Refining Stylized Facts from Factor Models of Inflation*

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Abstract

Factor models of disaggregate inflation indices suggest that sectoral shocks generate the bulk of sectoral inflation variance, but no persistence. Aggregate shocks, by contrast, are the root of sectoral inflation persistence, but have negligible relative variance. We show that simple factor models do not cope well with essential features of price data. In particular, sectoral inflation series are subject to features such as measurement error, sales and item substitutions. In factor models, these blow up the variance of sector-specific shocks, while reducing their persistence. We control for such effects by estimating a refined factor model and find that inflation variance is driven by both aggregate and sectoral shocks. Sectoral shocks, too, generate substantial inflation persistence. Both findings contrast with earlier evidence from factor models, but align well with recent micro evidence. Our results have implications for the foundations of price stickiness, and provide quantitative inputs for calibrating models with sectoral heterogeneity.

Keywords: Inflation persistence, sticky prices, factor model, sectoral inflation.

JEL Codes: E31, E32.

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1 Introduction

The extent and nature of price rigidities are important inputs for many macroeconomic considerations. A recent body of research aims to shed light on this issue by identifying the sources of volatility and persistence in disaggregate (sectoral) inflation rates (Boivin et al. 2009; Maćkowiak et al. 2009; Kaufmann and Lein, 2013). Based on a variety of estimated dynamic factor models for a number of different sectoral price data sets, two “stylized facts” emerge: (i) Sectoral inflation volatility is mostly due to sector-specific disturbances, while aggregate shocks explain only a small fraction of movements in inflation. (ii) Sectoral inflation persistence is driven by aggregate shocks. The response to idiosyncratic or sector-specific shocks, by contrast, is close to instantaneous. These empirical findings on the sources of inflation persistence and volatility are used to validate foundations of price stickiness. For instance, Maćkowiak and Wiederholt (2009, 2010) and Maćkowiak et al. (2009) argue for rational inattention as the root of price stickiness because it can replicate swift responses to sector-specific shocks and sluggish adjustment to aggregate shocks. Carvalho and Lee (2011) and Shamloo and Silverman (2010) show how models with time-dependent nominal rigidities can generate similar impulse responses.

It is well-known that factor models perform well in capturing aggregate dynamics. Studies that underline the favorable properties of factor models for the study of aggregate dynamics are Stock and Watson (1998), Forni et al. (2000) and Onatski and Ruge-Murcia (2013). However, factor models in the literature tend to treat aggregate and sector-specific sources of variance highly asymmetrically. On the one hand, aggregate dynamics are given ample flexibility; e.g. they can be driven by multiple factors, with different dynamic properties. On the other hand, sectoral dynamics are typically assumed to follow a scalar autoregressive process. The latter is an innocuous assumption for most of macroeconomics, in which the focus lies entirely on studying aggregate dynamics (e.g. Reis and Watson, 2010; Baumeister et al., 2013). The scalar process assumption for the sector-specific component is, however, instrumental to the relative properties of aggregate and sectoral shocks. It implies lumping all non-aggregate sources of volatility together into one (residual) sector-specific process. As a result, the variance and persistence of that process are not necessarily meaningful objects to validate theories against.

Due to two essential properties of price data, simple factor models of inflation indices can produce misleading statements about the relative importance and persistence of sector-specific vs. aggregate shocks. A first property is the presence of measurement error. It is uncontested that various forms of measurement error induce additional volatility in price data (Shoemaker, 2007; Eichenbaum et al., 2012). A second essential property of price data is
the presence of sales and product substitutions. A vast body of research on micro price data has shown that accounting for irregular price changes such as sales and product substitutions has a dramatic impact on measures of price rigidity (e.g. Nakamura and Steinsson, 2008; Kehoe and Midrigan, 2012; Eichenbaum et al., 2011). In view of these properties, the bulk of subsequent research on micro prices has aimed to control for the presence of sales and substitutions when evaluating the properties of (regular) price changes. These additional sources of volatility transmit to inflation indices.

Simple factor models are not well suited to handle these essential features of price data. Basically, each of these properties generates additional sector-specific inflation variance with low persistence. A simple factor model will lump such irregular price fluctuations together with (possibly persistent) sector-specific structural shocks. As a result, measurement error, sales and substitutions have the scope to drive the simple factor model exactly in the direction of the stylized facts (i) and (ii), by increasing the measured variance of sectoral shocks, while lowering their measured persistence.

We propose a refined factor model that controls for the properties mentioned above and use it to re-assess the stylized facts. The refined model nests the model of Boivin et al. (2009) and additionally allows for the presence of measurement error, sales and substitutions. We estimate the refined model on the same US personal consumption expenditure (PCE) data used by Boivin et al. (2009). The data favors the refined model over the simple model in several dimensions, including standard information criteria and out-of-sample forecast performance. In other words, sectoral inflation dynamics appear to be better described by a process that allows for multiple sector-specific components.

The fact that the refined model outperforms the simple factor model can be due to a variety of reasons. These include measurement error, sales and substitutions, but also multiple shocks affecting sectors and/or firms. Disentangling the exact source is a daunting task and definitive conclusions to that end require product-level data. Yet, irrespective of its source, the multicomponent nature of sector-specific shocks has implications for stylized facts (i) and (ii). We discuss these implications and their bearing on the sources of nominal rigidities in detail in Sections 6 and 7. In brief, the results of the refined model cast substantial doubt on whether either of the two stylized facts holds up to close scrutiny.

The paper is organized as follows. We start by reproducing (i) and (ii) using a simple factor model. Then, in Section 3, we show what can go wrong with factor models for inflation indices. Section 4 lays out essential features of price data as documented in the recent literature. Subsequently, in Section 5, we propose a refinement of the simple factor model and estimate it for US PCE data. After discussing the interpretation of our results in Section 6, we conclude. Online appendices provide additional detail and robustness exercises.
2 A simple factor model for sectoral inflation

Consider the following decomposition of sectoral inflation $\pi_{it}$ into a common and a sector-specific component

$$\pi_{it} = COM_{it} + SEC_{it}$$
$$= \lambda_i C_t + e_{it}. \quad (1)$$

Here, $COM_{it} = \lambda_i C_t$, and $C_t$ is a $N \times 1$ vector of common factors. These factors are distilled from a large cross-section of macroeconomic and/or sectoral time series, $X_t$. The factor loadings $\lambda_i$ measure the dependence of inflation in sector $i$ on aggregate, or common, conditions. The remainder, $e_{it}$, is a purely sector-specific scalar process. The dynamics of sectoral inflation originate from both the common component and the sectoral component, through

$$C_t = \Phi(L)C_{t-1} + v_t, \quad (3)$$
$$e_{it} = \rho_i(L)e_{it-1} + u_{it}. \quad (4)$$

With this kind of decomposition at hand, Boivin et al. (2009) and Maćkowiak et al. (2009) decompose the variance, $\sigma^2(\pi_{it})$, and persistence, $\rho(\pi_{it})$, of sectoral inflation into a common and a sector-specific part.\(^1\)

As a quantitative reference for what follows, we use the data of Boivin et al. (2009) to estimate the model (1)-(4).\(^2\) The data for $\pi_{it}$ are monthly PCE price indices for 190 sectors over the period 1976:1-2005:6. We extract 5 common factors $C_t$ from a total of 653 monthly series. In particular, $X_t$ consists of 111 macroeconomic indicators, 190 sectoral PCE and 154 Producer Price Index (PPI) inflation series as well as 190 sectoral PCE quantity series. In addition, $X_t$ contains 4 PCE price aggregates and the corresponding quantity aggregates. We set lag length to 13 for all lag polynomials, in analogy to Boivin et al. (2009), though results are very similar using standard lag-selection criteria.

Figure 1 plots the breakdown of PCE inflation variance and persistence into a common and a sector-specific component across all sectors. Comparing the upper and lower left plots, it is clear that inflation variance is primarily induced by sector-specific shocks. The variance

\(^1\)There are different ways to estimate such a decomposition. Boivin et al. (2009) take a two-step approach in which one first retrieves the common factors by principal components analysis, and subsequently estimates the observation equation (2) and the transition equations (3) and (4). Maćkowiak et al. (2009) opt for a Bayesian state-space model in which this is done jointly.

\(^2\)We closely follow Boivin et al. (2009), with two minor exceptions. First, we do not force the Fed Funds rate to be a separate factor. Second, we estimate the observation equation by maximum likelihood, which is useful for later reference. Neither difference is quantitatively important for what follows.
contribution of common shocks, by contrast, is concentrated toward zero. The right-hand plots of the figure show the decomposition of persistence across sectors. Sectoral shocks generally do not tend to cause much persistence. The distribution of persistence of the sectoral component is relatively flat, with the median sector having no persistence at all. The picture is dramatically different for the persistence of the aggregate component. Its distribution across sectors is strongly negatively skewed, with almost all sectors bunching up at very high levels of persistence.

Figure 1: Benchmark model - variance and persistence

Note: Inflation is standardized, such that $\sigma^2(\pi_{it}) = 1, \forall i$. Following Boivin et al. (2009), persistence is measured as the sum of the polynomial coefficients estimated for $COM_{it}$ and $SEC_{it}$. There is no natural lower bound on this persistence measure. To maintain visibility in the figures, we limit the scale to [-1,1]. The medians – green x’s – and histograms take into account all sectors.

These results are fully in line with those of Boivin et al. (2009) and Maćkowiak et al. (2009). In sum, from both the literature and our own simple factor model two seemingly robust conclusions emerge. For most sectors,

Stylized fact (i) : $\sigma^2(COM_{it}) < \sigma^2(SEC_{it})$
Stylized fact (ii) : $\rho(COM_{it}) > \rho(SEC_{it}) \approx 0.$
In words, sectoral inflation volatility is predominantly driven by non-persistent sector-specific shocks, while inflation persistence is due to the common component.

3 Factor models and measurement error

Factor models perform well in the presence of measurement error or misspecification, as shown in, among others, Stock and Watson (1998). This statement is, however, subject to an important qualification. The excellent performance of factor models concerns the identification of the common factors ($C_t$) and their loadings ($\lambda_i$). It does not pertain to inference on the residual. This qualification is not always addressed in applied work. At times, this may well be innocuous. In fields where residual properties matter for the interpretation of the results, it is not. The reason is that the mere presence of measurement error points to a clear form of misspecification in the simple factor model: $e_{it}$ is not a scalar process, but has multiple components.

To convey why the dimensionality of $e_{it}$ could matter for the study of inflation variance and persistence, consider the following example. Suppose inflation in sector $i$ is driven by an aggregate component, $COM_{it}$ as before, an AR(1) sector-specific shock $P_{it}$, with $\rho(P_{it}) > 0$, and an additional sector-specific component $S_{it}$. Let $S_{it}$ have positive variance, $\sigma^2(S_{it}) > 0$, and be orthogonal to $P_{it}$, $S_{it} \perp P_{it}$. Then

$$\pi_{it} = COM_{it} + SEC_{it} = \lambda_i C_t + P_{it} + S_{it}$$

$$\sigma^2(SEC_{it}) = \sigma^2(e_{it}) = \sigma^2(P_{it}) + \sigma^2(S_{it})$$

$$\rho(SEC_{it}) = \rho(e_{it}) = \rho(P_{it} + S_{it}) = \frac{\sigma^2(P_{it})\rho(P_{it}) + \sigma^2(S_{it})\rho(S_{it})}{\sigma^2(P_{it}) + \sigma^2(S_{it})}.$$ 

It is immediate that

$$\sigma^2(SEC_{it}) > \sigma^2(P_{it})$$

and if $\rho(S_{it}) < \rho(P_{it})$, then

$$\rho(SEC_{it}) < \rho(P_{it}).$$

Interestingly, the biases resulting from the presence of $S_{it}$ work exactly in the direction of the stylized facts obtained using simple factor models: very volatile and non-persistent sector-specific shocks. The literature studying micro price data suggests there are good a priori reasons to expect additional components $S_{it}$, with $\rho(S_{it}) \leq 0$, to be important. We now discuss those reasons.
4 Prices and measurement

In this section we discuss measurement of goods prices. In particular, we document the scope for classical measurement error, sales and item substitutions. We also spell out the inflation dynamics they imply.

The scope for measurement error in the collection of prices is widely recognized. Shoe-maker (2007) estimates that the sampling variance of collected prices is substantially larger than the variance of actual price changes for the median product in the data underlying the CPI – which corresponds to the PCE sectors we study. The basic problem is that only a small number of prices, slightly above 200 price quotes per CPI entry level item, are sampled at this level of disaggregation and frequency.\(^3\) In other words, at the level of disaggregation of the data we use, sampling error is a major concern.

Eichenbaum et al. (2012) point out several particular issues in price measurement that yield observed price changes even when the true price is unchanged. The largest issue, for this purpose, is the practice of measuring prices using unit value indices, i.e. as a ratio of sales revenue to quantity sold. This implies that a change in the composition of customers, and thereby in discounts, or any non-linearity in the contract will induce a change in the measured price. Another issue is uncorrected quality improvement. They document that these problems exist both in CPI data and most scanner data from retailers.

All but one of the above mentioned types of measurement error induce a classical uncorrelated term in the measured price level. The top left panel of Figure 2 illustrates the dynamics. The corresponding inflation dynamics is illustrated in the top right panel. This type of measurement error generates negative autocorrelation in inflation.

One can also argue for the existence of a classical measurement error in inflation, corresponding to permanent errors in the price level. In particular, any unrecorded change in quality, as noted by Eichenbaum et al. (2012), or size/quantity of a product will induce this type of error. The dynamics of this type of measurement error is illustrated in the bottom row of Figure 2.

The remainder of this section discusses two measurement issues that are particular for goods prices and have been widely emphasized in the micro price setting literature: sales and forced item substitutions (Golosov and Lucas, 2007; Klenow and Kryvtsov, 2008; Nakamura and Steinsson, 2008, 2009; Kehoe and Midrigan, 2012; Eichenbaum et al., 2011, Anderson et al., 2013).

Both sales and substitutions impart particular short-run dynamics on inflation. Sales are changes in a price that are undone after a brief period of time. They therefore generate

\(^3\)More details are provided in Appendix A (all appendices are online only).
negative autocorrelation in inflation. The simplest and most common sales definition used in the literature (e.g. by Nakamura and Steinsson, 2008) is the one-period symmetric ‘V-shaped’ pattern of the price level illustrated in the top row of Figure 2. The right-hand column of the same figure illustrates the corresponding inflation dynamics. A forced item substitution occurs when the price surveyor can not record the price of the exact same good as the previous period at a given location. It implies a change in the measured price that does not necessarily reflect an actual decision to change price, but nevertheless generates a one-off blip in observed inflation. This is shown in the bottom row of Figure 2.

Figure 2: Dynamics induced by measurement error (ME), sales, substitutions

The product-level price literature has also established that the scope for sales and substitutions is huge. Cross-sectional heterogeneity aside, estimates for the monthly frequency of sales range from 7.4% (Nakamura and Steinsson, 2008) to over 20% (Klenow and Kryvtsov, 2008; Kehoe and Midrigan, 2012), and 3.4% (Bils and Klenow, 2004) to 5% (Nakamura and Steinsson, 2009) for item substitutions. The size of price changes induced by sales is also large – the median sale is 2.6 times the size of the median regular price change according to Nakamura and Steinsson (2008). The size of the error induced by each item substitution is unobserved, and is therefore harder to quantify.
5 Refining the simple factor model

As documented in the previous section, there is large scope for several measurement issues to affect measured disaggregated prices. These measurement issues imply particular inflation dynamics, as documented in the right-hand panel of Figure 2. In the product-level pricing literature much work has been done to control for these issues, mainly regarding sales and substitutions. The importance of measurement error in prices is generally acknowledged in the literature. Bils and Klenow (2004) and Boivin et al. (2009) are but two examples where the effect of measurement error on measured persistence are discussed. But, the methods used in the literature studying sectoral inflation dynamics have not been well suited for – nor explicitly adjusted to – the presence of measurement error or other measurement issues.¹

To control for the possible effects of measurement error, sales and substitutions we refine the simple factor model. We will refer to this refined model as the benchmark model. Essentially, the benchmark model aims to nest the simple factor model while allowing for the possible dynamics induced by measurement errors, sales and substitutions.²

5.1 Specification

In eq. (2), as before, sectoral inflation $\pi_{it}$ loads on a number of common factors $C_t$ that evolve according to eq. (3). At the idiosyncratic level ($SEC_{it} = e_{it}$), inflation is still driven by a persistent process, $P_{it}$, but now also contains two additional components. The first additional component we allow for is an iid—component, $I_{it}$. Such a component can absorb measurement error in inflation or item substitutions, as in the bottom-right panel of Figure 2. The second additional component we introduce is a moving average component, $M_{it}$, that serves to absorb the pattern implied by sales or, alternatively, measurement error in the price level, as in the top-right panel of Figure 2. Thus, the sector-specific component, previously eq. (4), now becomes

$$e_{it} = P_{it} + I_{it} + M_{it} \quad (5)$$

¹A separate issue is to what degree measurement error and other measurement issues are reduced by aggregating from the product level to the sectoral level. We address this issue quantitatively and in detail in Appendix A.

²Two related recent studies, Beck, Hubrich and Marcellino (2011) and Andrade and Zachariadis (2012) extend the simple factor model to allow for geographical differences, such as global, country or region-specific factors. Here, the focus is on dynamics induced by essential features of price data.
where

\[ P_{it} = \rho_1(L)P_{it-1} + \varepsilon_{it} \]  \hspace{1cm} (6)  
\[ I_{it} = \xi_{it} \]  \hspace{1cm} (7)  
\[ M_{it} = \xi_{it} - \xi_{it-1} \]  \hspace{1cm} (8)

and

\[(\varepsilon_{it}, \xi_{it})' \sim N(0_{3 \times 1}, D), \quad D^{1/2} = \begin{bmatrix} \sigma_i^\varepsilon & 0 & 0 \\ 0 & \sigma_i^\xi & 0 \\ 0 & 0 & \sigma_i^\xi \end{bmatrix}.\]

We estimate the above factor model on the same data as Boivin et al. (2009). We compute the factors in a first step by principal components and subsequently estimate, for each sector, using maximum likelihood and the Kalman filter, the observation equation (2) accounting for (5)-(8).

Our motivation for using a two-step estimator is as follows. First, relative to Boivin et al. (2009), the central change in specification is idiosyncratic and uncorrelated across sectors. It is therefore unlikely to have an impact on identification of the common components \((C_t)\), especially since these are extracted from a very large cross-section. Second, the dimension of the model is so big that the number of parameters is so large that standard optimizers have trouble finding the optimum. Monte Carlo exercises on data-generating processes with dimensions equal to those of our data confirm the difficulty of one-step estimation. In view of this, we follow Boivin et al. (2009) and estimate the factors \(C_t\) in a first step.

Regarding the second step, the three (unobserved) components \(P_{it}, I_{it}\) and \(M_{it}\) have distinct persistence properties and mutually orthogonal shocks \(\varepsilon_{it}, \xi_{it}\) and \(\xi_{it}\).\(^6\) While this ensures theoretical identification, it does not reveal much about the empirical performance of the estimator in finite samples. In Appendix B we document the favorable properties of the multicomponent maximum likelihood procedure for various data-generating processes (DGP) of interest. In short, when the DGP has multiple components, the estimator identifies multiple components and recovers persistence estimates close to the DGP. Not surprisingly, for lower underlying persistence, the estimator has lower precision. Even when the DGP truly is a single component process, estimating a multicomponent process does not imply substantial biases. Importantly, on the other hand, estimating single component processes (ARs) on multicomponent data generates estimates not even in the ballpark of the true

\(^6\)Note that, theoretically, when \(\rho_1(.)\) has zero coefficients at all lags, there is an identification issue, as the likelihood then is flat in \(\sigma_i^\varepsilon\) and \(\sigma_i^\xi\). In practice, for the data we study, this turns out not to play an important role: these ridges are typically located away from the likelihood’s maximum.
persistence.\textsuperscript{7} 

Two further remarks are in order. First, the above definition of $M_{it}$ implies a quite restrictive definition for capturing sales. Nevertheless, this component will pick up a subset of sales and thereby alleviate the issues that follow from confounding several components into one scalar process. Our results go through for alternative, less restrictive specifications of $M_{it}$, which we document in Appendix D. Second, the dynamics of $I_{it}$ and $M_{it}$ are preserved when prices are aggregated from product level to sector level, as is shown in Appendix A.

5.2 Model selection

Observe that the benchmark factor model, through eq. (5), nests the simple factor model, via eq. (4). Standard model selection criteria can be used to choose between the simple model and the benchmark factor model. If the additional components $I_{it}$ and $M_{it}$ are of no importance, the increase in the likelihood of the benchmark factor model relative to the simple model will be marginal. Selection criteria penalizing for the additional number of parameters (i.e. $\sigma_{I}^{2}, \sigma_{M}^{2}$) will then favor the more parsimonious simple model. We compute the selection criteria for each sector, i.e. for the second step in our two-step estimation, since the first step is the same for both models. According to both information criteria used (AIC and SBIC), in 88% of the sectors the data is better described by the benchmark factor model than by the simple model. In only 12% of all sectors is there no notable improvement in terms of fit by allowing multiple components at the sectoral level.

Another way of assessing model performance is by means of out-of-sample forecasts. Table 1 contains information on root mean square (RMSE) and mean absolute (MAE) forecast errors. The first row shows the fraction of sectors that is better forecasted (in a mean square error sense) by the refined model than the simple model. It shows that the refined model produces better forecasts for approximately 60% of the sectors. This is true for all forecast horizons from 1 to 12 months. This superior performance is confirmed by a similar metric for absolute forecast errors: 60-70% of the sectors have smaller mean absolute forecast errors in the refined model, as shown in the second row of Table 1. Alternatively, one can combine forecast errors across sectors. The third row documents root mean square errors in the refined model minus those in the simple model, averaged across sectors. In a similar fashion, the fourth row documents the difference in mean absolute error, averaged over sectors. The refined model makes uniformly better forecasts for all horizons considered, irrespective of the forecast criterion used. Thus, out-of-sample forecasts confirm the message from the

\textsuperscript{7}Among other things, the appendix provides an example DGP with equal variances of the three components, a persistence of $P_{it}$ of 0.5 and the resulting estimated AR persistence centered around 0 - very much like the estimates in the previous literature.
information criteria: the refined (benchmark) model outperforms the simple model.

Table 1: Forecast comparison between models

<table>
<thead>
<tr>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.61</td>
<td>0.58</td>
<td>0.63</td>
<td>0.63</td>
<td>0.58</td>
<td>0.63</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>II</td>
<td>0.61</td>
<td>0.64</td>
<td>0.69</td>
<td>0.61</td>
<td>0.65</td>
<td>0.66</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>III</td>
<td>-0.003</td>
<td>-0.007</td>
<td>-0.009</td>
<td>-0.009</td>
<td>-0.010</td>
<td>-0.011</td>
<td>-0.011</td>
<td>-0.010</td>
</tr>
<tr>
<td>IV</td>
<td>-0.003</td>
<td>-0.009</td>
<td>-0.012</td>
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<td>-0.013</td>
<td>-0.012</td>
<td>-0.012</td>
<td>-0.012</td>
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Note: Rows: I: Fraction of sectors with RMSE(refined)<RMSE(simple). II: Fraction of sectors with MAE(refined)<MAE(simple). III: Cross-sectional average of (RMSE(refined)-RMSE(simple)). IV: Cross-sectional average of (MAE(refined)-MAE(simple)). Columns: Forecast horizon in months. Initial estimate based on first half of the sample (1976:1-1989:6), then rolls through the end of the sample. Forecasts made every month, for horizons up to 12 months. Parameter estimates updated annually. Lag length=1, for both $P$ and $C$, as forecast quality from either model (simple and refined) benefits strongly from parsimony.

Table 2 provides another view on the estimated benchmark factor model. It characterizes sectors by the relevance of their sector-specific components. A number of features stand out. First, all sectors have a persistent component. Second, for more than half of the sectors both $I$ and $M$ play a role. Third, only 11% of the sectors are well captured by a single component process. Thus, from this perspective too, the scope for additional components is substantial.

Table 2: Sectors and idiosyncratic components

<table>
<thead>
<tr>
<th>Components</th>
<th>% sectors</th>
</tr>
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<tbody>
<tr>
<td>$P$</td>
<td>11%</td>
</tr>
<tr>
<td>$I$</td>
<td>0%</td>
</tr>
<tr>
<td>$M$</td>
<td>0%</td>
</tr>
<tr>
<td>$P + I$</td>
<td>24%</td>
</tr>
<tr>
<td>$P + M$</td>
<td>13%</td>
</tr>
<tr>
<td>$I + M$</td>
<td>0%</td>
</tr>
<tr>
<td>$P + I + M$</td>
<td>53%</td>
</tr>
</tbody>
</table>

Yet another way of evaluating the relevance of the additional components is by means of significance of point estimates. Table 3 shows how the number of significant shock-variance estimates broadly confirms the scope for additional components in Table 2. Particularly, the fraction of sectors for which the variance of the $I$-component is significant is 69%, while

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8 For the purpose of this table, we consider a component irrelevant for a particular sector if it accounts for less than 1% of the variance in the sectoral component.

9 Not surprisingly, these are also the sectors for which the information criteria select the simple model over the extended model.
the $M$-component’s shock is significant in 61% of the sectors. Almost all sectors have a significant $P$-component.

Table 3: T-statistics: sector-specific shock variances

<table>
<thead>
<tr>
<th>Component</th>
<th>t-stat &gt; 1.96 (% sectors)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ : $\sigma_\varepsilon$</td>
<td>94%</td>
</tr>
<tr>
<td>$I$ : $\sigma_\varepsilon$</td>
<td>69%</td>
</tr>
<tr>
<td>$M$ : $\sigma_\xi$</td>
<td>61%</td>
</tr>
</tbody>
</table>

The additional components are also quantitatively important. Figure 3 decomposes the variance of the sectoral component into $P$, $I$ and $M$ for all sectors. A point at the origin implies that all the sectoral variance is attributed to the $I$ component. A sector located at the top corner signifies 100% of its sector-specific variance stems from the $P$ component, and analogously the right bottom corner signifies $\sigma^2(SEC_{it}) = \sigma^2(M_{it})$. If a sector is located on, say, the $I - P$ axis, this implies it has no $M$ component. A key message from Figure 3 is the enormous degree of heterogeneity across sectors. Further details about the variance decomposition are also documented in Table 4. First, in half of the sectors, most of the variance in $SEC$ is due to $P$. Conversely, the other half of the sectors have most of their sectoral variance coming from $I$ and $M$. Second, $I$ appears to be quantitatively more important than $M$.

Table 4: Variance decomposition - SEC

<table>
<thead>
<tr>
<th>Component</th>
<th>Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>0.51</td>
<td>0.52</td>
</tr>
<tr>
<td>$I$</td>
<td>0.29</td>
<td>0.32</td>
</tr>
<tr>
<td>$M$</td>
<td>0.11</td>
<td>0.16</td>
</tr>
</tbody>
</table>

The first two columns of Table 5 shows, for each component, the median and mean variance contribution to $\pi_{it}$ across sectors. As expected, the variance contribution of the common component is around 10-15%, consistent with the evidence in the literature. The remaining 85-90% inflation variance is driven by sector-specific shocks. As the next three rows in the table (and Figure 3) indicate, a non-negligible part of the sectoral variance is due to the $I$ and $M$ component. The median contribution of the persistent sectoral component $P$ to total sectoral inflation is 43%.

Because the benchmark model further develops the sector-specific component, one would expect the identification of the factors and the estimation of factor loadings to be largely unaffected (Stock and Watson, 1998). The biases we study should therefore have negligible impact on studies that solely focus on aggregate components, e.g. Reis and Watson (2010). In Appendix C we document the similarity in factor loadings between the simple model and the benchmark model.
6 Re-evaluating the stylized facts

The evidence thus far suggests the refined model outperforms the simple model by quite a margin. We now document the extent to which this affects conclusions regarding persistence and variance of the sectoral component compared to the aggregate one. It is worth noting that all our empirical results are robust to changes in sample period, data set and model specification. Details are documented in Appendix D.

6.1 Persistence

Section 3 shows how multiple components could lead to underestimating persistence for the simple example of an AR(1) data-generating process. For more elaborate processes (e.g. with longer lags) and persistence measures (e.g. sum of polynomial coefficients) the direction and size of the bias induced by sales and substitutions is less clear cut a priori. Whether persistence in the simple factor model is substantially biased is thus ultimately an empirical question. Figure 4 therefore compares persistence in the simple model (on the x-axis) to persistence in the benchmark factor model (y-axis). The result is overwhelmingly clear: 88% of all sectors lie above the 45°-line. In other words, the simple factor model substantially
underestimates the persistence of sectoral shocks. The two right-hand quadrants contain sectors that exhibit positive persistence in the simple factor model (about 50% of all sectors). For these, the median persistence estimate is 47% higher in the benchmark model than in the simple model. In the upper left quadrant, the benchmark factor model finds positive persistence, where the simple model fails to detect any. This quadrant contains 15% of all sectors. For the remaining sectors, in the bottom left quadrant, neither of the factor models finds any positive persistence.

These biases substantially alter the view on the persistence of sectoral shocks. The top row of Figure 5 first reprints the cross-section of persistence measures in the simple model. It is a rather flat distribution, with the median sector having zero persistence. This is the second stylized fact. The benchmark factor model (bottom row) shows that, actually, sectoral persistence is strongly negatively skewed. A lot of sectors cluster at very high levels of persistence. For the median sector, persistence is estimated at 0.4.

From this evidence on persistence we conclude that there is no need to disregard models that fail to deliver instantaneous responses to sector-specific shocks. The data suggest that sector-specific shocks do generate persistent inflation dynamics. But simple factor models fail to detect them because they confuse them with non-persistent sources of variance. We now turn to the interpretation of these additional components.

### 6.2 Variance

There are two extremes in how to interpret the additional components. On the one hand, they may be structural shocks. If this is the case, contemporary models do not explain the multi-faceted nature of sector-specific dynamics. While stylized fact (i) would still hold, models deemed to support it would not. On the other hand, the multiple components may be due to measurement issues. This second interpretation sees part of the sector-specific variance as non-structural and thus requires that it is abstracted from when evaluating

<table>
<thead>
<tr>
<th></th>
<th>Benchmark model</th>
<th>Accounting for measurement error</th>
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<tbody>
<tr>
<td></td>
<td>Median</td>
<td>Mean</td>
</tr>
<tr>
<td>COM</td>
<td>0.10</td>
<td>0.17</td>
</tr>
<tr>
<td>SEC</td>
<td>0.89</td>
<td>0.85</td>
</tr>
<tr>
<td>P</td>
<td>0.43</td>
<td>0.44</td>
</tr>
<tr>
<td>I</td>
<td>0.25</td>
<td>0.27</td>
</tr>
<tr>
<td>M</td>
<td>0.09</td>
<td>0.14</td>
</tr>
<tr>
<td>η</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
Figure 4: Persistence - Bias

structural models. Stylized fact (i) therefore should not guide validation of theories.

Section 4 provided evidence for the a priori plausibility of measurement error, sales and substitutions. In what follows, we perform a number of validation tests which support such an interpretation of the $I$ and $M$ components. But even if one does not abide this interpretation the mere presence of multiple components affects the type of economic environments factor models provide support for. We first discuss these implications.

6.2.1 Structural shocks

If one chooses to interpret $I$ and $M$ in a structural manner then it is not immediately obvious how some of the currently advocated models can explain them.

Consider first the Calvo model. Shamloo and Silverman (2010) and Carvalho and Lee (2011) show that (i) and (ii) can be explained with Calvo frictions once input-output linkages between sectors are incorporated in the model. Essentially, these allow sectors to behave differently conditional on aggregate shocks – where linkages matter – and on sector-specific shocks – where linkages matter only marginally. Interpreting the $I$ and $M$ component as
structural implies that further conditionalities need to be addressed. Particularly, it begs an explanation for conditionality within a sector: why is it that a sector sometimes responds slowly (as implied by $P$), while at other times it does so immediately or only temporarily (due to $I$ or $M$, respectively)? It is not obvious how a Calvo model would be able to generate such conditionality.

Second, contemporary models of rational inattention have argued that because sector-specific shocks are so volatile it pays off for agents to focus attention on them, implying a fast response to sector-specific shocks. Aggregate shocks, by contrast, receive less attention because they are much less volatile. Responses to them will therefore be sluggish. The multicomponent nature of sector-specific inflation dynamics challenges such an interpretation. From the perspective of an agent deciding on where to allocate her attention, incentives change. Particularly, inferring which of the sector-specific components fluctuates may place substantial additional required processing capacity on the agent. Aggregate shocks may therefore become a more attractive alternative focal point. In addition, the relative properties of the various sector-specific components are inconsistent with the most basic implication
of rational inattention: the most volatile component, $P$, is also the most persistent one for most of the sectors.

This does not necessarily mean that Calvo or rational inattention models are incapable of explaining these findings. However, in their current formulations they do not. Possible avenues to reconcile these theories with the multicomponent nature of sector-specific shocks include further heterogeneity in input-output structures, multi-product firms, and more.

6.2.2 Sales and substitutions

The prevalence of sales and substitutions in price data is one of the primary motivations for refining the simple factor model. We here validate this motivation by examining to what degree the presence of the $I$ and $M$ components in our benchmark factor model coincide with Nakamura and Steinsson’s (2008, henceforth NS) product-level CPI data evidence. Our focus is on extremes: we compare whether a sector has a sales or substitution component at all in our results to the prevalence of sales and substitutions in that ‘major group’ according to NS.\textsuperscript{11,12} As documented above in Table 2, sales and product substitution components, $M$ and $I$ respectively, are only present in a subset of the PCE sectors we study. In particular, Table 2 shows that 35% of sectors have no $M$ component while 24% of sectors have no $I$ component.

Table 6 documents the validation exercise of our $M$ and $I$ components vs. Nakamura and Steinsson’s sales and substitutions. A name of a sector in bold typeface in the table indicates that the presence/absence of our $M$ or $I$ component coincides with NS sales and substitutions, while a sector name in normal typeface instead indicates conflicting results compared to NS. Italics denote inconclusive comparison.

NS document that Utilities, Vehicle fuel, Services (excl. travel) and Travel have virtually no sales, and at the opposite end of the spectrum that Apparel, Household Furnishing and Food (processed and unprocessed) have the highest prevalence of sales.

\textsuperscript{11}The relationship between the variance of our sales (substitutions) component and the fraction of price changes that are sales (substitutions) is tenuous. Several factors, including heterogeneity across sectors in the relative size of sales price changes and in aggregation properties, distort the translation from micro price characteristics such as sales (resp. substitution) intensity to variance of $M$ (resp. $I$). For an intuitive reason why aggregation need not preserve the relation between our components and the micro data, consider the following example. Two sectors A and B each have 100 products sampled. In sector A all products have sales, while in sector B only 1 product is ever on sale. Sales in sector B will not average out across products, and will thus generate an $M$ component in the index of sector B. The index for sector A, by contrast, may well not be affected much by product-level sales, as they have the scope to average out across products. Thus, despite being a sales-intensive sector, sector A may not require an $M$ component. The opposite is true for sector B, despite having very few sales at the micro level. A similar logic applies to substitutions.

\textsuperscript{12}An additional factor that complicates comparisons is the imperfect mapping between PCE sectors and the CPI ‘major groups’ and ELIs that NS reports.
Comparing our results for which sectors lack a $M$ component we note that they coincide to a reasonable degree with NS sectors with least sales. Key utilities sectors (Electricity and Gas) have no $M$ component. Gasoline, on the other hand, does have $M$ component contrary to what NS sales results indicate.\[13\] In line with NS most travel sectors (Taxicab, Bus and Other) have no $M$ component. Services (excl. travel) is a very diverse group. We note that a roughly average fraction (31\%) of the PCE service sectors lacks a $M$ component, while NS report above average sales in services.

Switching to sectors which have lots of sales according to NS, we confirm that sectors within Apparel (clothes for men, women and children, respectively) have a sales component. Four of the five Household Furnishing sectors have a sales component. For food sectors a non-negligible fraction, 25\%, of them lack a sales component, contrary to the evidence in NS.

The analogous exercise for product substitution validates our method by lining up very well with NS. Their product-level data indicates that product substitution is most common in Apparel and Transportation goods (mainly cars), and least common in Vehicle fuel and Utilities. We find no substitution component in Gasoline or the utilities sectors Electricity and Water. Furthermore, and also in line with NS, we find a substitution component in all three clothes sectors and in all of the nine transportation good sectors.

To summarize, we find that our results on which sectors have sales and substitutions largely coincide with what NS find. This corroborates the a priori plausibility of the additional components $M$ and $I$ capturing sales and substitutions.

Since Bils and Klenow (2004) and Nakamura and Steinsson (2008), the micro price liter-

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Note: List of sectors with highest (lowest) prevalence of sales or substitutions according to Nakamura and Steinsson (2008). Bold typeface indicates sectors where our result coincides with NS. Italics denote inconclusive comparison and normal typeface instead indicates conflicting results compared to NS.

<table>
<thead>
<tr>
<th></th>
<th>Highest</th>
<th>Lowest</th>
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</thead>
<tbody>
<tr>
<td>Sales</td>
<td>Clothes, Hhs Furnishing, Food</td>
<td>Electricity, Gas, Travel, Services, Gasoline</td>
</tr>
<tr>
<td>Substitutions</td>
<td>Clothes, Transportation</td>
<td>Gasoline, Electricity, Water</td>
</tr>
</tbody>
</table>
nature has almost invariably filtered out substitutions and sales in its study of regular price changes. The reason is obvious: since the models being validated tend not to feature sales or substitutions, the moments of the data models aim to match should not capture them either. Clearly, to the extent that $I$ and $M$ are indeed substitutions and sales, using the simple factor model for model validation does not follow this principle.\footnote{For an example that does abide this principle, see e.g. Bouakez et al. (2009). They compare a model without sales to statistics from micro data which filter out sales.} It leads one to conclude that for the median sector, sector-specific shocks are almost an order of magnitude more important than aggregate shocks. This large difference dominates any cross-sectional heterogeneity. Taking the ratio of common to sectoral variance contributions in the simple model, it appears that only 5 out of 190 sectors (3\%) are more affected by aggregate shocks than by sectoral shocks. The top row of Figure 6 shows that result, with almost no mass below 1. By contrast, the benchmark model – which does control for sales and substitutions – estimates sectoral shocks to be three to four times as volatile as aggregate shocks for the median sector, as is apparent in the bottom row of Figure 6. Importantly, aggregate shocks are more important than sector-specific shocks for one sector in four. Thus, while sectoral shocks tend to dominate, this is certainly not true for all sectors.

The fact that sales and substitutions have particular dynamics does not imply that they generally should be ignored. They may contain valuable information and should therefore be understood more fully. However, the (macro-)theory of sales is only just developing (Midrigan, 2011; Guimaraes and Sheedy, 2011; Kehoe and Midrigan, 2012; Matějka, 2011) and theory is largely non-existing for substitutions. Unless one validates models that incorporate sales and substitutions, the evidence models are required to match should filter out their effects. Models should therefore not be required to imply a dominance of sector-specific shocks.

### 6.2.3 Measurement error

Measurement error in prices results in negative autocorrelation in inflation and can thus generate a $M$ component. Analogously, measurement error in inflation will result in an $iid$-component, similar to $I$. As such, measurement error is observationally equivalent to sales and substitutions. It is known from the micro price data literature that various forms of measurement error are prevalent (Shoemaker, 2007; Eichenbaum et al., 2012). The implication of measurement error for stylized fact ($i$) is straightforward: measurement error generates variance that should be ignored when evaluating structural models.

For some purposes it may actually be useful to quantify how much of the non-persistent sector-specific fluctuations is due to measurement error, rather than due to sales or substitu-
Figure 6: Variance ratios

Note: Due to the presence of sectors with virtually no variance in the common component, values above 10 are truncated at 10.

tions. For instance, many studies make conjectures about plausible degrees of measurement error, in order to verify whether it could drive their results (e.g. Bils and Klenow, 2004). To inform such questions, we here adapt our factor model to shed light on the importance of measurement error, relative to sales and substitutions.

One way to overcome the observational equivalence between sales and substitutions on the one hand, and measurement error in prices and inflation on the other, is to use quantities jointly with prices. A priori, there is no apparent reason to expect measurement error in prices to affect quantities. Sales and substitutions, by contrast, can be expected to influence quantities. In Appendix E, we describe an extension to the factor model that separates measurement error from sales and substitutions. Identification comes from the assumption that measurement error in prices does not affect quantities. We here summarize the results of that model specification briefly, while the appendix contains the results on variance and persistence across sectors.

The third column of Table 5 indicates that for the median sector, 11% of inflation variance
is due to measurement error ($\eta$). The differences in terms of variance decomposition across model specifications indicate that in the benchmark model (without quantities isolating measurement error), all three sectoral components appear to contain some measurement error. Also in the model that accounts separately for pure measurement error, the $I$ and $M$ components still appear very relevant. Importantly, the conclusions for the relative variance and persistence of common and sectoral shocks remain unchanged from our benchmark model.

7 Conclusion

The current litmus test for sectoral models of price setting is whether they can replicate (i) and (ii). We show that a simple change in specification beyond the simple factor model is not just empirically preferred, but also changes the facts (i) and (ii), both quantitatively and qualitatively.

First, one of the identified sectoral components exhibits substantial persistence. Thus, the generally found absence of sectoral persistence in (ii) is a result of measuring persistence of a composite process, thereby masking underlying persistence. Second, regarding the relative volatilities of sectoral and aggregate shocks in (i), the implication of our findings depends on the source of the multiple components. At a minimum, (i) requires a different interpretation. In particular, if the multicomponent nature is due to the presence of multiple structural shocks then the standard formulation of the rational inattention model (à la Maćkowiak and Wiederholt) does not explain it in any obvious way. Similarly, basic versions of models with time-dependent price setting (à la Calvo) also have a hard time matching the fact that within a sector some changes in prices are persistent while others are not. There are current efforts to understand how these types of frictions work in richer environments. If it is the presence of multiple structural shocks that causes the rejection of the simple factor model, then our estimates suggest that model development should aim not just at generating persistence in response to sector-specific shocks. In addition, it should also aim at providing reasons for why it coexists with non-persistent fluctuations within the same sector.

There is, however, another possible interpretation. Existing micro-evidence as well as validation exercises with our model support the case that at least part of the source of the additional components is due to measurement error, sales and substitutions. Put differently, the high-frequency components may well be the result of non-structural measurement issues.

\[^{15}\text{For instance, Pastén (2012) describes how rational attention allocation in multi-product firms may lead to less persistence in response to aggregate shocks and more persistence following sectoral shocks. Carvalho and Lee (2011) discuss the importance of complementarities in economies with input-output interactions.}\]
In this case, a simple factor model will misleadingly interpret all sector-specific fluctuations as structural and thus overestimate variance and underestimate persistence. Under this plausible alternative, our estimates indicate a ratio of sector-specific to aggregate volatility of three to four for the median sector. Moreover, heterogeneity prevails: for a quarter of the sectors in our data, aggregate shocks appear to be a more important source of fluctuations than sector-specific shocks. This stands in contrast to (i), which has led the field to disregard models that attribute a significant role to aggregate shocks. The relative volatility also matters for the calibration of price setting models. As discussed in Maćkowiak and Smets (2009), models of rational inattention (Maćkowiak and Wiederholt, 2010) and menu costs (Golosov and Lucas, 2007), for instance, often rely on sector-specific shocks that are an order of magnitude larger than aggregate shocks. Our refined factor model suggests that this is not necessarily what sectoral price data convey.

To summarize, our results suggest that the current litmus test – requiring models to replicate stylized facts (i) and (ii) – is somewhat misguided. It has implied refuting models that do not appear to be contradicted by the data.

The evidence presented in this paper also brings the micro and macro evidence on price sluggishness closer together. Initially, high frequency volatility in sectoral price series seemed puzzling from the perspective of inflation inertia at the macro level. Boivin et al. (2009) reconciled this (non-filtered) fast-micro and slow-macro evidence by invoking conditionality: it matters whether a shock is aggregate or sector-specific. Our results, by contrast, reveal that there is no conflict between the micro and macro evidence: Applying filters similar to those used in research on micro (product-level) price data, thereby taking account of measurement error, sales and substitutions, one obtains very similar results. Lower volatility and higher persistence are obtained when sales and substitutions are accounted for. This is apparent from micro studies such as Nakamura and Steinsson (2008), Kehoe and Midrigan (2012) and Eichenbaum et al. (2011) as well as from our benchmark factor model. In addition, there is a tremendous amount of heterogeneity between sectors in terms of persistence of sector-specific shocks, consistent with the micro-evidence on frequency of price changes (Nakamura and Steinsson, 2008). Furthermore, the findings contrast starkly with those obtained for non-filtered data, at both micro and macro level. In particular, non-filtered prices appear very volatile, and have low persistence. This is evident from the simple factor model (Boivin et al., 2009) and micro studies that do not control for sales (e.g. Bils and Klenow, 2004).

The results of the present paper also have implications for the appropriate design of core inflation indices. The fact that sector-specific dynamics are best characterized as multicomponent processes means that sectors should not be excluded from a core index based
on simple statistics such as unfiltered persistence or volatility.\textsuperscript{16} Such exclusion-based core measures are commonly used by central banks, most explicitly by Bank of Canada. The Federal Reserve’s motivation for focusing on PCE inflation excluding food and energy is a related short-cut in that direction.

References


\textsuperscript{16}Dolmas (2009) also concludes that simple filters mask important underlying persistence and discusses implications for core inflation indices. Bradley et al. (2014) criticize core CPI measures without using sectoral data. They find that core CPI only captures permanent price changes well for some time periods.


