would make the model inconsistent, see Appendix 1.

fourth, and most important, difference is the state-variable z_t that is associated with the drift parameter μ_1.

Market participants do not observe the state shifts directly. However, they do observe v_t and known the parameters μ_0, μ_1, β and λ. They are also aware that the transition between the two states is governed by a first-order Markov process:

$$(8)\qquad \begin{aligned} &\mathrm{P}\left[Z_t = 0 \middle| Z_{t-1} = 0\right] = p \\ &\mathrm{P}\left[Z_t = 1 \middle| Z_{t-1} = 0\right] = 1-p \\ &\mathrm{P}\left[Z_t = 1 \middle| Z_{t-1} = 1\right] = q \\ &\mathrm{P}\left[Z_t = 0 \middle| Z_{t-1} = 1\right] = 1-q, \end{aligned}$$

where, for instance, q denotes the probability of being in state one at time t conditional on being in state one at time t-1.

Define π_t as the market participants' subjective probability of being in state one at t conditional on the information they have available at time t, that is, $I_t = \{v_t, v_{t-1}, \ldots, v_0, \mu_0, \mu_1, \beta, \lambda, p, q\}$. The Markov property of the transitions then implies that their expectations about future probabilities are

$$(9)\qquad \mathrm{E}_t\, \pi_{t+j} = \pi + \delta^j (\pi_t - \pi),$$

where $\delta \equiv p+q-1$ and $\pi \equiv (1-p)/(1-q+1-p)$ is the unconditional probability of being in state one. Thus, the one-period-ahead (prior) probability is $\mathrm{E}_t\, \pi_{t+1} = (1-p)(1-\pi_t) + q\pi_t$. Equation (9) also implies that the conditional probability asymptotically (when $j \to \infty$) converges to the limiting unconditional probability, that is, current information is less valuable on longer forecasting horizons.

To close the model we assume that the subjective probability of the initial state is equal to the limiting unconditional probability, that is,

$$(10)\qquad \pi_0 = \pi.$$

Thereafter, market participants update their probabilities based upon subsequent observations of Δv_t according to Bayes' law.[5] Thus,

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[5] See, for instance, Zellner (1985) for an overview of Bayesian econometrics and Hamilton (1988, 1989) for applications with Markov chains.

(11) $$\pi_t = \frac{g(\Delta v_t | Z_t = 1)}{g(\Delta v_t)} \mathrm{E}_{t-1}\, \pi_t,$$

where $g(\Delta v_t) = g(\Delta v_t | Z_t = 0)(1 - \mathrm{E}_{t-1}\, \pi_t) + g(\Delta v_t | Z_t = 1)\mathrm{E}_{t-1}\, \pi_t$ and $g(\Delta v_t | Z_t = z_t) = f(\Delta v_t - \mu_0 - z_t\mu_1 + \beta / \lambda)$. Hence, the market's belief about the current state moves over time in response to the i.i.d. disturbance term, ε_t, and to realizations of the current state, Z_t. It is clear that equations (5), (6) and (7) together with the assumptions about the parameters imply that market participants will know for sure that $z_t = 0$ for observations of Δv_t below $\mu_0 + \mu_1 - \beta / \lambda$ and then assign the probability $\pi_t = 0$ to state one. A more gradual updating, in favor of state one, will take place above that threshold.[6]

In the analysis of the model we will proceed by examining the expression for the shadow floating exchange rate, defined below, and then determine the condition for a collapse of the fixed exchange rate regime. Next, we study the stochastic path of short-term interest rates and reserves.

Assume that $\bar{s}$ is the fixed exchange rate and define $\tilde{s}_t$ as the shadow floating exchange rate, the exchange rate that would prevail if market participants were to purchase all the central bank's reserves at t.[7] Alternatively, $\tilde{s}_t$ can be interpreted as the flexible exchange rate conditional on a collapse at time t. A fixed exchange rate regime will collapse at t if and only if $\tilde{s}_t \geq \bar{s}$, according to the simplest principles of currency arbitrage. If $\tilde{s}_t \geq \bar{s}$ market participants may buy reserves from the central bank at a price $\bar{s}$ and resell immediately on the post-collapse price $\tilde{s}_t$ and making a non-negative profit per unit of reserves of $\tilde{s}_t - \bar{s} \geq 0$. If $\tilde{s}_t < \bar{s}$, market participants would clearly not buy the central bank's remaining reserves at $\bar{s}$ for resale at $\tilde{s}_t$. Thus, the shadow floating exchange rate must equal or exceed the fixed rate for an attack or a collapse of the fixed exchange rate regime to occur.

The shadow free float exchange rate can be written as

(12) $$\tilde{s}_t = v_t + \alpha(\mathrm{E}_t\, \tilde{s}_{t+1} - \tilde{s}_t),$$

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6 At a general level, the model is somewhat related to the exchange rate model in Lewis (1989). However, state shifts are exogenous in that model. Markov chains have been applied to exchange rates by Engel and Hamilton (1990), but directly on the exchange rate and not on the fundamentals of the exchange rate.

7 The concept shadow floating exchange rate originates from Flood and Garber (1984b).

in correspondence with equation (1). The saddle-path solution, equation (2), enable us to express the shadow exchange rate as[8]

(13) $$\tilde{s}_t = \tilde{S}(v_t, \pi_t) = v_t + \alpha\mu_0 + \frac{\alpha}{1 - \frac{\alpha}{1+\alpha}\delta}\left(1 - p + \frac{\delta\pi_t}{1+\alpha}\right)\mu_1.$$

Thus, the shadow floating rate and hence the timing of the collapse is not determined solely by the variable v_t (as in the Flood-Garber model), but also by the subjective probability market participants assign to state one, that is, the state when μ_1 affects the fundamentals of the exchange rate. The parameters of the Markov chain are also crucial for the shadow floating rate. The first term within the brackets, $1-p$, is a constant and reflects the influence expectations about future shifts from state zero to state one have on the current shadow exchange rate. Moreover, the exchange rate is more sensitive to the probability market participants assign to state one when transitions between the states are relatively rare, that is, when p or q and consequently δ are high.[9] Hence, consistent with simple intuition, information about the current state is more valuable when states are less likely to be reversed.

If there is some persistence in the states, q and p are greater than 1/2, the shadow exchange rate will be contained in the following interval;

(14) $$v_t + \alpha\mu_0 + \frac{\alpha(1-p)}{1 - \frac{\alpha}{1+\alpha}\delta}\mu_1 \leq \tilde{s}_t < v_t + \alpha\mu_0 + \frac{\alpha}{1 - \frac{\alpha}{1+\alpha}\delta}\left(1 - p + \frac{\delta}{1+\alpha}\right)\mu_1$$

where the boundaries are given by $\pi_t = 0$ and $\pi_t = 1$, respectively. In *Figure 1* the boundaries of this band are plotted against velocity. It is worth noting that the collapse of the fixed exchange rate regime will occur somewhere in the interval v' and v''. However, the exact location of the collapse in that interval is determined by the probabilities market participants assign to state one.

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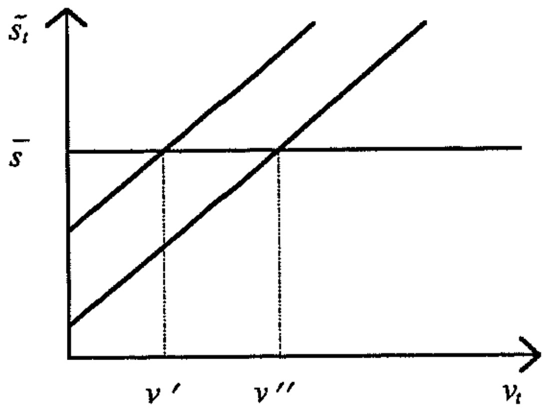
8 See Hamilton (1989) for present value calculations of variables with Markov trends.

9 There is no role for Bayesian updating in the model when $p = q = 0.5$. The shadow floating rate is then simply $\tilde{s}_t = v_t + \alpha\mu_0 + 0.5\alpha\mu_1$.

Figure 1. Band for the shadow floating rate



What about the interest rate differential? Let us assume that agents are risk neutral. The one-period interest rate differential can then approximately be written as

$$(15) \qquad i_t - i_t^* = \mathrm{E}_t(s_{t+1} - s_t).$$

The interest rate differentials during the free-float of the exchange rate are contained in the following band

$$(16) \qquad \mu_0 + \frac{1-p}{1-\frac{\alpha}{1+\alpha}\delta}\mu_1 \le i_t - i_t^* < \mu_0 + \frac{1}{1-\frac{\alpha}{1+\alpha}\delta}\left(1-p+\frac{\delta}{1+\alpha}\right)\mu_1,$$

where the lower boundary is the expected rate of depreciation when $\pi_t = 0$ and the upper boundary corresponds to $\pi_t = 1$. The interest differential during the fixed exchange rate regime must be below the upper boundary of this band and somewhere within the band at the time of the collapse. This follows from the result that a collapse occurs if and only if $\tilde{s}_t \ge \bar{s}$. To be more precise, the interest rate differential is $i_t - i_t^* = \mathrm{E}_t(s_{t+1} - \bar{s})$ as long as the exchange rate remains fixed. We may express $\mathrm{E}_t\, s_{t+1}$ as

$$(17) \qquad \mathrm{E}_t\, s_{t+1} = (1-\kappa_{t+1})\bar{s} + \kappa_{t+1}\,\mathrm{E}_t\left[\tilde{s}_{t+1} \middle| c_{t+1} = 1\right],$$

where κ_{t+1} is the probability of collapse at time $t+1$ and $\mathrm{E}_t\left[\tilde{s}_{t+1} \middle| c_{t+1} = 1\right]$ is the expected shadow free-float rate conditional on that a collapse will take place at $t+1$, both evaluated at time t.

The probability of a collapse at $t+1$ corresponds to the probability that there will be change in velocity at t+1, Δv^*_{t+1}, that is sufficient to push the shadow exchange rate above the fixed exchange rate. Formally:

$$\kappa_{t+1} = \mathrm{P}\left[\tilde{s}_{t+1} \geq \bar{s} \middle| I_t\right] = \mathrm{P}\left[\Delta v_{t+1} \geq \Delta v^*_{t+1} \middle| I_t\right] = (1-\mathrm{E}_t\,\pi_{t+1}) \int_{\Delta v^*_{t+1}-\mu_0+\beta/\lambda}^{\infty} f_\omega(\omega_{t+1})\,d\omega_{t+1} \tag{18}$$
$$+\mathrm{E}_t\,\pi_{t+1} \int_{\Delta v^*_{t+1}-\mu_0-\mu_1+\beta/\lambda}^{\infty} f_\omega(\omega_{t+1})\,d\omega_{t+1}.$$

The minimum change in velocity, Δv^*_{t+1}, at which the currency is floated, is obtained from the time t+1 counterparts of equations (11) and (13) for $\tilde{s}_{t+1} = \bar{s}$.

In equation (17), the expected shadow free-float rate conditional on a collapse at time $t+1$ ($c_{t+1} = 1$) can be expressed as

$$\mathrm{E}_t\left[\tilde{s}_{t+1} \middle| c_{t+1} = 1\right] = v_t + \mu_0 + (1-\mathrm{E}_t\,\pi_{t+1})\,\mathrm{E}_t\left[\varepsilon_{t+1} \middle| c_{t+1} = 1, Z_{t+1} = 0\right]$$
$$+\mathrm{E}_t\,\pi_{t+1}\left(\mu_1 + \mathrm{E}_t\left[\varepsilon_{t+1} \middle| c_{t+1} = 1, Z_{t+1} = 1\right]\right) + \alpha\mu_0$$
$$+(1-\mathrm{E}_t\,\pi_{t+1})\frac{\alpha}{1-\frac{\alpha}{1+\alpha}\delta}\left(1-p+\frac{\delta}{1+\alpha}\mathrm{E}_t\left(\pi_{t+1} \middle| c_{t+1} = 1, Z_{t+1} = 0\right)\right)\mu_1 \tag{19}$$
$$+\mathrm{E}_t\,\pi_{t+1}\frac{\alpha}{1-\frac{\alpha}{1+\alpha}\delta}\left(1-p+\frac{\delta}{1+\alpha}\mathrm{E}_t\left(\pi_{t+1} \middle| c_{t+1} = 1, Z_{t+1} = 1\right)\right)\mu_1.$$

The first four terms on the right-hand side of equation (19) denote the expected velocity at t+1, evaluated at t, conditional on a collapse at time t+1. The latter three terms denote the expected change in the exchange rate between t+1 and t+2, evaluated at time t and conditional on a collapse at t+1, times α.[10]

The probabilistic inference market participants draw, based on the observed behavior of Δv_t, has a strong influence on the interest rate differential through the perceived probability of collapse, equation (18), and the expected exchange rate in case of an collapse, equation (19). Thus, the interest rate differential may move up and down as a result of the

[10] The expressions within the expectations operators are evaluated in Appendix 2.

probabilistic inference, though velocity always increases and comes closer to the critical region where the fixed exchange rate regime is bound to collapse. There is therefore no one-to-one relationship between the level of velocity and the interest rate differential, as the case is in the Flood-Garber model.

The model also has some interesting predictions about the term structure of interest rates or differentials, to be more precise. Assume for simplicity that the foreign yield curve is horizontal. A log-linear Flood-Garber type of model, with a constant drift in fundamentals, implies that the yield curve is positively sloped throughout the fixed exchange rate regime. However, the slope decreases as fundamentals approach the point where the fixed exchange rate regime collapses, a point where the yield curve turns horizontal. The two-state model with probabilistic inference is able to generate a much richer class of yield curves. During the free float, the slope is negative when $\pi_t > \pi$, and positive when $\pi_t < \pi$.[11] During the fixed exchange rate regime, the slope is positive when velocity is far from the interval where the collapse occurs (bounded by v' and v''). However, close to or within that critical region, the effect from $\pi_t > \pi$ may dominate and the yield curve will then have a negative slope.

What about the foreign exchange reserves? To keep the exchange rate fixed at $s = \bar{s}$ the central bank must intervene to counteract the drift in velocity and changes in exchange rate expectations, that is, foreign exchange reserves are

$$(20) \qquad \begin{aligned} r_t &= \bar{s} - v_t - \alpha(\mathrm{E}_t\, s_{t+1} - \bar{s}) && \text{for} \quad \tilde{s}_t < \bar{s}, \\ r_t &= 0 && \text{for} \quad \tilde{s}_t \geq \bar{s}. \end{aligned}$$

Thus, the central bank's foreign reserve holdings may move up or down together with the interest rate differential before the fixed exchange rate regime collapses.

[11] During the free float, the interest rate differential with maturity τ is approximately; $i_{t,\tau} - i^*_{t,\tau} = \mathrm{E}_t(s_{t+\tau} - s_t)/\tau =$ $(\tau\mu_0 + \sum_{j=1}^{\tau} \mathrm{E}_t\, \pi_{t+j}\mu_1 + \gamma \mathrm{E}_t(\pi_{t+\tau} - \pi_t)\mu_1)\frac{1}{\tau} = \left(\tau\mu_0 + \tau\pi\mu_1 + (\delta/1-\delta)(1-\delta^\tau)(\pi_t - \pi)\mu_1 + \gamma(\delta^\tau - 1)(\pi_t - \pi)\mu_1\right)\frac{1}{\tau}$, where $\gamma = \alpha\delta/(1+\alpha-\alpha\delta)$.

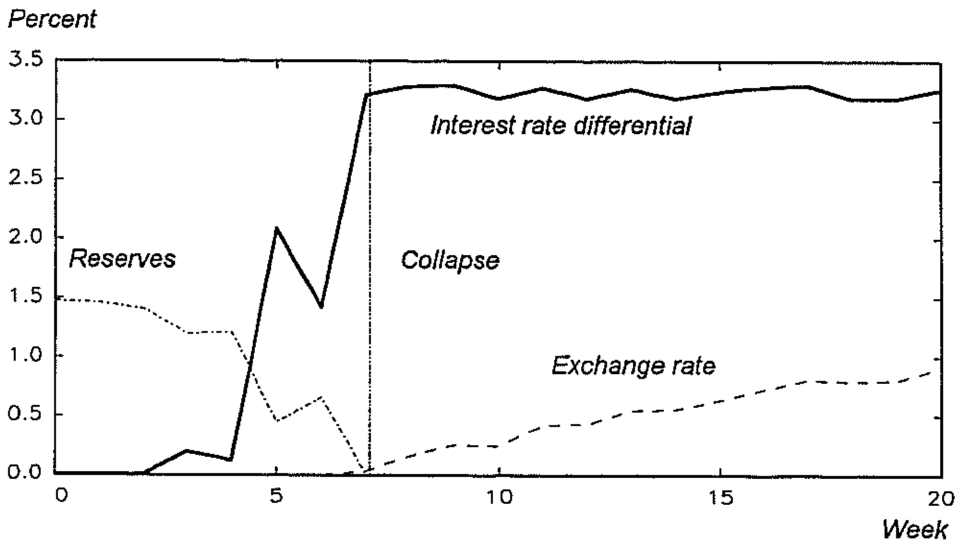
3 Simulations

In the previous section it was shown that interest rate differentials, reserves and the timing of the collapse are partly determined by the probabilistic inference market participants draw about the states of the process that drives fundamentals. In this section I will simulate the model to gain some more insights about the mechanisms involved and their resemblance to the Swedish currency crisis.

I will study one-week interest rates and weekly changes of reserves, so time is measured in weeks. The parameters of the Markov chain are $p = 0.75$ and $q = 0.75$. Thus, the degree of persistence is the same in the two states. The drift parameters are $\mu_0 = 0.02$ and $\mu_1 = 0.025$ per year, respectively. Hence, the total drift in state one is 0.045 per year. The semi-elasticity of money demand, α, is set to 0.3535 years, in accordance with the estimate Lindberg and Söderlind (1994) obtain on Swedish data from 1986 to 1990. The velocity at $t = 0$ is $v_0 = -0.0148$ and we set the fixed exchange rate equal to zero, $\bar{s} = 0$, for convenience. The parameters of the gamma distribution are $\beta = 2$ and $\lambda = \beta / \mu_0$ (with μ_0 measured per week). Thus, there will be realizations of Δv_t over the interval $[0, \infty)$ in state zero and over $[0.025/52, \infty)$ in state one.

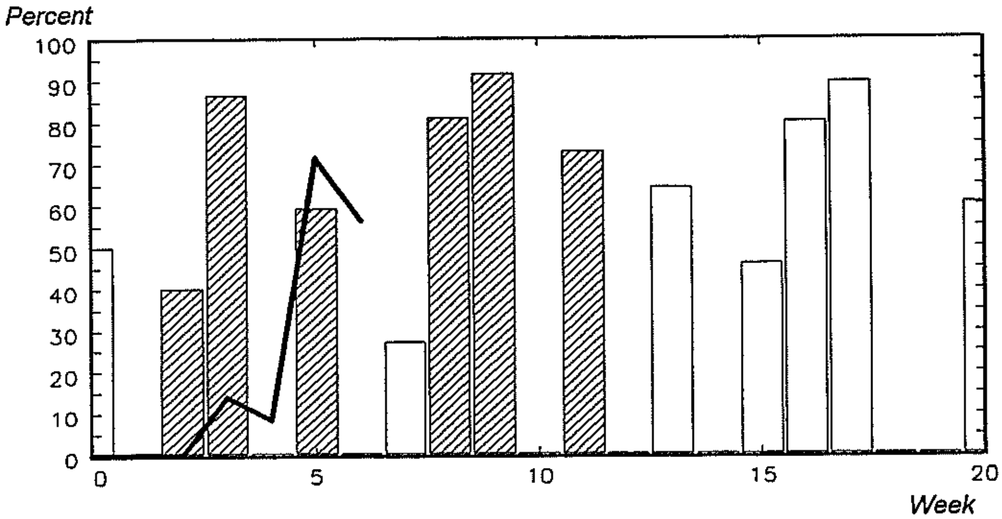
Figure 2 shows an example of a collapse scenario with the interest rate differential (percent per year), foreign exchange reserves (log percent) and the exchange rate (log percent) plotted against time.

Figure 2. Collapse scenario



The market participants learning about the states is displayed in *Figure 3*. The bars in the figure represent the probabilities agents assign to state one. The bars are shaded when the actual state is one. The collapse probabilities are marked by a solid line.

Figure 3. Learning process



The probability of an imminent collapse is low and the interest rate differential is just slightly above zero during the first two weeks since the gamma distribution with $\beta = 2$ is heavily skewed to the right and has most of its probability mass close to zero. Increasing β, but keeping β / λ unchanged, would raise the probability of a collapse within a week and hence the interest rate differential on that maturity. It is also worth noting that the state shifts together with the probabilistic inference are able to create two peaks in the one-week interest rate during the third and fifth week. At those instances, market participants assign a high probability to state one, a probability that falls when they receive new information the following week. The collapse occurs after seven weeks, marked by the vertical line in figure 2. The occurrence of the collapse that particular week is partly due to the high probability market participants assign to state one, though state zero is in effect; cf. figure 3.

The path of the interest rate differential has its mirror image in the central bank's holdings of foreign exchange reserves. For instance, there is an inflow of capital and an increase of the foreign exchange reserves when the interest rate differential falls during week four and six. It is also illuminating study how the exchange rate behaves during the free float. For instance, there are episodes when the exchange rate appreciate as a result of the learning process, though velocity only moves in one direction.

What about the yield curve? The interest rate differentials on longer maturities (not plotted in the figure) are above the one-week differential to the time of the collapse. Hence, in this particular case there is a positive slope of the yield curve throughout the fixed exchange rate regime. However, the yield curve may take a negative slope close to the point where the collapse occur, as noted in the previous section.

The type of collapse scenarios the model is able to generate has some resemblance with the Swedish experience. There were several currency crises before the final collapse of the Swedish exchange rate band in November 1992, at which interest rate differentials rose and capital flowed out from the country. However, the sharp peaks in the short term interest rate differential, especially during 1992, can not be explained without adding time-varying devaluation expectations to the model. It is also clear the collapse of the Swedish exchange rate band should be studied in a model with sticky-prices in which the central bank is concerned about the real effects of the defence of the fixed exchange rate regime.

4 Summary and concluding comments

This paper investigates the role uncertainty about future fundamentals may play in the collapse of fixed exchange rate regime. A model is presented in which the process that drives fundamentals switches between two states that differ with respect to their short-run compatibility with the fixed exchange rate regime. Market participants do not observe the state shifts directly, but are able to draw probabilistic inference based on the observed behavior of the data. It is concluded that the shifts in states and the learning process influence the timing of the collapse and induce a richer pattern in interest rate differentials and foreign exchange reserves.

At a general level, I believe that state shifts and learning processes add some realism to exchange rate models. For instance, Lewis (1989) finds that learning processes may help to explain the failure of unbiasedness tests of forward exchange rates. The occurrence of repeated state shifts may also explain why the conditional exchange rate distribution often show up with fat tails (excess kurtosis) in more conventional models, e.g., Baillie and Bollerslev (1989).

Appendix 1. The case when velocity may decrease

Flood and Garber (1984b) argue that it would be permissible to allow for negative realizations in the process that drives fundamentals in a discrete time log-linear model. The possibility of decreases in velocity would clearly add some more realism to the model. However, it would also make the model inconsistent, as demonstrated below.

For simplicity, assume that velocity follows the stochastic process

(21) $$v_t = v_{t-1} + \omega_t,$$

where the disturbance term ω_t is normally distributed, $\omega_t \sim \mathrm{N}(0, \sigma^2)$. The central bank will defend the fixed exchange rate, given by

(22) $$\bar{s} = r_t + v_t + \alpha(\mathrm{E}_t\, s_{t+1} - \bar{s}),$$

to the point where foreign exchange reserves are exhausted. Hence, the exchange rate remains fixed as long as

(23) $$r_t \geq 0.$$

The shadow floating exchange rate is simply

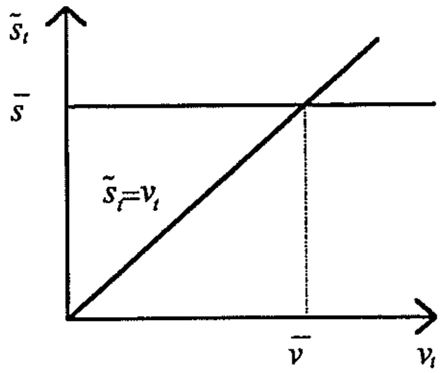
(24) $$\tilde{s}_t = v_t,$$

since there is no drift in velocity. In section 2 it was concluded that the fixed exchange will collapse at t if and only if

(25) $$\tilde{s}_t \geq \bar{s},$$

according to the simplest principles of currency arbitrage. Thus, equation (24) together with proposition (25) suggests that the fixed exchange rate regime would collapse the first time velocity reaches $\bar{v} = \bar{s}$; cf. *Figure 4*.

Figure 4. Shadow floating rate



Moreover, restriction (23) implies that foreign exchange reserves should be non-negative when the currency is floated; $r_t = \bar{r} \geq 0$. The reserve level, $\bar{r}$, corresponding to $\bar{v}$, is obtained from equation (22);

$$(26) \qquad \bar{r} = \bar{s} - \bar{v} - \alpha\left(\mathrm{E}_t\left[s_{t+1} \middle| v_t = \bar{v}\right] - \bar{s}\right),$$

where $\mathrm{E}_t\left[s_{t+1} \middle| v_t = \bar{v}\right] > \bar{s}$, since $s_{t+1} = \bar{s}$ for $\varepsilon_{t+1} \leq 0$ and $s_{t+1} = \tilde{s}_{t+1} = \bar{v} + \varepsilon_{t+1}$ for $\varepsilon_{t+1} > 0$. Hence, $\bar{r} < 0$, since $\bar{s} = \bar{v}$. However, negative reserves are clearly at odds with the restriction $r_t \geq 0$. Thus, a contradiction arises in the model as a result of the possibility of decreases in velocity, i.e., negative realizations of ω_t.

Appendix 2. The expected exchange rate conditional on a collapse

The terms within the expectation operators in equation (19) can be expressed as follows:

The expectations about the random disturbance are

(27) $$E_t\left[\varepsilon_{t+1} \middle| c_{t+1}=1, Z_{t+1}=0\right] = -\frac{\beta}{\lambda} + \int_{\Delta v^*_{t+1}-\mu_0+\beta/\lambda}^{\infty} \omega_{t+1} f_\omega(\omega_{t+1})\, d\omega_{t+1} \Big/ \int_{\Delta v^*_{t+1}-\mu_0+\beta/\lambda}^{\infty} f_\omega(\omega_{t+1})\, d\omega_{t+1}.$$

(28) $$E_t\left[\varepsilon_{t+1} \middle| c_{t+1}=1, Z_{t+1}=1\right] = -\frac{\beta}{\lambda} + \int_{\Delta v^*_{t+1}-\mu_0-\mu_1+\beta/\lambda}^{\infty} \omega_{t+1} f_\omega(\omega_{t+1})\, d\omega_{t+1} \Big/ \int_{\Delta v^*_{t+1}-\mu_0-\mu_1+\beta/\lambda}^{\infty} f_\omega(\omega_{t+1})\, d\omega_{t+1}.$$

According to Bayes' law, equation (11), the probability of being in state one at $t+1$, is a function of the market participants prior information, $E_t\, \pi_{t+1}$, and the sample information, Δv_{t+1}, they receive at $t+1$. Label this function $\pi_{t+1} = \Pi_{t+1}(\Delta v_{t+1}; E_t\, \pi_{t+1})$. Hence, we can write the expected value, evaluated at time t, of the probability assigned to state one at $t+1$, conditional on a collapse at $t+1$ and on $Z_{t+1}=0$ as

(29) $$E_t\left(\pi_{t+1} \middle| c_{t+1}=1, Z_{t+1}=0\right) = \int_{\Delta v^*_{t+1}}^{\infty} \Pi_{t+1}(\Delta v_{t+1}; E_t\, \pi_{t+1}) f_\omega(\Delta v_{t+1}-\mu_0+\beta/\lambda)\, d\,\Delta v_{t+1} \Big/ \int_{\Delta v^*_{t+1}}^{\infty} f_\omega(\Delta v_{t+1}-\mu_0+\beta/\lambda)\, d\,\Delta v_{t+1}.$$

The same expression, but conditional on $Z_{t+1}=1$, is

(30) $$E_t\left(\pi_{t+1} \middle| c_{t+1}=1, Z_{t+1}=1\right) = \int_{\Delta v^*_{t+1}}^{\infty} \Pi_{t+1}(\Delta v_{t+1}; E_t\, \pi_{t+1}) f_\omega(\Delta v_{t+1}-\mu_0-\mu_1+\beta/\lambda)\, d\,\Delta v_{t+1} \Big/ \int_{\Delta v^*_{t+1}}^{\infty} f_\omega(\Delta v_{t+1}-\mu_0-\mu_1+\beta/\lambda)\, d\,\Delta v_{t+1}.$$

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