How can monetary policy take account of uncertainty and risk?

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The academic discussion of monetary policy frequently employs a conceptual framework based on assumptions suggesting that consideration of risks and uncertainty has no effect on the decisions taken. Decision-makers wishing to take such consideration therefore need a new conceptual framework. In this article, we analyse two different versions of an extended conceptual framework in which uncertainty and risks can influence the decisions taken. The common factor for both of these methods is that a central bank is not certain which description of the world is correct. The focus thus lies on an uncertainty over which economic relationships or forecast models are most appropriate to use when a decision is to be taken. These two different methods have theoretical differences but we consider that, in a practical analysis of how monetary policy can be conducted, the differences between these two methods need not be very great. The discussion in this article is based on theoretical reasoning, as illustrated by stylised models, and makes no claim to be a realistic description of how monetary policy functions in practice.

How can monetary policy take account of uncertainty and risk?

In discussions of monetary policy decision-making, it has been pointed out, several times, that risks or consideration of uncertainty influence the decisions taken. One example of this is Greenspan (2005), who states that US monetary policy at the start of the century was characterised by consideration of the risk that inflation would be lower than shown by the forecasts.[1] In the United Kingdom, Ian McCafferty (2014) has discussed the monetary policy decisions taken by the Bank of England in the autumn of 2014, in which he himself participated as a member of the Monetary Policy Committee (MPC). McCafferty has described how his considerations were centred around uncertainty over how the supply side of the economy was functioning at the time of the decision. Poloz (2013), Governor of the Bank of Canada, argues that monetary policy decisions in Canada have focused on balancing the risk of low inflation against increased risks of imbalances in the financial system. In Sweden, Ingves (2014) has argued for attaching particularly great importance

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1 Greenspan considers that the decisions had the nature of risk management and that the monetary policy actually conducted was more expansionary than it would have been had the outcome of inflation been known.

to particularly unfavourable forecast scenarios when decisions are taken. Bernanke (2007) discusses the general principle that monetary policy decisions need to consider that the state of the economy and its functioning are not known. Bernanke considers that actual monetary policy decisions will therefore differ from decisions taken when the conditions are known.

Following the financial crisis, interest in academic circles has increased over how account can be taken of uncertainty and risks in monetary policy decisions. In some cases, the discussion has focused on whether (and how) monetary policy can consider risks and uncertainty associated with financial stability. This problem was also discussed prior to the financial crisis by Moessner (2006), but more recently the issue has been given increased attention, among others by Woodford (2012), Williams (2012), Ajello, Laubach, López-Salido and Nakata (2015) and Svensson (2012). However, this academic discussion of risks and uncertainties is of a general nature and is not specifically tied to risks to financial stability. Brock, Durlauf and West (2003) discuss a general principle for how monetary policy can manage decisions subject to risk and uncertainty. Hansen and Sargent (2008) discuss another type of general principle for decision-making subject to uncertainty, in line with methods developed by engineers.

In this article, we discuss the limitations of the traditional, academic conceptual framework for monetary policy in terms taking account of risks and uncertainty. The traditional framework is characterised by what is known as *certainty equivalence*. The consequence of this is that the decisions taken under uncertainty are the same as the decisions taken under known circumstances. We will describe how this happens in more detail in the next section. Decision makers willing to let consideration of risks and uncertainty influence their decisions may require a different conceptual framework. In this article, we analyse two different versions of an expanded framework that takes account of risk and uncertainty. The common factor for both of these methods is that the decision maker is uncertain which description of the world is correct. The focus lies on an uncertainty over which economic relationships, or forecast models, are most appropriate to use when a decision is to be taken. The two different methods have theoretical differences, but our view is that the differences probably need not be so great in practical application.

A SIMPLE CONCEPTUAL FRAMEWORK FOR MONETARY POLICY

Even if both decision makers and academics sometimes focus on risks and uncertainty, it is not self-evident whether and how monetary policy decisions reflect this. Of course, one reason may be that no particular uncertainty is actually present. But another reason could be that there are methodological problems for how risks and uncertainties impact on decisions. This may mean that a decision maker intends to take account of uncertainty in a decision but that the conceptual framework used is unclear about how this is actually done.

In the analysis of monetary policy, it is not unusual to use a conceptual framework that has its origin in control theory.[2] Using this conceptual framework, it is the decision maker's task to minimise the variation in a number of target variables, under the assumption that the forecasts for the various variables can be described by linear relationships. In this conceptual framework, there is only one source of uncertainty, namely random disturbance terms that additively influence the relationships used to make forecasts. The traditional conceptual framework is based on underlying assumptions that result in *certainty equivalence* and that mean that the same decisions will be taken, regardless of whether it is known with certainty how the target variables will develop over time or whether an uncertain forecast must be made for them.[3] This may sound strange, so the term certainty equivalence therefore deserves a more detailed explanation.

We use a simple example to explain how certainty equivalence functions. We assume that inflation is the target variable and that the central bank wishes to minimise any variation in deviations in inflation (π) from the inflation target (π^*). We use a loss function to describe this:

(1) $$L_t = (\pi_t - \pi^*)^2$$

If inflation is equal to the inflation target, the value of the function is zero and, if inflation deviates from the target, the value will increase as this deviation widens. Large target deviations thus result in very high values in the loss function.[4] We also assume that inflation is only due to resource utilisation (x) in the economy.

(2) $$\pi_t = x_t$$

In turn, the central bank's nominal policy rate (i) affects resource utilisation according to equation (3) below:

(3) $$x_t = ai_t$$

In equation (3), a is a coefficient that is less than zero. By combining equations (2) and (3), we obtain a new equation (4), in which we can see that the central bank has a direct influence on inflation:

(4) $$\pi_t = ai_t$$

2 See, for example, Jacobs (1996) for an introduction.
3 This contradicts the view that decisions taken under conditions of uncertainty differ from decisions taken when the circumstances are understood, as Bernanke (2007) considers.
4 When inflation deviates from the inflation target, large deviations are much worse than small ones, as the deviations are squared. A deviation of two units is therefore not twice as serious as a deviation of one unit, but four times as serious.

The question now is which interest rate level the central bank should maintain to keep the variation in the deviation of inflation from target as small as possible. We can calculate this by using the expression (4) in the loss function (1), and then minimising the rewritten loss function with regards to the policy rate. This calculation shows that the decision leading to the central bank achieving the smallest variation in inflation's deviation from the inflation target is holding the rate in proportion to the inflation target according to the following rule:[5]

$$i_t = \frac{\pi^*}{a} \tag{5}$$

Let us now introduce uncertainty into our reasoning. The uncertainty of the forecasts is illustrated in the usual manner, which is to say by introducing a random disturbance term into equation (4). The disturbance term, ε_t, has the expected value of zero and a given standard deviation. With uncertainty in the inflation forecast, the inflation relationship can now be written as:

$$\pi_t = ai_t + \varepsilon_t \tag{6}$$

With uncertainty in the forecast, it is no longer only resource utilisation that steers inflation, but inflation is also affected by the disturbance term ε_t. The central bank that still wants to minimise the variation of the deviation in inflation from target is no longer certain what inflation will be. The question is whether this changes how the central bank makes its decisions.

As the loss function is quadratic and linear relationships are used to make forecasts, and as the central bank cannot affect the disturbance term ε_t, the answer is that the term does not actually make any difference whatsoever to the decisions taken by the central bank. In the same way as above, the best strategy is to keep the policy rate in proportion to the inflation target in accordance with the rule below:[6]

$$i_t = \frac{\pi^*}{a} \tag{7}$$

The degree of inaccuracy in the forecast, which is to say how large the variation of the disturbance term ε_t is, therefore is of no importance at all to the central bank. When the forecast is unbiased, uncertainty will not have an effect on the actual decision. This is known as *certainty equivalence* and thus means that the decisions taken are the same,

5 Holding the rate in proportion to the inflation target as specified by equation (5) results in the lowest loss possible. In this case, the loss is zero. This can be seen by substituting equation (5) and equation (4) in the loss function (1) and expand it. In this simple model, the optimal rate is negative and resource utilisation is positive when the inflation target is positive as a is negative. But the point of the model is not to be realistic but to be simple and to illustrate the significance of certainty equivalence.

6 Once again, this can be checked by substituting the equations (7) and (6) in the loss function (1) and expand it. The smallest possible loss is now no longer zero but equal to the variance of the disturbance term ε_t.

entirely regardless of whether it is known with certainty how the target variables will develop over time or whether an uncertain forecast must be made for them.

A MODEL FOR FORECASTS AND MONETARY POLICY DECISIONS

In the section above, we saw that forecast uncertainty as represented by additive disturbance terms has no effect on the decisions taken by a central bank in a simple forecast model.[7] Certainty equivalence also applies to more complicated forecast models according to the same assumptions, which is to say that the loss function is quadratic, that the forecasts are based on linear relationships and that the only source of uncertainty is additive disturbance terms with a known variation.

A more complicated forecast model for resource utilisation and for the deviation of inflation from the inflation target ($\pi - \pi^* = \hat{\pi}_t$) is described by Giordani and Söderlind (2004).[8] In this model, the central bank adjusts its nominal interest rate (the policy rate) in relation to the long-term sustainable interest rate ($i - i^* = \hat{\imath}_t$) to minimise the variation in its target variables. The model is written as:

(8) $$x_t = E_t x_{t+1} - \gamma(\hat{\imath}_t - E_t \hat{\pi}_{t+1}) + \xi_t^D$$

(9) $$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \alpha x_t + \xi_t^S$$

(10) $$\xi_t^D = \rho_D \xi_{t-1}^D + \varepsilon_t^D, \quad \text{in which } \varepsilon_t^D \text{ is } N(0,1)$$

(11) $$\xi_t^S = \rho_S \xi_{t-1}^S + \varepsilon_t^S, \quad \text{in which } \varepsilon_t^S \text{ is } N(0,1)$$

Equation (8) describes the resource utilisation that the central bank influences by varying its nominal interest rate. One important component of the forecast model is that resource utilisation is determined by the expectations held for resource utilisation in the next period, $E_t x_{t+1}$. Equation (9) describes the relationship between resource utilisation and how much inflation deviates from the target. Here too, an important component is that current inflation deviations depend on how the deviations are expected to develop in the next period, $E_t \hat{\pi}_{t+1}$. The terms ξ^S and ξ^D are persistent supply and demand shocks respectively. In this case too, the choice of parameters follows Giordani and Söderlind (2004), where $\beta =$ 0.99, $\alpha =$ 0.64, $\gamma =$ 0.5 and $\rho_S = \rho_D =$ 0.8. The parameter β designates a subjective discount factor, γ is the intertemporal substitution elasticity and the parameter α specifies how much resource utilisation influences inflation. The parameters ρ_S and ρ_D specify how persistent

7 Given that decision makers minimise the variation of their target variables and that the forecasts are described by linear relationships.

8 Resource utilisation here means the deviation of demand from long-term sustainable production, $y_t - y^* = x_t$.

the supply and demand shocks are, respectively. The equations (8) to (11) can also be written in a compact form:[9]

(12) $$A_0 w_{t+1} = A_1 w_t + Bu_t + C\varepsilon_{t+1}$$

Giordani and Söderlind (2004) assume that the central bank's target variables are inflation's deviation from the inflation target and resource utilisation but that the central bank also wishes to avoid sudden and dramatic changes to monetary policy and therefore includes the deviation of the policy rate from its long-term sustainable level in the loss function. The central bank thereby has the following loss function:

(13) $$E_{t|t_0}\left(\Sigma_{i=0}^{\infty}\, \beta^i\left(\hat{\pi}_{t+\mathrm{i}}^2 + \lambda x_{t+\mathrm{i}}^2\ \mu \hat{\imath}_{t+\mathrm{i}}^2\right)\right)$$

in which λ = 0.5 and μ = 0.2.[10] In terms of equation (12), we can instead write the loss function as:[11]

(14) $$E_{t|t_0}\left(\Sigma_{i=0}^{\infty}\, \beta^i\left(w_{t+\mathrm{i}}' R w_{t+\mathrm{i}} + u_{t+\mathrm{i}}' Q u_{t+\mathrm{i}}\right)\right)$$

According to the traditional, academic conceptual framework, the central bank's problem concerns finding a decision rule for monetary policy so that the value of the loss function (14) is minimised at the same time as the forecasts for inflation and production develop according to (12). As we saw above, this means that uncertainty does not affect the actual decision. In other words, it makes no difference to the central bank's rate setting that we have forecast uncertainty, $C\varepsilon_{t+1}$, in equation (12).

In the same way as in the simple example in the section above, the solution to the central bank's problem is a rule for how the rate is to be set. A monetary policy decision following this rule or policy is *optimal* in the sense that the central bank achieves the smallest possible variation in the target variables.[12] In Diagram 1 below, we see examples of forecast deviations and monetary policy reaction in the event of a positive demand shock. A demand shock may conceivably arise as the result, for example, of a temporary tax change that allows households and companies to increase consumption and investment for a time.

9 $w_{t+1} = \begin{pmatrix} \xi_{t+1}^D \\ \xi_{t+1}^S \\ E_t x_{t+1} \\ E_t \hat{\pi}_{t+1} \end{pmatrix},\ u_t = \hat{\imath}_t,\ A_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \gamma \\ 0 & 0 & 0 & \beta \end{pmatrix},\ A_1 = \begin{pmatrix} \rho_D & 0 & 0 & 0 \\ 0 & \rho_S & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & -\alpha & 0 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ 0 \\ \gamma \\ 0 \end{pmatrix},\ C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix},\ \varepsilon_{t+1} = \begin{pmatrix} \varepsilon_{t+1}^D \\ \varepsilon_{t+1}^S \end{pmatrix}$

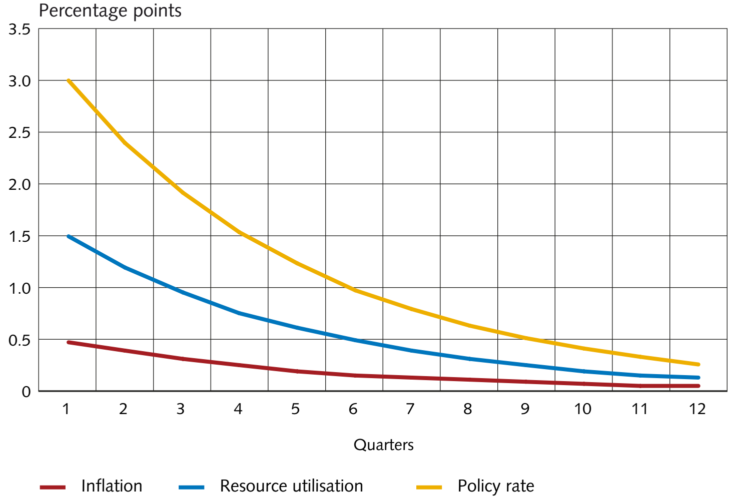
10 The numerical values of λ and μ follows Giordani and Söderlind (2004).

11 In which $R = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix}$ and $Q = \mu$.

12 Here, we solve the time-consistent (discretionary) problem for the central bank.

Diagram 1. Forecast deviations for inflation and resource utilisation, as well as the monetary policy reaction after a demand shock

Percentage points



Source: Author's own calculations

In Diagram 1, we see that the demand shock leads to resource utilisation initially deviating from forecast by 1.5 percentage points. Higher resource utilisation leads to inflationary pressures arising and, in response to this, the central bank raises its interest rate by about 3 percentage points. Despite this, the forecast deviation for inflation is about 0.5 percentage points after a demand shock. Monetary policy continues to counteract the forecast deviations that have arisen and, after just over 12 quarters, the effects of the shock have abated.

A METHOD FOR MANAGING UNCERTAINTY IN MONETARY POLICY DECISIONS

As we discussed in the introduction, there are several examples in which decision makers at central banks have said they have considered risks and uncertainty. But, in these cases, the usual conceptual framework does not reflect the actual decision-making situation. Neither is the uncertainty a matter of the perceived magnitude of the forecast errors. Rather, the uncertainty is a matter of whether the average relations used to make forecasts are a good description of how the economy is functioning at the time of the decision. A decision-maker may also have misgivings about whether the forecasts consider all important aspects of the economy. It may also be a matter of which basic relationships may be used to make the best possible forecasts of inflation and resource utilisation, for example. This type of uncertainty is usually called model uncertainty.[13] One example of this is if the policy rate, in certain situations, has a stronger effect on resource utilisation and inflation than the

13 The term 'model uncertainty' is used here as a broad designation of the uncertainty that arises over whether a specific dynamic system (that is used to make forecasts) is a satisfactory description of real economic developments. See the introduction in Dulerud and Paganini (2000) for further discussion.

decision maker believes applies on average. In such a situation, the monetary policy, with the average reaction pattern, may result in an exaggerated variation in the central bank's target variables.

The usual conceptual framework is poorly adapted to manage this kind of uncertainty. The fundamental problem is that, in the usual conceptual framework, a small change in the forecast methods can have clear consequences for the decisions taken. This type of problem was already being discussed among control engineers in the 1970s and 1980s. See the discussion in Doyle (1978), for example.[14] To address the way that changes in underlying assumptions effect the forecasts and thereby have consequences for the decisions taken, a branch of control theory was developed which had the management of decisions in an uncertain situation as its primary purpose.[15] This branch of control theory is usually called 'robust control', as it is aimed at managing a situation in which the decision maker wishes to pursue a policy that is robust regarding specification errors.[16] In this type of control theory, it is not unusual for decisions to be aimed at managing the worst conceivable forecasts.[17]

So far in this discussion, we have not made any distinction between the terms 'risk' and 'uncertainty'. However, in discussions of decision-making, it is also common for a distinction to be made between the terms. Hansen and Sargent (2008) present the background to this distinction. Put briefly, 'uncertainty' normally refers to uncertainty that is difficult to quantify or even have an idea of, whereas the term 'risk' is usually reserved for a type of uncertainty that can be identified and quantified, or where it is at least possible to form a subjective idea of the sample space. In the continuing discussion, we will try to maintain this distinction between the terms.

One method for managing decisions taken in conditions of uncertainty has been reported by Hansen and Sargent (2008) and by Giordani and Söderlind (2004). The concept is that a decision maker considers that the forecast model used may have a specification error and that this error, in addition, is unknown. We can illustrate this by suggesting there may be an alternative model that could form the correct description of the relationship between the target variables, and that the forecasts from the main model are consequently being influenced by an unknown disturbance term, ϑ_{t+1}. Forecasts from this alternative model can then be written in terms of the main model:[18]

14 This article is often considered to have one of the best abstracts of any piece of research.

15 See Dulerud and Paganini (2000), Costa, Fragoso and Marques (2010) and Hansen and Sargent (2008) for a discussion of these methods.

16 Specification error means that the dynamic system (which is used to make forecasts) is *not* a satisfactory description of actual economic events.

17 One possible example of this is that safety systems for a nuclear power station are not constructed for a situation in which everything is functioning normally. Instead, the systems are adjusted so that safety can be maintained, even when parts of the safety system have been disabled. The construction of extra safety measures that are not normally used can be seen as insurance. The opposite attitude, only being able to maintain safety systems under normal circumstances, could have catastrophic consequences in the event of an accident that disables the safety systems.

18 See the discussion in chapter 2 of Hansen and Sargent (2008).

(15) $$A_0 w_{t+1} = A_1 w_t + Bu_t + C(\varepsilon_{t+1} + \vartheta_{t+1})$$

As earlier, the monetary policy decision is based on minimising the variation in the target variables. At the same time, the decision maker wants monetary policy to be robust if the main model should turn out to have a specification error. This means that the decision maker simultaneously maximises the variation in the target variables that are caused by the disturbance term. This problem can then be described as:

(16) $$\underset{\{u_i\}}{min}\, \underset{\{\vartheta_{i+1}\}}{max}\, E_{t|t_0} \sum_{i=0}^{\infty} \beta^i \left(w'_{t+i} R w_{t+i} + u'_{t+i} Q u_{t+i} - \theta \vartheta'_{t+i+1}\, \vartheta_{t+i+1} \right)$$

The decision maker attempts to find a monetary policy that will function in the 'worst case' forecast model, which is unknown.[19] If the worst case model should turn out to be correct, the monetary policy decision will then be appropriate for managing a situation with the target deviations that have been applied. If the economy instead develops in a more positive direction, this robust policy may be far from optimal. The decision maker is thus willing to pay an 'insurance premium' when conditions are normal to be able to manage any problems arising when developments instead become very unfavourable.[20] How much emphasis the decision maker places on managing any specification error is determined by the parameter θ.[21] This approach to decision-making is focused on ensuring that decision are robust in the event that the normal forecast model is wrong about the relationship between the target variables.[22]

In Diagram 2 below, we see forecast deviations and the monetary policy response if a positive demand shock should arise. We see both the case in which the usual conceptual framework is used and also how a decision maker should act when there is uncertainty in order to achieve a robust decision.

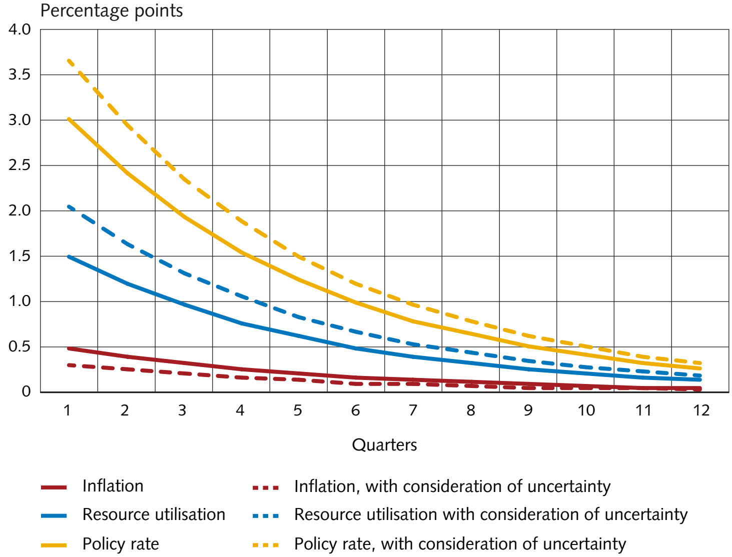
19 The reasoning behind the 'worst case' forecast model is that the forecast deviations arising if this model should be correct would be very difficult for the central bank to counteract with monetary policy.

20 See Hansen and Sargent (2008), page 40 onwards, for further discussion.

21 A low value for θ means that the decision-maker has placed great emphasis on a possible specification error in the forecast model. A high value means a fairly low emphasis on specification errors in the model. The numerical example below uses $\theta = 500$.

22 In the numerical simulation that follows, we only solve the time consistent (discretionary) case.

Diagram 2. Forecast deviations for inflation and resource utilisation, as well as the monetary policy reaction after a demand shock, with and without consideration of uncertainty
Percentage points



Source: Author's own calculations

In Diagram 2, we can be seen that the demand shock leads to resource utilisation deviating from the forecast by 1.5 percentage points when the central bank does not take account of uncertainty, while the forecast deviation in the 'worst case' forecast is 2.0 percentage points. We can also see that monetary policy reacts more strongly to the forecast deviations when the decision maker takes account of uncertainty and acts robustly, and the policy rate is now raised a little over 3.5 percentage points. At the same time, the forecast deviation for inflation is slightly less when the central bank considers the uncertainty. The central bank now attaches importance to the demand shock possibly having a greater effect on resource utilisation than in the normal case and therefore reacts more strongly to the shock. The consequence of this is that inflation is slightly lower than would otherwise be the case.

The method thus is a matter of managing forecasts that risk leading to very high losses for the target variables and that the decision makers find particularly difficult to manage. For monetary policy decisions, this means, for example, that decision makers tend to focus on forecasts in which shocks to the economy have greater effects or are more persistent, as opposed to circumstances in which shocks are small and tend to vanish quickly.

However, considering risks and uncertainties in a decision need not always be the same as managing the worst conceivable forecast. In theory, the method may be an attractive principle for monetary policy decisions, but, in practice, this may mean that the central bank risks focusing entirely on highly unlikely forecasts. It cannot be ruled out that this, in turn, may lead to poor average goal fulfilment. Another problem with the method is that, in practical, decision-making situations, a decision maker may often need to manage and communicate a specific uncertainty (that is, the decision maker rather needs to manage and communicate a risk), while the uncertainty that this method defines may be seen as somewhat theoretical and abstract.

This method works for general uncertainty. With a loss function, the worst case forecast is then made, followed by a robust policy capable of managing the forecast deviations arising in that case. However, there are other methods for managing risks and uncertainty in decision-making that reduce how sensitive a decision is to the analytical assumptions that a decision maker needs to take a stance towards.[23] In the next section, we describe one of these methods.

A METHOD FOR ANALYSING MONETARY POLICY AND RISKS

Unlike the method presented in the section above, there is a method that focuses more on managing and quantifying specific risks in decision-making. This has been described by Costa, Fragoso and Marques (2010). The method is formulated to manage situations in which the decision maker needs to specify in advance which risks are to be considered and how large they are. Svensson and Williams (2008) then expanded the method to also deal with forecast models with forward-looking expectations. The method has the advantage of being fairly simple but simultaneously flexible enough to illustrate a series of different types of risk that may be relevant for a decision maker. See Svensson and Williams (2008) for a discussion.[24]

As with the section above, we can illustrate the risk of using a forecast model with a specification error with this method. The idea is to allow the risk to be represented by the possibility of the economy shifting between these two (or more) different forecast models. This shift is described by a Markov chain with transition probabilities:[25]

(17) $$P = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix}$$

In the equation (17), p_{00} describes the probability that the forecasts will be generated by a model in the next period, given that this model has been used as a starting point for forecasts. Reversed, p_{01} then describes the probability that the forecasts will be generated by another model over the coming period. Consequently, at any point in time, the decision maker may realise that the forecast model being used is incorrect and that another model instead provides the correct description of the actual relationship between the

23 See, for example, Onatski and Williams (2003) or Brock, Durlauf and West (2003).

24 The method is not normally subject to certainty equivalence as the degree of risk will influence the optimal policy. There may, however, be special cases in which even this method may lead to decisions that are subject to certainty equivalence.

25 For a discussion of Markov processes (chains), see Stroock (2014). The transition probabilities are here assumed to be unchanged by the state and over time. However, there is nothing to prevent these transition probabilities being functions of the (lagged) state or of time. The appendix shows how both the formation of expectations and monetary policy depend on these transition probabilities.

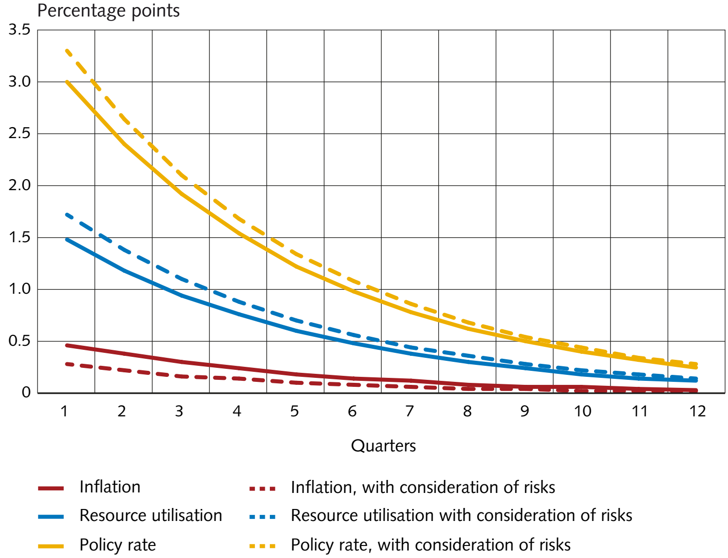
target variables. As p_{00} and p_{01} are probabilities, this means that if p_{00} = 50 per cent, the probability for $p_{01} = 1 - p_{00}$ is also 50 per cent.

In this way, forecasts from different models can represent various mode forecasts, which is to say the most probable forecast given a certain forecast model.[26] A final average forecast is then obtained by weighing the various mode forecasts together with their respective probabilities. The Markov chain means that it can be defined, in a simple manner, how the risk of shifting between the mode forecasts changes over the forecast horizon.

To refer back to the previous example, we can allow a forecast to be generated by the equation (12), which is to say $A_0 w_{t+1} = A_1 w_t + B u_t + C\varepsilon_{t+1}$, while an alternative forecast model is represented by equation (15), which is to say $A_0 w_{t+1} = A_1 w_t + B u_t + C(\varepsilon_{t+1} + \vartheta_{t+1})$.

The probability of shifting between the forecast models is set at 50 per cent in all cases. Even if the central bank uses equation (12) to make forecasts, it will react more strongly to shocks than otherwise, as it takes account of the possibility that it is an incorrect forecast model and that the alternative forecast model (15) instead provides the correct description of the relationship between the target variables.

Diagram 3. Forecast deviations for inflation and resource utilisation, as well as the monetary policy reaction after a demand shock, with and without consideration of risks
Percentage points



Source: Author's own calculations

In Diagram 3, we see that a demand shock leads to resource utilisation deviating from forecast by 1.75 percentage points when there is a 50 per cent risk of the worst case forecast being correct. The forecast deviation is thus a little higher than in the case in which the central bank does not take account of risks. The expected forecast deviation for inflation in the case in which the central bank takes account of risks is around 0.25

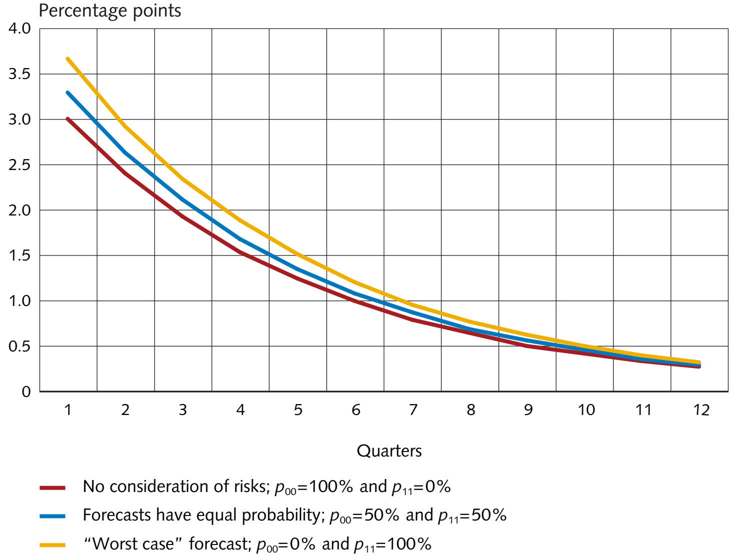
26 The mode value of a stochastic variable is the most probable outcome.

percentage points and, just as before, the deviation is around 0.5 percentage points when the central bank does not take account of any risks.[27] In Diagram 3, we also see that monetary policy reacts more strongly when the decision maker takes account of the risk of the worst case forecast being correct. In this case, the policy rate is raised by 3.25 percentage points, which is higher than when the central bank does not take this risk into account.

The monetary policy reaction is due to the possibility that the central bank will have to change its forecast model.[28] This means that the monetary policy decision no longer has to be subject to certainty equivalence but depends on risks, here in the form of a future change of forecast model. This is illustrated in Diagram 4 below.

Diagram 4. Monetary policy reaction after a demand shock, with different probabilities for the forecasts

Percentage points



Source: Author's own calculations

The difference between the different monetary policy reactions to a demand shock is thus a matter of differing probabilities. In the diagram, we can see that, when the probability is 100 per cent that the worst case forecast is correct, the monetary policy response becomes the same as when the central bank is acting under genuine uncertainty, according to the method presented in the previous section.[29] With this method, we can thus replicate the monetary policy subject to uncertainty via the probabilities in the Markov chain. However,

27 Precisely as in Diagram 2, the central bank takes into account that the demand shock may have a greater effect on resource utilisation than in the normal case and therefore reacts more strongly to the shock. The consequence of this is that inflation is slightly lower than it would otherwise be.

28 That is to say, change forecast model because the alternative model is the correct description of the relationship between the target variables.

29 The 100 per cent probability of the worst case forecast model corresponds to transition probabilities p_{00} = of 0 and p_{11} = 100 per cent.

this requires that the worst case model is known in advance. If this method is to be used as a tool to manage risks in decisions, a decision must thus be taken as to which risks are present and their extent. The probabilities may be empirically founded, but may also be entirely subjective. Blake and Zampolli (2006) discuss the possibility of letting both the central bank and the market participants have subjective perceptions of the risks that, in addition, may differ from the actual, objective risks.

WHAT CAN HAPPEN WHEN MONETARY POLICY INFLUENCES RISKS?

So far, we have assumed that the probability of the risks can be taken as given. For a small, open economy, there are many international risks that the central bank must manage and that, with good reason, may be considered as given. On the other hand, it cannot always be assumed that all risks are always independent of monetary policy. This is an important issue as the conditions for monetary policy can change rapidly if it influences the risks itself. Such an influence could have consequences for variations in inflation and resource utilisation if the central bank adjusts its policy rate. Among other things, this can affect the scope of the central bank's reaction via the policy rate when various shocks occur in the economy.

As an illustrative example, we allow the demand shock to be affected by the policy rate, so that monetary policy has an impact on the actual shock in a case in which the alternative model is correct:[30]

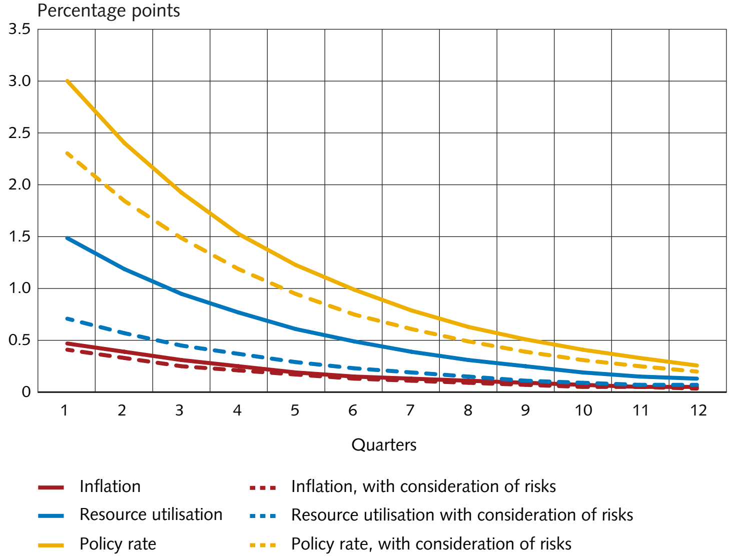
$$\xi_{t+1}^{D} = \rho_D \xi_t^D + \psi(s_t)\hat{\imath}_t + \varepsilon_{t+1}^{D} \quad (18)$$

in which $\psi(s_t = main\ model) = 0$ and $\psi(s_t = \text{“}worst\ case\text{”}\ model) = -0.1$. This parameter is aimed at illustrating that monetary policy in itself can have a direct impact on the shocks the economy is exposed to. In Diagram 5 below, we see that, when ψ could be negative, the central bank instead reacts slightly less to a demand shock than would otherwise be the case, when account is not taken of risks.

30 This reasoning is based on the discussion in Svensson, Lars E. O. (2003), section 7. Another way in which monetary policy can affect risks is through the direct or indirect influence of the central bank's policy rate on the transition probabilities in the Markov chain.

Diagram 5. Forecast deviations for inflation and resource utilisation, as well as the monetary policy response after a demand shock, with and without taking risks into account

Percentage points



Source: Author's own calculations

Just as previously, the central bank raises the policy rate by 3 percentage points when risks are not taken into account. As there is a 50 per cent risk that the worst case forecast is correct, the policy rate is now raised to a lesser extent, by about 2.4 percentage points. The reason for this is that, according to equation (18), when a demand shock occurs, an interest rate higher than the equilibrium rate will lead to the shock being smaller than it would if the alternative model is correct. The tendency of the worst case forecast to give a greater forecast deviation is more than compensated for by monetary policy, at the same time, counteracting the demand shock more powerfully. We can see this in Diagram 5, where the forecast deviation for resource utilisation now becomes slightly lower, around 0.75 percentage points, compared with the case in which risks are not taken into account.

THERE ARE NO EASY ANSWERS FOR HOW RISKS SHOULD BE MANAGED

The influence of risks on monetary policy is basically a matter of the forecast relationship assumed by the central bank.[31] Neither need taking account of risks in monetary policy be reduced to a question of the extent of the central bank's reaction to the shocks affecting the economy. There are cases in which consideration of risks is instead expressed by weighing into a decision factors that would be of little significance to the central bank's target variables under normal circumstances (in this case, inflation and resource utilisation).

31 Leitemo and Söderström (2008) show that, for a small, open economy, taking account of uncertainty can mean that the central bank occasionally reacts more aggressively to shocks but that the result can occasionally be the reverse. It all depends on which shocks occur and on how the links the forecasts are based on are constructed.

A forecast model that makes good forecasts of resource utilisation and inflation under normal circumstances can make very poor forecasts in special situations, such as, for example, if financial frictions are deemed to be influencing the economy in a way that the simple model does not take into account. Another, possibly more complicated model with links between the financial and real economies could then provide better forecasts. In such a case, taking account of risks in a monetary policy decision could involve factors that, under normal circumstances, play a minor role being weighed in, as these play an important role in particular situations.

By analysing various forecast models, account can be taken of a series of different risks in a decision. For example, we can analyse risks linked to the persistence of inflation shocks, an increased variability for inflation shocks or the relationship between demand pressure and inflation by allowing different forecast models to represent it.[32] In the same way, we can also analyse the consequences for monetary policy decisions when there is a risk that the effects of monetary policy will be weaker or stronger than normal. The risk of entering a crisis situation can also be examined by varying the magnitude of the shocks in different forecast models. Risks associated with financial frictions can be analysed by representing these financial frictions in a forecast model. Risks concerning the level of resource utilisation and between two (or more) opposing points of view on how the economy works can also be analysed in a similar way.

WHAT EXACTLY ARE THE DIFFERENCES BETWEEN VARIOUS METHODS FOR MANAGING DECISIONS SUBJECT TO UNCERTAINTY AND RISK?

In a theoretical sense, there is one important difference between considering uncertainty and considering risks in a decision. In one case, as presented in this article, decisions that are subject to uncertainty are best managed by focusing entirely on the worst conceivable forecast, for example the forecasts that give the greatest variation in the target variables. This way of reasoning is a way of taking decisions under genuine uncertainty, which is to say when the decision maker is unable either to assess the scale of the risks or even to identify the various risks existing. However, using this principle, the central bank will also always focus its decision on the worst conceivable forecast, even if it is not a particularly likely forecast.

The other method concerns managing risks in decision-making and is based on the decision maker assessing the scale of the risks and the effects these have on the economy.

However, the differences between these methods need not be so great when a decision is made in practice. In theory, the worst case forecast may be a consequence of the risk preferences a central bank may conceivably have.[33] In practice, the worst case forecast

32 For example, the risk that the relationship between resource utilisation and inflation has become lower is represented by a model with a flatter Phillips curve. See Blake and Zampolli (2006) or Demers (2003) for an example of this.

33 See Hansen and Sargent (2008), page 40 onwards, for further discussion.

is often limited by a rule of action.[34] One such rule of action focuses on trying to find a value for the parameter θ (see equation (16)) with the assistance of statistical methods.[35] In other words, an indirect decision is taken as to what the worst-case forecast may be by limiting the results in advance. Sims (2001) also discusses the problems in seriously taking account of all the uncertainty existing over economic relationships. The tendency to limit the sample space when identifying the worst-case forecast is thus always present. In an actual decision-making situation, the different points of view may therefore have a closer resemblance than they do in theory.

CONCLUSION

In this article, we have addressed the issue of whether and how monetary policy decisions can deal with uncertainty and risks. However, the presence of risks does not always have to be a decisive factor for monetary policy. In many cases, it is difficult to quantify the risks and neither is it clear how monetary policy should actually deal with risks. Neither can it be ruled out that a satisfactory strategy for managing risks could be to act as if under certainty equivalence. However, entirely disregarding risks in this way has the disadvantage that the central bank may enter into situations in which a monetary policy decision seems to be well-balanced according to one forecast, a conclusion that needs not necessarily hold true if there is uncertainty over the relationships used to make the forecasts. It may then be attractive to use methods in which account can be taken of uncertainty in decision making. Completely focusing on decisions for managing uncertainty could, on the other hand, lead to average target fulfilment suffering. In the longer term, it cannot be ruled out that this approach will lead to other types of uncertainty arising.

A method lying between both of these approaches could then be a practical compromise. Different forecast models could represent various possible descriptions of the world. The risks in the decision can be highlighted by weighing monetary policy in the different descriptions of the world together with their respective probability. This will make it possible for monetary policy to be characterised by consideration of risks but, at the same time, these risks must be defined and quantified. It will also increase possibilities for monitoring and evaluating the account that has been taken of the risks in the decisions.

34 See the discussion in Sims (2001) and Hansen and Sargent (2008).

35 A method based on error detection probabilities is often used. See Chapter 9 of Hansen and Sargent for further discussion.

APPENDIX: THE METHOD OF SOLVING A MONETARY POLICY PROBLEM IN WHICH RISKS ARE REPRESENTED BY A MARKOV CHAIN

The starting point is that a central bank minimises the value of the following loss function:

(19) $$E_t\left(\Sigma_{i=0}^{\infty}\beta^i\left(w'_{t+i}Rw_{t+i} + u'_{t+i}Qu_{t+i}\right)\right)$$

where $w = \begin{pmatrix} z \\ \tilde{z} \end{pmatrix}$ and $z = \begin{pmatrix} \varsigma^D \\ \varsigma^D \end{pmatrix}$ are predetermined variables and $\tilde{z} = \begin{pmatrix} x \\ \hat{\pi} \end{pmatrix}$ are forwardlooking variables. The central bank employs a vector with control variables, u, and matrices R and Q contains weightings for the central bank's target variables. The parameter β is a subjective discount factor that specifies how the central bank weighs target deviations in the near future against those later on. Target variables and other state variables are evolving as follows:

(20) $$A_{01}(s_t)\,z_{t+1} = A_{11}(s_t)\,z_t + A_{12}(s_t)\,\tilde{z}_t + B_1(s_t)\,u_t + C_1(s_t)\varepsilon_{t+1}$$

(21) $$E_t(A_{02}(s_t)\,\tilde{z}_{t+1}) = A_{21}(s_t)\,z_t + A_{22}(s_t)\,\tilde{z}_t + B_2(s_t)\,u_t + C_2(s_t)\varepsilon_{t+1}$$

To illustrate the possibility that the economy will function differently in different states, it is assumed here that the parameters are statedependent, where s_t signifies the state. We assume here that there are only two states in the economy ($N = 0.1$) which is assumed to follow a Markov chain with transition probabilities:

(22) $$P = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} \quad \text{where } p_{ij} \equiv p(s(t+1) = i|s(t) = j).$$

The value function in a given state i is:

(23) $$(w_t'V(i)_t w_t + \omega) = \min_{\{u_i\}}(w_t'Rw_t + u_t'Qu_t + \Sigma_j^N p_{ij}E(w'_{t+1}V(j)_{t+1}w_{t+1} + \omega))$$

The time consistent (discretionary) solution involves monetary policy and the private sector's expectations having the following decision rules:

(24) $$u(i)_t = -F(i)z_t \quad \text{for } i = 0,1$$

(25) $$E_t(\tilde{z}_{t+1}) = G(i)z_t \quad \text{for } i = 0,1$$

where $F(i) = [\tilde{Q}(i) + \Sigma_j^N p_{ij}\tilde{B}(j)'V(j)_{t+1}\tilde{B}(j)]^{-1}[\tilde{R}(i) + \Sigma_j^N p_{ij}\tilde{B}(j)'V(j)_{t+1}\tilde{A}(j)]$,

$G(i) = \bar{A}(i) - \bar{B}(i)F(i)$,

$\tilde{A}(i) = A_{11}(i) + A_{12}\bar{A}(i)$,

$\tilde{B}(i) = B_0(i) + A_{12}\bar{B}(i)$,

$$\tilde{R}(i) = \bar{A}(i)' R \bar{B}(i),$$

$$\tilde{Q}(i) = Q + \bar{B}(i)' R \bar{B}(i),$$

$$\bar{A}(i) = [A_{22}(i) - \Sigma_j^N p_{ij} G(j) A_{12}(j)]^{-1} [\Sigma_j^N p_{ij} G(j) A_{11}(j) - A_{21}(i)],$$

$$\bar{B}(i) = [A_{22}(i) - \Sigma_j^N p_{ij} G(j) A_{12}(j)]^{-1} [\Sigma_j^N p_{ij} G(j) B_0(j) - B_1(i)]$$

The decision rules, (24) and (25), are thus affected by the probability of changing state from the state currently prevailing.

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