Forward Guidance and Heterogeneous Beliefs*

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Abstract

Central banks' announcements that future interest rates will remain low could signal either a weak future macroeconomic outlook – which is bad news – or a future expansionary monetary policy – which is good news. In this paper, we use the Survey of Professional Forecasters to show that these two interpretations coexisted when the Fed engaged into date-based forward guidance policy between 2011Q3 and 2012Q4. We then extend an otherwise standard New-Keynesian model to study the consequences of such heterogeneous interpretations. We show that it can strongly mitigate the effectiveness of forward guidance and that the presence of few pessimists may require keeping rates low for longer. However, when pessimists are sufficiently numerous forward guidance can even be detrimental.

Keywords: Monetary policy, forward guidance, communication, heterogeneous beliefs, disagreement.

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1 Introduction

> The FOMC has not been clear about the purpose of its forward guidance. Is it purely a transparency device, or is it a way to commit to a more accommodating future policy stance to add more accommodation today?
>
> Charles I. Plosser, March 6, 2014.

When facing the Zero Lower Bound (ZLB) on its nominal policy rate, a central bank can still affect current allocations by making statements about future policy rates, indicating that they will remain very low for a significant length of time.[1] In the aftermath of the Great Recession, several central banks implemented such forward guidance policies with somewhat mixed success: they succeeded in lowering expected future interest rates[2] but their resulting impact on the macroeconomy seemed limited or even sometimes lead agents to expect future contractions.[3] As Charles Plosser's quote above suggests, one possible reason is that a central bank's announcement that future interest rates will remain low for a given period of time is ambiguous (see also Woodford, 2012): it is consistent with anticipation of bad economic fundamentals – forward guidance is then said to be *Delphic* – or with anticipations of an expansionary monetary policy – forward guidance is then said to be *Odyssean*.[4] In this paper, we show that these two interpretations of the same policy coexisted when the Fed engaged into date-based forward guidance announcements between 2011Q3 and 2012Q4 and we investigate how such heterogeneous interpretations can change the effectiveness and the design of forward guidance policies.

We make three contributions. First, we show that the date-based forward guidance of the Fed coincides with striking patterns of heterogeneity in the expectations of both professional forecasters and households. These patterns are consistent with the fact that some individuals understand the policy as *Odyssean* and some interpret it as *Delphic*. Second, we extend a New-Keynesian model allowing for such heterogeneous beliefs about the type of policies conducted by the central bank and the fundamentals of the economy. We show that, in line with the heterogeneity in expectation surveys, different interpretations of the same path of future policy rates, implying different stances of policy, can coexist in an equilibrium. We also underline

[1] See Krugman (1998), Eggertsson and Woodford (2003) or, more recently, Werning (2012). In pratice, the lower bound can be below 0% and rates can be slightly negative as in Switzerland and in the euro area.

[2] See e.g. Swansson and Williams (2014).

[3] See e.g. Campbell et al. (2012), Del Negro et al. (2015) or Campbell et al. (2016).

[4] This terminology has been introduced by Campbell et al. (2012). See also Ellingsen and Söderström (2001) for a seminal contribution where monetary policy decisions are either related to new information about the economy or to changes in the preferences of the central banker.

that this can explain why forward guidance did not have a large macroeconomic impact other than a large drop in expected future interest rates. Third, we use the model to analyze the optimal forward guidance policy at the ZLB, i.e. the number of periods for which the central bank commits to keep its interest rate at zero. We show that with too many pessimistic agents – that is private agents who only interpret zero interest rates for longer as a an indication that the liquidity trap will be longer – Odyssean forward guidance policy can be detrimental: instead of a boom, it may lead to worse expectations of the macroeconomic outlook and consequently to lower current aggregate demand. Overall, our approach can rationalize the evidence in the literature that forward guidance lowered expected future interest rates with limited macroeconomic impact and even sometimes signalled future contractions.

We start by documenting new facts on the heterogeneity of individual expectations in the US Survey of Professional Forecasters (SPF). Disagreement about 1-year and 2-year ahead short-term interest rate forecasts dropped to historically low levels during the period where the Fed conducted date-based forward guidance. Starting from August 2011, the FOMC announced that its policy rate would remain "at exceptionally low levels [...] at least through mid 2013." This date-based forward guidance was therefore coincident with an exceptional coordination of views about the path of future short-term interest rates over the next 2 years. However, while professional forecasters broadly agreed that future short-term interest rates will remain low for the next two years, they revised differently their expectations of future macroeconomic conditions. Some *optimists* foresaw an improvement and revised *upward* their forecasts of inflation and consumption growth 2 years ahead. Some *pessimists* expected a worsening and revised these forecasts *downward* and significantly below the revisions of optimists. This coexistence of an agreement about future interest rates together with different revisions of other macroeconomic forecasts is specific to the date-based forward guidance episode. Indeed, *optimists*' revisions in inflation and interest rates were positively correlated before date-based forward guidance started but they became negatively correlated over the period where it was conducted. In contrast, *pessimists*' revisions were positively correlated both before and after this new policy. Overall, these patterns are consistent with *optimists* interpreting the date-based forward guidance policy as a promise of future accommodation, i.e. as Odyssean, while *pessimists* interpret it as a signal of worse future macroeconomic outlook, i.e. as Delphic.

We then analyse individual answers in the Michigan Survey of Consumers during the period of date-based forward guidance and underline that households' expectations exhibit patterns that are comparable to the ones observed in the SPF and, in addition, that optimistic expectations lead to higher individual demand. More precisely, the proportion of households expecting interest rates would not increase in the next 12 months reached an historical high in late 2011. Among these households, *optimists* expected relatively higher inflation and better economic conditions over the next 12 months and were more inclined to purchase durable goods than *pessimists*, who expected relatively lower future inflation and worse economic conditions. So, during the date-based forward guidance period, households broadly agreed that future interest rates will stay low but had different views on the macroeconomic outlook. In addition to the results obtained in the SPF, the evidence in the Michigan survey suggests that the heterogeneity of macroeconomic expectations was likely to translate into different current consumption decisions.[5]

Our second contribution is to introduce an otherwise standard New-Keynesian model where agents may (agree to) disagree both on the nature of forward guidance policy and on future fundamentals.[6] We use this model to analyse the macroeconomic consequences of the kind of disagreement outlined in the survey data. In this setup, households face a common discount factor shock which drags the economy to the ZLB. Private agents observe the current discount rate shock and the resulting current allocation. But they do not know the number of periods this shock will last. The central bank reacts to the shock by implementing a date-based forward guidance policy, i.e. by setting the nominal interest rate at zero for a number of periods. However, private agents cannot observe whether such future interest rates involve some periods of accommodation after the end of the trap. This information is not revealed before the economy reaches the actual end of the trap. We first show that, in this economy, private agents can agree on the path of future nominal interest rates without agreeing on the length of the trap, provided they disagree on the type of policy implemented by the monetary policy authority. Optimistic agents anticipate a shorter liquidity trap together with an accommodative stance of monetary policy after its end (Odyssean forward guidance). Pessimistic agents expect the trap to last for the period of time the central bank keeps its policy rate at zero (Delphic forward guidance). Consistent with the patterns in survey data, despite agents agree on the path of future interest rates, their disagreement about the potential future accommodation at the end of the trap leads them to disagree on future aggregate demand and inflation. Optimistic agents who interpret the policy as Odyssean anticipate higher future inflation and consumption while pessimistic agents who interpret the policy as purely Delphic expect lower future inflation and consumption. Moreover, the different interpretations of forward guidance induce different actions which offset each other: optimists consume more

[5] Bachmann et al. (2015) provide evidence that the fraction of households answering that the time is favorable to buy durable goods in the Michigan survey is positively correlated with current consumption.

[6] See Wiederholt (2014) for an analysis of dispersed information about current fundamentals at the ZLB.

today in anticipation of a boom tomorrow; pessimists consume less today in anticipation of a recession tomorrow. These offsetting choices hampers the efficiency of date-based forward guidance even in situation where the private sector agrees on a path of low future interest rates.

Finally, we use the model to investigate how the heterogeneity of beliefs affects the efficiency of forward guidance. When a sufficiently large fraction of agents are pessimistic and interpret forward guidance as *Delphic*, the implementation of an Odyssean forward guidance may be inefficient and even detrimental compared to the implementation of a pure Delphic announcement. This is because these latter agents drag current aggregate consumption down through direct and general equilibrium effects. In contrast, if the fraction of agents who are optimistic and interpret forward guidance as *Odyssean* is sufficiently large, such policy can still stimulate consumption and raise inflation expectations when the central bank commits to low rates for longer.[7]

Related literature Gürkaynak et al. (2005) show that FOMC announcements have strong effects on asset prices and in particular expected future policy rates while Romer and Romer (2000) provide evidence FOMC decisions convey Fed-specific information about the macroeconomic outlook so that private agents update their forecasts accordingly. Campbell et al. (2012) confirm such results in a sample that includes the Great Recession.[8] Their results are consistent with market participants interpreting FOMC's announcements as being *Delphic* rather than *Odyssean*. Our analysis complements these empirical exercises by analyzing the dispersion, rather than the average, of individuals' macroeconomic forecasts. We show that the two interpretations of forward guidance announcements coexisted, we provide an explanation why this is so, and we analyse the aggregate consequences of this heterogeneity.

Our paper is also related to the literature on the effectiveness of forward guidance. Carlstrom et al. (2012) and Del Negro et al. (2015) underline that standard DSGE models predict incredibly high positive impacts of forward guidance policies on future inflation and activity. According to these models, announcements such as those made by the Fed should have led to a boom in demand much greater than what has been observed, a result Del Negro et al. (2015) dubbed the "forward guidance puzzle". These papers consider the impact of policies that are unambiguously perceived as sequences of deviations from the normal times reaction function of the central bank. We provide evidence that agents had different forecasts of such deviations

[7] Note that forward guidance may work through additional channels than the ones considered in the framework of this paper, e.g. by reducing the uncertainty associated with future economic outcomes.

[8] See also Del Negro et al. (2015) and Campbell et al. (2016) for similar evidence on the more recent period.

and we show that this heterogeneity of interpretation reduces the aggregate impact of such policies. Existing complementary explanations of the "forward guidance puzzle" rely on some form of "discounting" in the aggregate Euler equation due to borrowing constraints (McKay et al., 2015), higher order beliefs (Angeletos and Lian, 2016), or bounded rationality (Gabaix, 2016). These references do not find a potential detrimental effect of forward guidance.

In addition, and in contrast to frequent policy discussions (e.g. Filardo and Hofmann, 2014), our results underline that gauging the efficiency of forward guidance announcements by merely looking at the reaction of expected future policy rates can be misleading as agents may disagree on the meaning of such a low future interest rate path. As Woodford (2012) emphasizes, for forward guidance to be effective, private agents should not only believe that interest will remain low in the future but they also should understand that the reason *why* they will is that the central bank will temporarily allow for more inflation than in normal times. Engen et al. (2014) provide survey evidence that it took time to convince private agents that the unconventional policies of the Fed implied a policy stance that would be more accommodative than it had been in the past.

Our paper is linked to the literature studying how the ZLB affects optimal monetary policy. Krugman (1998), Eggertsson and Woodford (2003) and Werning (2012) study the optimal policy at the ZLB in an infinite horizon model, emphasizing the associated commitment problem. Coibion et al. (2012) determines the optimal inflation target in the presence of occasional liquidity traps. Bassetto (2015) studies the optimal communication problem of central banks' forward guidance policies. Gaballo (2016) shows that Delphic announcements can become welfare detrimental when they are liable to subjective interpretations. In contrast to these studies, the distinctive feature of our setup is that we allow for heterogeneous beliefs about the commitment type of the central bank. Bodenstein et al. (2012) investigate quantitatively how imperfect credibility of future policy rate announcements in Sweden and in the US lowered the impact of forward guidance policies. They conclude that imperfect credibility should be compensated by extending the period of low interest rate providing future monetary stimulus. In contrast, we show that the impact of forward guidance may not only be muted compared to the full commitment case, but that it can even be detrimental when the credibility of commitment is not shared broadly enough among agents. Recently, Bilbiie (2016) characterized the optimal policy using close-form solutions in alternative formulations of the liquidity trap.

We investigate disagreement in the case of liquidity traps provoked by exogenous shocks to the natural rate of interest as in Eggertsson and Woodford (2003) or Werning (2012). Similar insights can be obtained in the context of endogenously low natural rate of interest as in Eggertsson and Krugman (2012) or Eggertsson and Mehrotra (2014) as soon as agents can disagree on future monetary policies.

Several papers study monetary policy under limited information. Melosi (2010) considers a dispersed information setup where monetary policy decisions signals central bank's information to the private sector. Paciello and Wiederholt (2014) analyse optimal monetary policy when firms choose the attention they pay to aggregate conditions. Several papers investigate the role of imperfect information at the ZLB. Wiederholt (2014) develops a model where agents have dispersed beliefs about the current aggregate shock generating the liquidity trap. He then explains why inflation expectations remained well anchored, even in the presence of deflationary risks. He also highlights the potentially detrimental effect of communication policies that may lead agents to revise downward these expectations. Bianchi and Melosi (2015a) emphasize that transparency on future deviations from active stabilization is welfare improving as it anchors medium to long-run macroeconomic expectations. Bianchi and Melosi (2015b) study the consequences of the private sector's uncertainty on whether a stabilizing monetary/fiscal policy mix regime will be abandoned when the economy reach the ZLB. Kiley (2014) illustrates how imperfect information can reduce the efficiency of forward guidance in New-Keynesian models. In contrast, we derive the optimal monetary policy at the ZLB when agents perfectly observe market variables but they disagree about the unobserved length of the trap. Finally, Michelacci and Paciello (2016) investigates the consequences of ambiguous announcements on the effects of monetary policy, when agents are ambiguity-averse.

Finally, we rely on survey expectations to infer how private agents form their expectations. We mainly work with surveys of professional forecasters but find comparable results for households survey. Carroll (2003) shows professional forecasters influence households' expectations. Coibion and Gorodnichenko (2015) find that both professional forecasters and households expectations result from the processing of imperfect information. Andrade et al. (2016) show that, in normal times, professionals understand monetary policy in the sense that they rely on a Taylor rule to draw their interest rate forecasts. Carvalho and Nechio (2014) provide similar evidence from qualitative households' forecasts. Whether households adjust their current consumption in reaction to changes in their inflation expectations, which monetary policy should contribute to determine, is open to debate (see Bachmann et al. (2015) and D'Acunto et al. (2015)). In contrast to these contributions, we focus on how private agents understand future monetary policy when the economy is at the ZLB and the central bank deviates from its normal times rule.

The paper is organized as follows. We present the stylized facts specific to forward guidance in Section 2. We introduce a New-Keynesian model with heterogenous beliefs in Section 3 and we characterize the optimal monetary policy in Section 4. Section 5 concludes by discussing the policy implications of our results.

2 Stylized facts on date-based forward guidance

In this section, we present new facts on the cross-sectional dispersion of forecasts observed in surveys of expectations during the period when the Fed conducted a date-based forward guidance policy.

On August 8, 2011, the FOMC stated that "The Committee currently anticipates that economic conditions [...] are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013". Following this first announcement of date-based forward guidance, the Fed extended the horizon twice. On January 25, 2012 the horizon was extended to "... at least through late 2014". And on September 13, 2012, it was extended to "...at least through mid 2015". The FOMC had already implemented forward guidance before Summer 2011; starting its December 2008 meeting the Fed issued "open-ended" statements and referred to a period of "exceptionally low interest rates" that will last "for some time" or "for a long period of time". So the horizon during which interest rates would be kept at zero was less precise. Date-based forward guidance ended on December 12, 2012, when the FOMC moved to a "state-based" forward guidance whereby it committed to keep policy rates at zero as long as inflation and unemployment rates did not reach specific numerical thresholds (and inflation expectations remained anchored) and the reference to a specific date for keeping rates at exceptionally low levels was dropped altogether.

As we detail below, this period of date-based forward guidance coincided with striking patterns in the heterogeneity of forecasts observed in the US Survey of Professional Forecasters (SPF). Professional forecasters had two contrasting views on the future macroeconomic outlook: some revised future inflation and consumption growth upward while some others revised downward. Moreover, although data characteristics prevents us to conduct exactly the same analysis than for the SPF, individual households' expectations observed in Michigan Survey of Consumers feature comparable patterns over the same period and the heterogeneity in expectations translated in heterogeneous decisions. Before describing in detail how the cross-section of forecasts evolved when the FOMC started to conduct date-based forward guidance, we first document what happened to the average of individual forecasts in the US SPF at that time.

2.1 Average expectations in the US SPF

Figure 1 displays the evolution of the average of individual short term interest rate forecasts 1 quarter, 1 year and 2 years ahead. Three specific subperiods are highlighted: 2008Q4-2011Q2 which corresponds to the time when the US economy reached the ZLB and the Fed conducted its "open-ended" forward guidance; 2011Q3-2012Q3 which corresponds to the "date-based" forward guidance period; and 2012Q4-2013Q2 which corresponds to the "state-based" forward guidance.

This figure shows that 1-quarter ahead short-term interest rate forecasts reached levels close to zero in 2009 that is when the US economy hit the ZLB. 1-year and 2-year ahead short-term interest rates forecasts were already low when date-based forward guidance policy started, but they went further down during that period to finally reach levels close to zero and comparable to the 1-quarter ahead forecasts.

As Figure 2 illustrates, over the same date-based forward guidance period, 1-quarter, 1-year and 2-year ahead consumption growth (resp. inflation) forecasts only slightly decreased (resp. increased). This makes our first fact.

Fact 0. *Date-based forward guidance was coincident with a drop in the mean forecasts of the short-term nominal interest rates to historically low (and close to zero) levels up to 2 years, a limited increase in the average forecast of inflation and a limited decrease in the average forecast of consumption growth.*

These patterns are reminiscent of results stressed in previous studies documenting the reaction of macroeconomic expectations to various forward guidance announcements (Campbell et al., 2012; Del Negro et al., 2015; Campbell et al., 2016): such policy lowered expected future short-term interest rates but the reaction of inflation, output or consumption growth were much smaller and sometimes negative. One reading is that forecasters had on average a Delphic interpretation of date-based forward guidance: announcements of future low interest rates were interpreted as signalling worse future macroeconomic conditions.

2.2 Agreement on future short-term interest rates

Let us now analyse the cross-sectional dispersion of professionals' forecasts. Figure 3 presents the evolution of the interquartile range in the cross-section distribution of individual 1-quarter, 1-year and 2-year short-term nominal interest rate forecasts since 2002.

This figure reveals that, in addition to the lowering in average interest rate forecasts documented above, the date-based forward guidance period was associated with a sharp drop in the dispersion of short-term nominal interest rate forecasts. In particular, this dispersion on medium-term (1-year and 2-year) nominal interest rate forecasts dropped to their lowest value since 2002 (and actually since at least 1982 when the survey started to collect such interest rate forecasts). So, the commitment to keep interest rates at zero until an explicit date was associated with an exceptional coordination of opinions about future interest rate on an horizon broadly consistent with the length of the initial date-based forward guidance announcement. By contrast, the "open-date" announcements used from 2008Q4 to 2011Q2 were only associated with a strong coordination of opinions on 1-quarter interest rate forecasts.[9]

We can summarize this paragraph's findings as follows:

Fact 1. *When "date-based" forward guidance started, professional forecasters' disagreement on future short-term interest rates 1-year and 2-year ahead declined sharply and reached an historical low.*

To summarize, Fact 1 together with Fact 0 indicate that, during the date-based forward guidance period, forecasters revised their short-term interest rate forecasts to eventually agree on a period of low interest rates over the next two years. Therefore, the policy announcement has been perceived as credible. Note that this does not necessarily imply that agents also agree for more long term horizons. We next investigate to which extent such agreement on interest rates reflected an agreement on the future macroeconomic outlook.

2.3 Heterogeneous forecasts of inflation and consumption

We investigate how professional forecasters updated their medium-term (2 years ahead) macroeconomic forecasts in the SPF released for 2011Q4, 2012Q1 and 2012Q4, i.e. after each of the three date-based forward guidance announcements.[10]

[9] As this can be observed in Figure 3, this drop in medium-term interest rate forecasts was already initiated during the "open-ended" forward guidance episode for the 1-year horizon (-50% of disagreement for the peak of 2008) but accelerated during the "date-based" forward guidance episode (-75% during the period). For the 2-year horizon, the drop mainly happened during the "date-based" episode (-22% during the "open-date" period and -67% during the "date-based" period).

[10] Specifically, we look at individual revisions of forecasts between two subsequent surveys observed after each of the "date-based" forward guidance announcements (which were made on August 9, 2011; January 27, 2012 and September 13, 2012): *(i)* revisions between the the 2011Q3 survey (conducted between July 29 and August 8, 2011) and the 2011Q4 survey (conducted between October 27 and November 8, 2011); *(ii)* revisions between the 2011Q4 survey and the 2012Q1 survey (conducted between January 27 and February 7, 2012); *(iii)* revisions between the 2012Q3 survey (conducted between July 27 and August 7, 2012) and the 2012Q4 survey (conducted between October 26 and November 6, 2012). The relatively low frequency of the SPF data

Table 1 presents the average revisions of 2-year consumption growth, inflation and interest rate forecasts observed at these dates. Different subgroups of forecasters are considered. “Optimists” who have both revisions of inflation and consumption growth above the average revision across forecasters observed at that date. “Pessimists” who have both revisions of inflation and consumption growth that are below the average. And “Pessimists and others” which are forecasters that are not optimists. The fraction of optimistic forecasters was non-negligible during the date-based forward guidance period. It increased from 19% to 36% with later announcements. This also suggests that it took time to convince a significant fraction of the population that forward guidance intended to provide further accommodation, as Engen et al. (2014) also emphasize.

Table 1 reveals that Optimists and Pessimists are two statistically different groups. In each period considered, optimists' revisions of consumption growth and inflation forecasts were *significantly* above the ones of pessimists. This also applies, although with a lower degree of significance, to the ones of the rest of forecasters as a whole. By contrast, every group revised similarly (downward or non-significantly differently from zero) their interest rate forecasts. That optimists (resp. pessimists) revised their inflation and consumption forecasts upward (resp. downwards) when revising interest rate downward is consistent with the anticipation that monetary policy will be more accommodative (resp. fundamentals will be weaker) in the future.

This difference in revisions between groups emerges even more strikingly once we look at implied shadow Taylor rates. Specifically, we computed by how much each forecaster should have revised their 2-year forecasts of short term-interest rate, if they were to infer these forecasts from their pre-ZLB perceived Taylor rule.[11] The revision in the resulting shadow Taylor rate is positive for optimists, which contrasts with their negative revisions of expected interest rates. In 2011Q4, for example, while on average optimists revised their interest rate forecasts down by -0.41pp, their view on the future macroeconomic outlook corresponded to an average increase of 0.35pp in the shadow Taylor rate. This difference is statistically significant. So optimists expected more accommodation over the forecast horizon than what

prevents from observing the reaction of individual forecasts on a specific day. See Del Negro et al. (2015) and Campbell et al. (2016) for evidence using higher frequency data.

[11] We use the panel of individual forecasts in the SPF to estimate a perceived Taylor rule using data prior to 2008Q3. The postulated rule includes an interest rate smoothing term and reactions to an inflation and to an output growth gaps. The target inflation and potential consumption growth are obtained from individual 10 year ahead forecasts. The estimation features individual fixed effects. We then infer revisions in the individual shadow Taylor rate using these estimated perceived Taylor rules and individual revisions in macro-economic forecasts.

their projections of macroeconomic fundamentals would have called for in normal times. By contrast, pessimists revised their interest rate forecasts by an average -0.38pp, very close to their -0.37pp downward revision of their shadow Taylor rates.[12] In 2012Q1 and 2012Q4, when interest rates cannot be revised further down because of the ZLB, both "Pessimists" and "Pessimists and others" forecasters revise down their inflation forecast in sharp contrast with the revisions of "Optimists" which is consistently positive.

The contrast between revisions in interest rates and shadow Taylor rates suggests that date-based forward guidance announcements introduced a change in the perception on how interest rates relate to fundamentals for optimists but not for pessimists. This intuition is confirmed by the correlation between revisions of inflation forecasts and interest rate forecasts , which becomes negative during the date-base forward guidance only for "optimists". In contrast, as shown in the bottom panel of Table 1, this correlation had been positive until 2011Q3 for all groups of forecasters and it remained positive for non-optimists. Put differently, only after Q3 2011, optimists' joint revisions in interest rates and inflation becomes consistent with an Odyssean understanding of forward guidance: future interest rate cuts entail higher future inflation. By contrast, the correlation remained positive for pessimists (and others) which is consistent with a Delphic interpretation of forward guidance: future interest rate cuts signal lower future inflation.

This leads to the following fact:

Fact 2. *Two (significantly different) groups of forecasters emerged during the date-based forward guidance period. Despite a similar downward revision in expected interest rates, a group of optimists revised their inflation and consumption forecasts up consistently with the expectation of a future policy accommodation, whereas a group of pessimists revised their inflation and consumption forecasts down consistently with the expectation of a weaker future economic outlook.*[13]

Levels of disagreement. The combination of similar views about future interest rates together with disparate views on future inflation and consumption that started with date-based forward guidance also show up in the disparities in the levels of individual 2-year

[12]Let us note that the average of consumption revision in 2011Q4 is negative as documented by Fact 0, despite the presence of optimists. This finding is consistent with Del Negro et al. (2015) who argue that the shift to date-based forward guidance was understood as *delphic* on average, which means that it conveyed a (bad) news on fundamentals. Yet our evidence also shows that a significant share of forecasters expected that two years from now a combination of low interest rates and higher inflation.

[13]To avoid any confusion is worth to specify that "similarly" here means that the average revisions in the two groups are not statistically different, and not that everybody is revising to the same extent.

ahead forecasts. More precisely, compared to historical standards, the beginning of date-based forward guidance is associated with a significant excess disagreement about these medium-term forecasts.

This is illustrated by Figure 4 which displays the residuals from a regression of the disagreement about 2-year ahead forecasts of consumption (resp. inflation) on the disagreement about 2-year ahead forecasts of short-term nominal interest rates controlling for the disagreement about 1-quarter ahead consumption and inflation forecasts and estimated on a pre-crisis sample (1982Q2-2008Q4). The beginning of the date-based forward guidance policy is a striking outlier. Before August 2011 the residuals are not significantly different from zero. Disagreement about future consumption (resp. inflation) stays in the range of what its correlation with disagreement on 1-quarter consumption (resp. inflation) forecasts and disagreement on interest rate would predict. By contrast, disagreement on future consumption (resp. inflation) becomes significantly higher than its predicted value over the date-based forward guidance period. Controlling for short-term disagreement, disagreement about medium-term consumption (resp. inflation) started to be much higher than what disagreement about future interest rates would imply at the time the date-based forward guidance started to be conducted.[14]

2.4 Further evidence using household surveys

We also investigate how expectations of US households evolved when date-based forward guidance was conducted by exploiting the Michigan Survey of Consumers. More specifically, we analyse households' expectations about *(i)* the evolution of interest rates over the next 12 months (increase, stay constant, decrease), *(ii)* the evolution of prices over the next 12 months (average inflation rate), *(iii)* whether it is a good time to buy durables (good, neutral, bad) and *(iv)* the expected overall aggregate business conditions over the next 12 months (good, neutral, bad).[15]

[14]A concern is that this excess disagreement results from an increase in uncertainty. Yet, our control for disagreement on short-term inflation and consumption forecasts, which are often used as a proxy for uncertainty, should partly capture such an increase. As we document in Appendix B, there is no clear evidence that date-based forward guidance coincided with an increase in uncertainty as captured by alternative measures of macroeconomic uncertainty introduced recently. Furthermore, one still observes excess disagreement in 2-year consumption and inflation forecasts during the date-based forward guidance period if one includes such alternative measures as additional controls in the regression exercise.

[15]Each month, about 500 households are surveyed. The sample is designed to be representative of the US population. We exploit the responses of the 60% of individuals that are first time respondents to the survey. The remaining 40% are interviewed twice but with a 6 months period between the two interviews. Due to this repeated cross-section structure it is not possible to compute revisions of forecasts between 2 subsequent

We start with households' expectations of future interest rates. Figure 5 plots the share of respondents anticipating that interest rates will either decline or stay constant over the next 12 months over a 2002-2014 sample. The chart underlines that this share jumped to levels above 60% during the date-based forward guidance period.[16] So the majority of households in the Michigan Survey of Consumers interpreted forward guidance announcements as indicating that interest rate will not increase (at least) over the next year.[17]

In a second step, we analyze the heterogeneity of expectations across surveyed households. We split the sample of respondents expecting stable or lower interest rates into the three categories: optimists if they expect better aggregate business conditions and have inflation expectations above the average; pessimists if they expect worsening business conditions and have inflation expectations below average; and others. Table 2 reports the average expectations of each of these groups observed in the month following the three date-based forward guidance announcements of August 2011, January 2012 and September 2012. The results reveal that among households who anticipated stable or lower interest rates, the ones who expected higher inflation and better economic conditions also considered that the time was more favorable to purchase durable goods. By contrast, pessimists expect lower inflation and a smaller fraction among them consider that it is time to purchase durable goods.[18]

survey rounds. Moreover, several questions asked to households call for qualitative rather than quantitative answers. These data limitations prevent us to conduct an analysis as detailed as the one we can conduct with the SPF. The other way around, the question on current durable good consumption in the Michigan survey as no equivalent in the SPF. As Bachmann et al. (2015) emphasize, consumption of durable goods follows total consumption so that answers to this question is a good proxy for the current total consumption decisions.

[16]This surge in the proportion of households expecting that interest rate will not increase is mainly driven by households expecting interest rates will stay constant which reaches an all-time high (above 50%) during the date-based forward guidance period. This contrasts with the 2009 episode where the high share of households foreseeing that interest rate will not increase is mostly due to a large majority of people expecting a drop in future interest rates.

[17]This complements the results of Carvalho and Nechio (2014). They show that, in normal times, some households in the Michigan survey understand monetary policy: they adjust their interest rate expectations in a way that is consistent with a Taylor rule and their views on the macroeconomic outlook. We find that a substantial share of households reports interest rate forecasts consistent with the date-based forward guidance policy implemented at the ZLB.

[18]Note that observing a fraction of optimistic households who declare to consume more when they anticipate higher inflation and better economic conditions does not contradict the results in Bachmann et al. (2015) who find that, *on average*, during the ZLB period, households who report higher inflation expectations in the Michigan survey are likely to consume less. Moreover, in the Appendix, we also report results that are very similar to theirs over the date-based forward guidance episode. Namely, we drop the expected inflation criteria in the definition of optimistic households and consider the larger group of agents who expect an improvement

Overall, we get the following stylized fact:

Fact 3. *During the period of date-based forward guidance, among households expecting interest rates not to increase over the next 12 months, the ones anticipating better economic conditions and higher inflation were more likely to purchase durable goods than the ones anticipating worse economic conditions and lower inflation.*

To sum up, the period of date-based forward guidance is characterized by striking patterns in the cross-section distribution of professional forecasters or households expectations. Some of them had optimistic expectations consistent with the belief that the policy announcement was a signal of future accommodation – Odyssean interpretation of forward guidance. Some other had pessimistic expectations consistent with the belief that the policy announcement was a signal of bad news on future fundamentals – Delphic interpretation of forward guidance – (Fact 2). Such an heterogeneity of individual forecasts was coincident with a consensus (or a lower disagreement) on the path of future interest rate (Fact 1), and a mild response of average survey expectations of these two variables (Fact 0). Finally, such heterogeneous readings of forthcoming low interest rates implied different perception on whether it is a good time to purchase durable goods, which other studies have shown is correlated to current consumption decisions (Fact 3).

In the next Section, we introduce a model of disagreement on forward guidance policies that is consistent with these stylized facts.

3 A model of disagreement on policy

In this section, we extend a standard New-Keynesian model in the spirit of Eggertsson and Woodford (2003) by allowing for heterogenous beliefs. Our aim is to model a situation in which disagreement on policies can coexist with an agreement on future interest rates. We show that when this is the case, the economy exhibits properties that are qualitatively similar to the facts uncovered above.

The key feature that differentiates our analysis from the rest of the literature stems from the natural assumption that the ability of the central bank to sustain a commitment to a

in future activity. The results show that this broader class of optimistic households is more likely to purchase durable goods but also expect lower inflation than the average household. These optimists behave like in Werning (2012)'s model of forward guidance in which such policy increases consumption today by generating expectations of a boom in activity tomorrow. Finally while this broader class of optimists by definition accounts for a larger fraction of the sample of households surveyed, it does not represent the whole sample: again optimists coexisted with pessimists during the period of date-based forward guidance.

future policy can only be assessed ex-post. As a result, although agents may agree on a future path of policy rates, they may disagree – and, in particular, agree to disagree – on the reasons that justify such a path. This is especially true once rates reach the zero lower bound (ZLB); at that point, an announcement that rates will be low for longer can be either interpreted optimistically as a signal of a more accommodative future stance, or pessimistically as a signal of weaker future fundamentals. The possibility of two contrasting interpretations of the same policy rate path is central to our results.

In the remainder of this section, we first present the key equations and assumptions of the modeling framework. We then characterize different equilibria, stressing the possibility of disagreement on future policy. We finally derive some positive implications and relate them to the facts of the empirical section. The optimality of the monetary policy at the ZLB in the presence of heterogenous beliefs is discussed in the following Section.

3.1 NK-economy with heterogeneous beliefs

Our model is a standard New-Keynesian model extended to account for heterogeneous beliefs. To streamline the presentation, we directly discuss the three key equations of the model – the IS curve, the New-Keynesian Phillips curve and the monetary policy rule – expressed in log-linear deviations from the steady-state. The detailed microfoundations underlying these equations as well as the detailed derivations are postponed to Appendix.

Consumption equation. There is a unit mass of atomistic agents indexed by $i \in (0,1)$; they are homogeneous in any respect, except that they may hold different beliefs. Their consumption decisions comply with the standard Euler equation (expressed in log-linear deviations from the steady-state):

$$c_{i,t} = -\gamma^{-1}(E_{i,t}[\xi_{t+1}] - \xi_t + r_t - E_{i,t}[\pi_{t+1}]) + E_{i,t}[c_{i,t+1}], \tag{1}$$

where $c_{i,t}$ denotes the consumption of agent i at time t, γ is the inverse of the inter-temporal elasticity of substitution $E_{i,t}[(\cdot)]$ represents the conditional expectation of an agent i type at time t, ξ_t is a common shock hitting at time t the discount factor in agents' utility function, r_t is the nominal interest rate and π_{t+1} is inflation at date $t+1$.

It is important to remark that heterogeneity in beliefs can potentially induce different individual consumption paths, and hence different wealth profiles. To deal with this problem, in the micro-foundations of our model, detailed in Appendix C, we include a risk sharing mechanism that endogenously make agents equalize their wealth as soon as they agree on the future course of the economy. As a result, in our setup differences in wealth can only be temporary; that is, agents have the same steady state level of consumption.

Shock. As it is standard in the literature of optimal policy at the ZLB, we focus on a particular sequence of discount factor shocks that drags the economy in a liquidity trap, i.e. a situation where the natural rate of interest is below its steady state for a number of periods. Formally, we consider a series of shocks $\{\xi_\tau\}_{t=0}^{\infty}$ to the households' discount factor such that $\xi_\tau - \xi_{\tau+1}$ takes value $-\xi$ with $\tau = 0, ...T-1$ and zero afterwards. This generates a trap of *length* $T \in \mathbb{N}$ which starts at time $t = 0$ and ends in $t = T$ which is the first period out of the trap.[19]

We assume that agents perfectly observe the discount factor shock at time $t = 0$, but they are uncertain about the length of the trap T. Note that such end of the trap has the feature of a "news" on a future shock. As such it can be only assessed *ex-post*. We describe how agents form expectations on the length of the trap T later in this section.

Phillips Curve. The optimal choices of firms and households lead to a New-Keynesian Phillips curve which links the current aggregate inflation to current aggregate consumption and to the average expectation of future inflation across different individuals i. Namely, we get (see Appendix C for details):

$$\pi_t = \kappa c_t + \beta \int_0^1 E_{i,t}[\pi_{t+1}]di, \tag{2}$$

with κ the slope of the Phillips curve and β the agents' discount factor. Three conditions are needed for the derivation to hold: i) agents perfectly observe the current allocation; ii) firms' shares are held by agents in equal proportion; and iii) agents participate to the same labor market. In particular, the second condition makes sure that firms maximize profit using the same stochastic discount factor and that the relevant expectation for pricing is the average one across agents' type.[20]

Monetary policy. The central bank sets a path of nominal interest rate $\{r_t\}_{t\geq 0}$ to maximize the average utility across households. The objective of the monetary authority and the derivation of its optimal policy are addressed in Section 4. At this stage, it is sufficient to note that the monetary authority could potentially offset any change in real interest rates by appropriately setting r_t so that the term appearing in brackets in (1) equals zero at any

[19] There is no loss of generality in considering deterministic traps. In a more general framework, the shock can follow a Markov process as in Eggertsson and Woodford (2003).

[20] We show in the Appendix C.5 that our results are robust to changes in these assumptions.

time. In this case, there are no fluctuations (neither in consumption nor in inflation) and the allocation is optimal.

However, we assume nominal interest rates face a zero lower bound (ZLB) so that they cannot go negative. This constraints policy actions. In particular, when a negative discount shock is large enough the best the monetary authority can do is to maintain interest rates at zero for a given period of time, *i.e.* to set the interest rate in deviation from steady-state to $r_t = -\log R$ with $-\log R = \log \beta$ the (real) natural rate of interest in steady-state.

Therefore, for our purposes and without loss of generality, we restrict our attention to the following policy representation:

$$r_t = -\log R \text{ for } t \leq T_{cb} \tag{3}$$

$$= \phi \pi_t, \text{ otherwise.} \tag{4}$$

where T_{cb} represents a lift-off date for interest rates, which is a choice of the authority and where $\phi > 1$ to ensure determinacy.[21]

Let $E_{cb,0}[T]$ denote the authority's expectation of the length of the trap. A conventional monetary policy would conduct to a lift-off date satisfying $T_{cb} = E_{cb,0}[T]$. The monetary authority provides the maximal stimulus it can during the trap and then raises interest rates once out of the trap. In line with the standard *Taylor principle*, this policy ensures reaching the steady state as soon as possible.

At the ZLB however, following such a conventional monetary policy may not be optimal. As shown by Krugman (1998), Eggertsson and Woodford (2003) and Werning (2012), the second-best policy then prescribes to keep policy rates at zero for longer than required by a strict application of the Taylor principle. Indeed, the authority can stimulate current consumption by promising to keep short-term rates at zero for some periods after the trap ends that a liftoff date $T_{cb} \geq E_{cb,0}[T]$. This policy generates an expansionary stimulus after the end of the trap hence boost inflation and consumption in the future. In turn, the expectation of a future boom stimulates current consumption and reduces the impact of the crisis.

As is well-known, this *unconventional* policy is time-inconsistent. Once the recovery occurs, inflation is no longer socially desirable and the authority is tempted to renege on her promise and to set $r_t = \phi \pi_t$ from T onward as prescribed by a Taylor rule satisfying the Taylor principle. Therefore, implementing this second-best policy at the ZLB requires a central bank being able to commit. When instead the authority cannot commit, the optimal (second best) policy corresponds to the one implemented in normal times as soon as the economy exits from the trap. We define the following two types of monetary authority

[21] Note that, in our framework, there is no loss of generality not including output gap terms in the rule.

Definition 1. *When the authority is Odyssean, it can commit to extra periods of accommodation, that is, it sets* $T_{cb} \geq E_{cb,0}[T]$ *with possibly a strict inequality; when instead the authority is Delphic, it cannot commit to extra periods of accommodation, that is it necessarily sets* $T_{cb} = E_{cb,0}[T]$.

We borrow this terminology to Campbell et al. (2012). Odyssean refers to the fact that the announced date of liftoff results not only from the length of the trap but also from a commitment of the authority to some periods of extra accommodation (i.e. the authority ties its hands as Odysseus before he meets the sirens). Conversely, Delphic refers to the fact that the policy only reveals the date at which the authority thinks the trap will end.[22]

Information. We first assume that the commitment ability is unobservable to agents and cannot be signaled by the authority. During the trap, it is optimal for both types of monetary authority to keep interest rates at zero so that the Odyssean monetary authority cannot be distinguished from the Delphic one.[23]

Agents have different priors on the commitment ability of the central bank defined as follows:

Definition 2. *A fraction* $\alpha \in (0, 1)$ *of pessimists (i=p) believe that the authority is Delphic, that is, she sets* $T_{cb} = E_{cb,0}[T]$*, whereas a fraction* $1 - \alpha$ *are optimists (i=o) and believe that the authority is Odyssean, that is she sets* $T_{cb} \geq E_{cb,0}[T]$ *with possible strict inequality.*

Second, we assume that agents are also uncertain about the end of the trap. For simplicity of exposure, we consider that agents have an uninformative prior about the length of the trap.

Note that, despite the current allocation is perfectly observed, it is not sufficient for agents to infer both the commitment ability and the length of the trap. These are only revealed when the trap actually ends and the effective policy is observed. Therefore, provided there is no evidence that contradicts their beliefs, agents can agree to disagree about both the commitment ability of the monetary authority and the length of the trap.

[22] Note that the Odyssean authority has some Delphic component as T_{cb} also provides information on the length of the trap. A pure Odyssean policy (as in e.g. Campbell et al. (2012)) would only convey information on policy beyond the end of the trap.

[23] An Odyssean central banker would be willing to signal its type through its current actions, which are observable. However, such signaling is not necessarily implementable. The reason is that the Delphic type could strategically mimic the Odyssean type, preserving the advantage of not paying the cost of an ex-post inefficient boom (see Barthélemy and Mengus, 2016, for an analysis of such imitation gains). Signaling through communication is not necessarily more effective as it can be cheap talk as shown by Bassetto (2015). Therefore, our assumption of heterogeneous beliefs about the commitment ability of the central bank is consistent with a pooling equilibrium with agents having heterogenous priors.

Disagreement on policy with agreement on a liftoff date of interest rate. An equilibrium at time 0, for a given lift-off date T_{cb}, is a current rate of inflation, and a profile of current and future expected consumption for each agent, that satisfy (1)-(4). In the following, we focus on a given subset of equilibria which are in line with the facts observed during the date-based forward guidance of the Fed that we described in Section 2.[24]

In particular, we focus on equilibria where agents agree on the lift-off date T_{cb}. This agreement can originate from a credible announcement of a lift-off date by the authority. Although we think that this is a natural interpretation, we do not model such coordination explicitly as our analysis concerns the properties of certain equilibria, and not the conditions under which those emerge. A comprehensive approach to equilibrium selection through institutional communication would require to study the communication game between the monetary authority and private agents and search for equilibria where credibility can be sustained by reputation concerns. We consider that this goes beyond the scope of the present paper and we refer the interested reader to Bassetto (2015) and Wiederholt (2014).

In any case, it is important to note that the credibility of an announcement does not imply that the monetary authority is able to commit. The announcement of a period of zero interest rates can be credible either because it involves a commitment or because the authority is forced to do so given economic conditions. It is exactly this kind of ambiguity that is the object of our analysis.

We aim at clarifying under which conditions an agreement on future policy actions can conceal a disagreement about the type of policy adopted. In particular, our focus is on how agents *revise* their beliefs on the length of the trap once the authority announces a path of future interest rates. This requires i) that agents place some weight on the central bank having better information on the length of the trap and ii) that announcements are credible. We are going to assume both conditions are satisfied in what follows.

Even if all agents may believe and understand the announcement, they may still revise differently their projections on future consumption and inflation as long as they disagree on the commitment ability of the central bank. Pessimists believe that the announced lift-off date speaks about the length of the trap: from their point of view the announcement is *Delphic*. Formally, they believe that $T_{cb} = E_{p,0}[T]$. Optimists instead believe that the announced date of liftoff implies some periods of extra accommodation: from their point of view the announcement is *Odyssean*. Formally, they believe that $T_{cb} \geq E_{o,0}[T]$, potentially with strict

[24] In general, many equilibria can exist in the non-linear version of a New-Keynesian model, including self-fulfilling liquidity traps as in Mertens and Ravn (2014) among others. See Appendix C, for a formal definition of an equilibrium in the fully-fledged model is given in the Appendix C.

inequalities. Therefore, we have $E_{p,0}[T] \geq E_{o,0}[T]$.

3.2 Characterization of the effects of heterogeneous beliefs

We first analyse the two polar cases where agents are all pessimists ($\alpha = 1$) or all optimists ($\alpha = 0$) and then present the effects of heterogeneity. In doing so, we take the central bank's policy (T_{cb}) as being exogenous.

Case $\alpha = 1$. When all agents believe the authority has no commitment ability, they all interpret the lift-off date as the expected end date of the trap. In such a case, agents have homogeneous beliefs that the trap will last for T_{cb} periods and that the authority will keep interest rates at zero from $t = 0$ until $T_{cb} - 1$ included.

The expected current consumption is given for each i by:

$$\text{for } t \in (0, T_{cb} - 1): \qquad E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R - \xi + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}], \tag{5a}$$

$$\text{for } t \geq T_{cb}: \qquad E_{i,0}[c_{i,t}] = 0 \tag{5b}$$

where the expected inflation path is determined by the Phillips curve (2).

Figure 6 illustrates the path for aggregate consumption and inflation (green lines) in an example where the length of the trap is 12 quarters, the central bank announces it will keep interest rates at zero until $T_{cb} = 12$, and everybody understands that this lift-off date corresponds to the end of the trap. In that case, consumption and inflation increase monotonically and reach their steady state values after 12 quarters.

Case $\alpha = 0$. When instead all agents believe the authority has commitment ability, they all interpret that the time until lift-off implies some periods of extra accommodation where the interest rate will be at zero after the end of the trap. In such a case, agents have homogeneous beliefs that the trap will last less than T_{cb} periods, $E_{i,t}[T] < T_{cb}$, and that the authority will maintain interest rates at zero from $t = 0$ to $T_{cb} - 1$ included, even though this implies that inflation is above its steady state level between $E_{i,t}[T]$ and $T_{cb} - 1$.

The expected current consumption is given for each i by:

$$\text{for } t \in [0, E_{i,t}[T] - 1]: \qquad E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R - \xi + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}], \tag{6a}$$

$$\text{for } t \in [E_{i,0}[T], T_{cb} - 1]: \qquad E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}], \tag{6b}$$

$$\text{for } t \geq T_{cb}: \qquad E_{i,0}[c_{i,t}] = 0, \tag{6c}$$

where, again, the inflation path is determined by the Phillips curve (2). In fact, an Odyssean policy amounts to stimulate current consumption promising lower short-term rates once the trap ends.

Figure 6 illustrates the resulting path for aggregate consumption and inflation (blue lines) in an example where the length of the trap is again 12 quarters but the central bank announces it will the liftoff date is $T_{cb} = 17$. In contrast to the previous case, consumption and inflation converge to the steady state non-monotonically. In particular, once the trap is over at date $t = 12$, low interest rates generate a boom, which induces more current consumption. The steady state is reached later than in the previous case, but now the path remains on average closer to the steady state. Therefore, this policy can, through an optimal choice of T_{cb}, deliver higher welfare than what following a conventional Taylor rule would imply.

However, as we already mentioned, once the trap ends at time $t = 12$, the boom is no longer socially desirable and the authority is tempted to renege on its promise and to set $r_t = \phi\pi_t$ onward, which corresponds to the time-consistent solution with perfect stabilization at steady state, after the end of the trap. Therefore, the second-best policy at the ZLB requires a commitment ability to solve for this time-inconsistency problem.

Case with heterogeneous beliefs, $0 < \alpha < 1$. We now describe a case where, although agents have homogeneous beliefs about the interest rate path and observe the current allocation, they still disagree on the length of the trap to the extent they disagree on the commitment ability of the central bank.

Suppose all agents believe in the announcement that the lift-off date is a certain T_{cb}. As already discussed, this leads to agreement on the interest rate path for this given period of time.[25] And they entertain different beliefs on the reason why the interest rate will be at zero for a given period of time. In this sense, agents observe the distribution of beliefs but they have no reason to update their own opinion and, thus, they may agree to disagree.

However, it is common knowledge that at time $E_{o,t}[T]$, which is, according to optimists, the date at which the trap ends, only one of the two types will be right. In case the trap is over at $E_{o,t}[T]$ optimists will be right and pessimists will be wrong. Otherwise, the opposite occurs. In either case, the heterogeneity of beliefs cannot last beyond $E_{o,t}[T]$. All this is common knowledge among agents.

[25] In an extension of the model where the preference shock follows a Markov process, optimists' subjective beliefs on the probability of exiting the trap is higher than the ones of pessimists. This leads to a limited disagreement (rather than to strict agreement as in the deterministic case) between optimists and pessimists on the whole path of nominal interest rates.

Such temporary disagreement has an impact on the current and expected allocations as established by the following proposition (See Appendix C for a formal proof).

Proposition 1 (Heterogeneity)**.** *Suppose agents agree on T_{cb} and there exists a fraction $1-\alpha$ of optimists for whom $E_{o,0}[T] \leq T_{cb}$. The expected path for individual consumption is given respectively by (6) with $i = o$ for the optimists, and (5) with $i = p$ for the pessimists, where the inflation path is expected by each type according to:*

$$E_{o,0}[\pi_t] = \kappa E_{o,0}[\alpha c_{p,t} + (1-\alpha)c_{o,t}] + \beta E_{o,0}[\alpha E_{p,t}[\pi_{t+1}] + (1-\alpha)E_{o,t}[\pi_{t+1}]], \tag{7}$$

where

$$\text{for } t \in [0, E_{o,0}[T]-1]: \quad E_{o,0}[c_{p,t}] = E_{p,0}[c_{p,t}] \quad \text{and} \quad E_{o,0}[E_{p,t}[\pi_{t+1}]] = E_{p,0}[\pi_{t+1}], \tag{8a}$$

$$\text{for } t \geq E_{o,0}[T]: \quad E_{o,0}[c_{p,t}] = E_{o,0}[c_{o,t}] \quad \text{and} \quad E_{o,0}[E_{p,t}[\pi_{t+1}]] = E_{o,0}[\pi_{t+1}], \tag{8b}$$

for optimists, and symmetrically for pessimists.

The interpretation of beliefs is intuitive. Each type understands that until date $E_{o,0}[T]$ no information can lead the other type to change her beliefs. Hence, in the short run, agents agree on the path of both inflation and consumption and they only disagree for periods after the date ($E_{o,0}[T]$), when optimistic expect the end of the trap. At that date, as the truth finally unfolds, each type expects that the other will conform to her own expectations. After that date, optimists believe that monetary policy will engineer a boom resulting in higher inflation and higher consumption, and that pessimists will finally share the optimist view. Conversely, pessimists expect that the economy will still be experiencing the negative shock and that optimists will finally share the pessimistic views. In sum, disagreement on the commitment ability of the authority tends to yield disagreement over medium-term inflation and consumption expectations, whereas it will have no impact on short-term expectations.

Figure 7 plots the dynamics of consumption and inflation when beliefs on the nature of policy are heterogeneous. In this example, the trap lasts again 12 quarters but 75 per cent of households are convinced that the monetary policy is Odyssean and the other 25 per cent believes that the policy is Delphic ($\alpha = .25$). With this fraction of optimists and pessimists, the optimal policy of the central bank is still to implement an Odyssean forward-guidance. Yet, to compensate for the presence of pessimists, it keeps interest rates at zero for more periods after the end of the trap (6 periods instead of 5 when $\alpha = 0$) and announces a later date of liftoff (at date $t = 18$ instead of 17 when $\alpha = 0$). This results in a larger and longer boom at the end of the trap.

Figure 7 also illustrates that the heterogeneity of beliefs about the effects of policy at the end of the trap induces different current individual actions despite every agent agrees on current future allocations until the end of the trap: optimists consume more in the short run as they expect higher consumption and inflation in the medium run than pessimists. Optimists expect pessimists to consume less than them in the short run as they know that pessimists do not share their beliefs. But they expect pessimists to revise their beliefs at date T and, then, to consume more in the future, catching up with optimists. This expected revision of pessimists' beliefs contributes to the optimists' anticipation of a future boom. Symmetrically, pessimists expect optimists to consume more than them in the short run, but they also expect them to revise their expectations downward at date T, pushing the economy to a new recession and a longer trap.

Heterogeneous beliefs and rigid prices. To give some further understanding on how heterogeneity affects current aggregate demand, we extend our analysis to the case where prices are rigid over the whole period, as in Werning (2012). This implies that "the expected inflation channel" of forward guidance is off. Figure 11a plots the dynamics of aggregate consumption and inflation in an illustrative example where the trap lasts for 12 periods. The results looks pretty similar to our benchmark case. The only difference is that agents do not disagree on current real rates since inflation and nominal interest rates are both pegged. This leads then to share the same *growth rate* of consumption. However, they still disagree on future real interest rates and, thus, on future consumption, so that they have different *levels* of current consumption. Hence, even if disagreement has limited effects on inflation expectations or if these expectations are not affecting current households decisions, it would still dampen the effects of an Odyssean policy as long as agents form different expectations on future consumption.

3.3 Positive Implications

Four positive implications of our model relate to the empirical facts discussed in the previous section. We list them in the form of claims.

Claim 1. *Agents agree on an extended period of zero nominal rates.*

In all the equilibria that we study, the central bank will keep the interest rate at zero for an extended period of time (T_{cb}) and all agents agree on this. This replicates Fact 1. Yet, in the model, the fact that all agents expect zero interest rates until date-T_{cb} does not mean that they agree on the reasons why interest rates will remain so low.

Claim 2. *Agents disagree on the reasons why interest rates stay at zero until date-T_{cb}: optimists are expecting a policy accommodation from period T to T_{cb} while pessimists do not expect any policy accommodation before T_{cb}.*

This second claim is consistent with Fact 2 that we showed in SPF data: optimistic forecasters expected a policy accommodation (a negative deviation from their perceived Taylor rule) during the period of expected zero interest rates, while pessimistic forecasters did not. Let us also note that disagreement in the model concerns medium-term macroeconomic outcomes but agents agree on short-run variables. This echoes our findings that disagreement over 2 years ahead inflation and consumption increased well beyond its predicted values by 1 quarter ahead disagreement.

What are then effects of these heterogeneous beliefs? As previously discussed, they affect date-0 consumption decisions through the expectation of better economic conditions:[26]

Claim 3. *Agents who expect higher future consumption are also consuming more at date $t = 0$.*

This claims parallels Fact 3 on the Michigan survey, where households that anticipate better future economic conditions are also more likely to consider that the time is good to purchase durable goods.

Finally, Figure 7 illustrate that, when beliefs are heterogeneous, date-based forward guidance is not as effective as in a world where all agents are optimistic, i.e. all agents consider that the central bank is pursuing an Odyssean policy. Pessimists attenuate the current impact of future monetary accommodation. More generally, the higher the fraction of pessimist, the lower the impact of a date-based forward guidance policy on the macro-economy. In fact, when the fraction of pessimists is high enough the impact can become even negative, as pessimists misinterpret extra periods of accommodation for a more severe recession. We will discuss this issue formally in the next section. But, so far, what we obtained leads to the following claim:

Claim 4. *(Forward Guidance Puzzle) An announcement that coordinates agents' agreement on an expected interest rate path may have limited or negative effects on current and expected aggregate economic activity and inflation.*

This claim then explains how Fact 0 – forecasts of aggregate consumption and inflation have not reacted substantially or even with the wrong sign over the date-based forward guidance period – can coexist with a coordination on a long period of zero interest rates as stated

[26] Again, this may or may not involve the expectation of future higher inflation. As in Werning (2012), it is sufficient that agents expect higher future consumption.

by Fact 1. More generally, the mechanism that we emphasize can potentially explain why the effects US forward guidance policy on the economy has been much weaker than predicted by state of the art New Keynesian models as Del Negro et al. (2015) underlined (see Appendix C.4 for further analysis).

In the end, the model that we have built replicates the main stylized facts outlined in section 2. We next investigate how this potential heterogenous understanding of a date-based forward guidance announcement affects the optimal monetary policy at the ZLB.

4 Optimal policy with heterogeneous beliefs

In this section, we take the point of view of a central bank that has the commitment ability (so pessimists are wrong) and we discuss how the coexistence of optimists and pessimists affects the design of an optimal Odyssean forward guidance; that is, how optimal policy changes with α.[27]

To determine optimal policy, conditional to its own belief on the length of the trap T, the authority chooses a lift-off date according to (4) that maximizes the expected utility of agents, taking as given agents' optimal consumption, pricing decisions and beliefs. To avoid creating additional multiplicity of equilibria, we shall assume that the central banker knows α and does not need to infer it.[28]

In Appendix C.6, we proceed similarly to Gali (2008) to approximate the resulting welfare objective by a quadratic function. We show that, in the special case where marginal utility is equally elastic to consumption and labor – i.e., agents' coefficient of relative risk aversion γ equals the inverse of the Frisch elasticity of labor supply ψ – this functions is:

$$\mathbb{W} = \int_0^1 \mathbb{W}_i di \equiv -\varpi\theta^{-1} \int_0^1 \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\lambda c_{i,t}^2 + \pi_t^2\right)\right] di, \tag{9}$$

which looks very much like the textbook welfare approximation typical of New-Keynesian models with homogeneous beliefs.[29]

[27]We consider α as a given constraint. We provide a short discussion on how a monetary authority could affect the fraction of pessimists in the Conclusion.

[28]Given that the central bank observes α, it can commit to implement an Odyssean forward guidance whenever it is optimal. This way, we prevent the emergence of a coordination failure between the central bank and optimists. Indeed, if optimists behaved as if they were pessimists, then it would be optimal for the Odyssean central bank not to implement its commitment policy. In turn, this makes self-fulfilling optimists behaving like pessimists. Note that this kind of equilibrium is not a consequence of the heterogeneity of beliefs we consider in our model and can also emerge in the classical framework of Eggertsson and Woodford (2003).

[29]In the more general case where this particular restriction does not hold, the heterogeneity of beliefs leads

In this particular specification, general features of the optimal policy can be derived for a given length of the trap T and a fraction of pessimists α. Our main result, described in the proposition below, is that the optimal number of periods of accommodation after the end of the trap is a non-monotonic function of the share of pessimists.

Proposition 2. *In the special case where the coefficient of risk aversion equals the inverse of the Frisch elasticity, there exist two values $\underline{\alpha}$ and $\bar{\alpha}$ where $\underline{\alpha} > \bar{\alpha}$ such that, for a given T, the optimal policy $T_{cb}(\alpha, T)$ is:*

- *increasing in α, i.e. $T_{cb}(0, T) < T_{cb}(\alpha, T)$, for $\alpha < \underline{\alpha}$;*
- *decreasing in α, i.e. $T_{cb}(\alpha', T) > T_{cb}(\alpha'', T)$, for $\underline{\alpha} < \alpha' < \alpha'' < \bar{\alpha}$;*
- *equal to T, i.e. $T_{cb}(0, T) > T_{cb}(\alpha, T) = T$, for $\alpha > \bar{\alpha}$.*

Proof. See Appendix D.2. □

Proposition 2 suggests that a coordination of beliefs on an extended period of low interest rates can be detrimental when misunderstood. Indeed, a misinterpretation can exacerbate the consequences of the ZLB and therefore have more dramatic effects than just mitigating the effect of Odyssean forward guidance. As a result, the central bank can be better off not implementing an Odyssean forward guidance policy, no matter whether it is willing and able to commit to it.

On the other hand, when only a small share of agents misunderstand the Odyssean forward guidance, the central bank is better off reinforcing this policy. In such a case, a further drop in pessimists' consumption (as they wrongly interpret additional periods of low interest rate as a sign of a longer trap) is more than compensated by an increase in optimists' consumption. Thus, when the fraction of pessimists is sufficiently low, the optimal policy calls for increasing the period of low interest rates T_{cb}.

Proposition 2 states the existence of a "bad news" channel of forward guidance in presence of heterogeneous beliefs. By fixing a lift-off date beyond the end of the trap, the central bank induce pessimists to consume less than if the lift-off date is the end of the trap. This occurs because pessimists interpret the policy as a "bad news" and the authority has no instrument to confute such a belief.[30]

to additional terms for which the distribution of beliefs play an important role. We refer the interested reader to the Appendix C.6 for more theoretical details. Our derivation in the special case is also close to the one of Bilbiie (2008) who looks to a case in which agents have limited asset market participation.

[30] The "bad news" channel is not the only channel at work, but the most important. See Appendix C.7 for details.

Numerical illustration. Here, we present numerical simulations illustrating how results obtained in the special case where $\psi = \gamma$ extend to the general case $\psi \neq \gamma$.

Figure 8 plots the number of periods of extra accommodation as a function of the fraction of pessimists. We contrast the optimal policies after a large shock ($\xi = -0.01$) in the upper panel and after a small shock ($\xi = -0.007$) in the lower panel. In both cases we consider a shock lasting for 20 periods.[31]

In each panel, there are three types of curves: solid, dashed and dotted. The solid line corresponds to the optimal policy when $\lambda = 0$. This is a limit case when the authority only cares about inflation. In this case, the relation is hump-shaped as described in Proposition 2: the presence of pessimists forces the central bank to extend its monetary stimulus, until the contractionary effects that are growing with the share of pessimist outweigh the benefits of additional stimulus. Then, the central bank starts reducing the length of its stimulus and eventually reaches a point where it prefers not to implement Odyssean forward guidance.

The dotted and dashed lines represent the optimal policy when $\lambda = 50$ (a case where the policy maker's loss function puts a large weight on the variance of output gap) and $\psi = \gamma$ or $\psi = \gamma/4$ respectively. The two curves illustrate that the optimal length of extra accommodation becomes a monotonically decreasing function of α for a sufficiently high ratio γ/ψ. This illustrates that, when deviating from the condition $\psi = \gamma$, additional welfare cost terms appear due to heterogeneity and these terms reduce the incentive of the authority to generate disagreement by further reinforcing Odyssean forward guidance.

Finally, let us comment on how policy reactions vary with the size of the shock. Ceteris paribus, with larger shocks, the contractionary effect of pessimists increases. For sufficiently low fractions of pessimists, the optimal number of extra-accommodation after the end of trap increases when the shock is larger. Yet, the threshold value of (the fraction) of pessimists beyond which the central bank prefers not to use its commitment ability decreases when the size of the shock increases.

5 Conclusion

In this paper, we have shown a form of disagreement among professional forecasters and households on future monetary policy in the period when the Fed implemented a date-based

[31]Figure 11 presents similar results obtained with alternative specifications of the Phillips curve. In Panel (a) we shut down the NK Phillips curve setting redetermined prices. In Panel (b) we assume that firms are owned by optimists, that is, we set firms' expectations equal to optimist expectations (instead of being equal to the average expectations as in the standard case).

forward guidance policy. We also showed that such a form of disagreement may lead date-based forward guidance to be detrimental. The core of our analysis relies on the assumption that agents are unsure about whether future monetary policy is going to be Odyssean, i.e. accommodative, or Delphic, i.e. forced to the ZLB by future fundamentals. To conclude, let us now discuss how the central bank could overcome this uncertainty on its future policy.

Can a Central Bank signal its future policy type by current actions? In our benchmark model, policies are constrained by the ZLB during the trap and, in particular, before the $E_{0,o}(T)$. As a result, a pure Delphic or an Odyssean policy implies similar policy rates until the end of the trap, thus sustaining multiple interpretations by private agents. Could the Odyssean central banker signal her type by changing interest rates?

Such a signaling before the end of the trap actually happens is difficult because the Delphic type can easily replicate the same signal, without bearing the "time inconsistency" costs of forward guidance. In addition, signaling using interest rates would imply raising the current nominal rate, which may have extremely costly effects in comparison with the benefits of forward guidance. More generally, Barthélemy and Mengus (2016) show that signaling Odyssean forward guidance could only take place before the liquidity trap begins.

Alternative signaling devices. Other tools for such a signaling may be available such as communication, transparency on central banks' beliefs (e.g. by releasing forecasts) or unconventional monetary policy instruments (as quantitative easing or loan policies such as the targeted long-term refinancing operation (TLTRO) implemented by the ECB).

Communication. One mean to limit the fraction of pessimists would be to communicate on her commitment. As argued by Woodford (2012), the announcement of a clear commitment by the central banker can be a way to make costly *ex post* deviations from the central bank's commitment ("to cause embarrassment" to borrow Woodford's words) and, so, to convince pessimists to change their views on the central banker's type. Yet, such announcements can be also made by no-commitment central bankers. And again, the latter will not bear the cost of reneging it while it pockets the *ex-ante* gains related to increasing the proportion of optimists.

In the end, communication on commitment is plagued by cheap talk problems: to the extent that it costs nothing more to the no-commitment central banker, such communication provides no information on central bankers' types. Such cheap talk aspect of forward guidance announcements is developed in Bassetto (2015).

Transparency. The second mean to increase the proportion of optimists is to communicate on fundamentals and to try to coordinate agents on shorter liquidity traps than the horizon of the zero interest rate policy. This can be achieved by releasing forecasts of macro-economic variables, as frequently done by central banks, or by committing to temporarily overshoot the inflation target (the Fed, the Bank of England and more recently the Bank of Japan have made such announcements). Yet, this policy can also be mimicked by no-commitment central bankers and it involves only some unpalatable costs when deviating *ex-post* from the commitment.

Quantitative policies and long-term provision of liquidity at fixed rates. Quantitative policies and the purchase of long maturity bonds at very low rates, or supplying liquidity at long horizons at zero interest rates amount to "putting your money where your mouth is". It can provide a strong signal on the central bank's willingness not to raise policy rates in the future. Indeed, such policies can imply a cost to the central bank in case it deviates from its commitment: a rise in interest rate may lead to a depreciation of purchased assets and so to capital losses to the central bank (see Bhattarai et al., 2014, for an investigation of this mechanism). Yet, such a signaling device hinges on the central bank's aversion for capital losses and the extent to which it cannot be rescued by the fiscal authority in case of negative equity.

Forecast revisions	Optimists	Pessimists	Pessimists and others
2011Q4			
Share of individuals	19%	29%	81%
Consumption	.32 (.28) [**/#]	-.20 (.19)	-.05 (.41)
Inflation	.19 (.22) [**/#]	-.22 (.14)	-.12 (.55)
Nominal rates	-.41 (.46)	-.38 (.30)	-.42 (.44)
Revisions of shadow Taylor-rate	.35 (.25) [***/###]	-.37 (.14)	-.16 (.37)
2012Q1			
Share of individuals	22%	23%	78%
Consumption	.79 (.33) [***/##]	.13 (.24)	.19 (.24)
Inflation	.48 (.29) [***/###]	-.26 (.29)	-.12 (.30)
Nominal rates	-.37 (.55)	-.04 (.08)	-.04 (.07)
Revisions of shadow Taylor-rate	.86 (.55) [**/#]	-.17 (.31)	.05 (.35)
2012Q4			
Share of individuals	36%	24%	64%
Consumption	.20 (.19) [***/###]	-.26 (.22)	-.21 (.26)
Inflation	.19 (.23) [***/#]	-.32 (.32)	-.17 (.36)
Nominal rates	-.04 (.15)	.02 (.02)	-.02 (.06)
Revisions of shadow Taylor-rate	.23 (.30) [***/##]	-.36 (.27)	-.27 (.26)
Corr(rev. inflation, rev. rates)			
2009Q1-2011Q3	.41 (.07)	.15 (.07)	.24 (.07)
2011Q4-2012Q4	-.26 (.20)	.38 (.25)	.22 (.15)

Table 1: Average revisions of 2 years ahead forecasts across groups of forecasters.

This table reports the cross-section average of individual revisions for consumption, inflation and short-term nominal interest rates forecasters 2 years ahead for different groups of forecasters. Forecasters are classified as 'Optimists' when both their inflation and consumption revisions are above the cross-sectional mean at the specified date, as 'Pessimists' when both inflation and consumption revisions are below the cross-sectional mean. The group 'Pessimists and others' gathers all forecasters excluding the optimists. Standard deviations are reported in parenthesis. t-stats significance levels are reported in brackets: ***, **, * indicate that the average revisions of optimists are significantly different from the average revision of pessimists at the 1%, 5% and 10% levels; ###, ##, # indicate that the average revisions of optimists are significantly different from the average revision of pessimists and others at the 1%, 5% and 10% levels.

	Optimists	Pessimists	Pessimists and others
Averages observed in 2011m9			
Fraction of respondents	5%	50%	95%
Good times for durable	.50	.27	.25
Inflation	6.64	1.77	3.51
Averages observed in 2012m2			
Fraction of respondents	13%	28%	87%
Good times for durable	.55	.30	.36
Inflation	5.50	1.37	3.10
Averages observed in 2012m10			
Fraction of respondents	15%	30%	85%
Good times for durable	.46	.24	.29
Inflation	7.34	1.95	3.37

Table 2: Average of qualitative forecasts across groups of households.

This table computes the cross-sectional mean for current durable consumption (DUR), expected inflation over next 12 months and expected short-term nominal interest rates (RATEX) over next 12 months when forecasters are sorted classified according to their expected inflation/business conditions/nominal interest rate over next 12 months. Optimistic forecasters expected higher inflation that the cross-sectional mean and had a positive view of the business/financial conditions over the next 12 months. Pessimists expected lower inflation than the cross-sectional mean and had a negative view of the business/financial conditions over the next 12 months. Others include all households except optimists. All forecasters considered expect constant or decreasing nominal interest rates over the next 12 months.

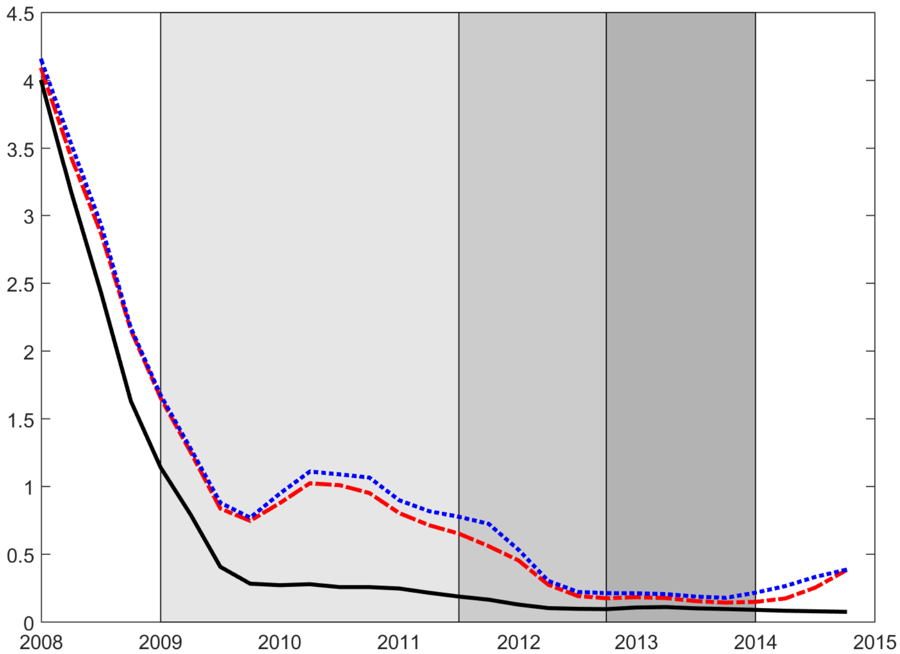
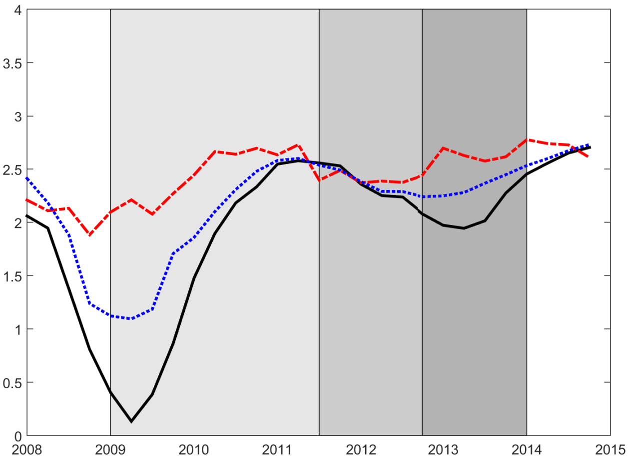


Figure 1: Average of individual short-term interest rate forecasts.

The chart displays the evolution of a moving average over the last 4 quarters of the average of individual forecasts of the 1-quarter (black line), 1-year (red line), and 2-year (blue line) ahead individual mean point forecasts for 3-month T-Bill interest rate. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

(a) Consumption



(b) Inflation

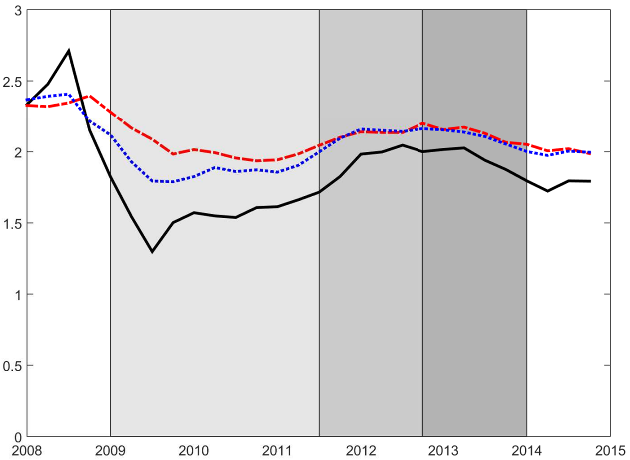


Figure 2: Average of individual consumption growth and inflation forecasts.

The figure shows the evolution of a moving average over the last 4 quarters of the average of individual forecasts of 1-quarter (black line), 1-year (red line), and 2-year (blue line) ahead individual mean point forecasts for real consumption growth and CPI inflation. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

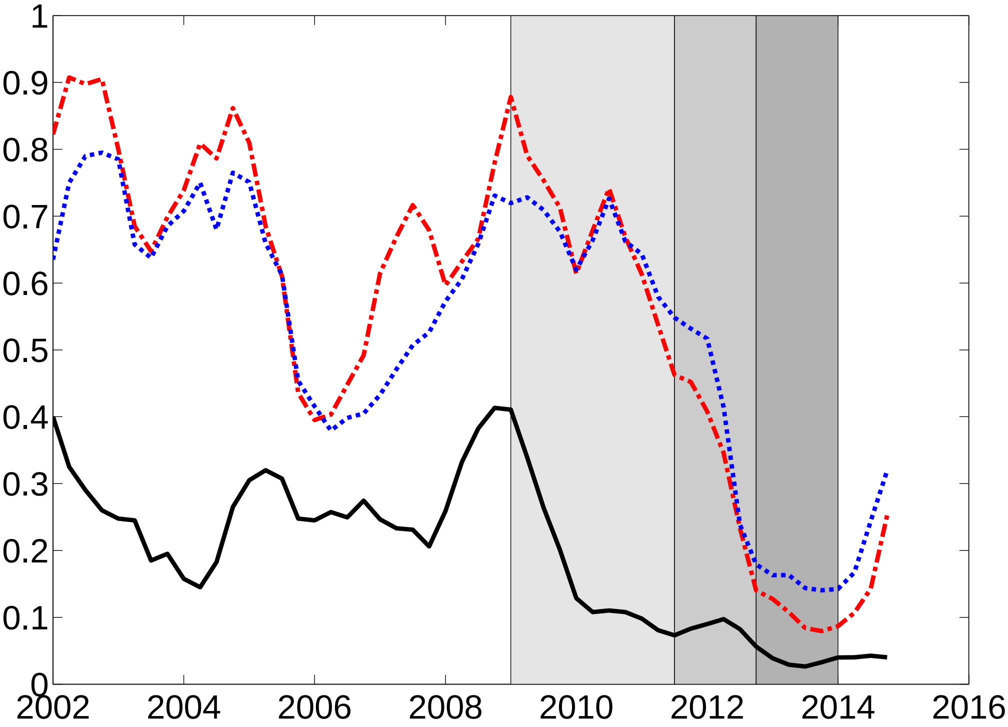
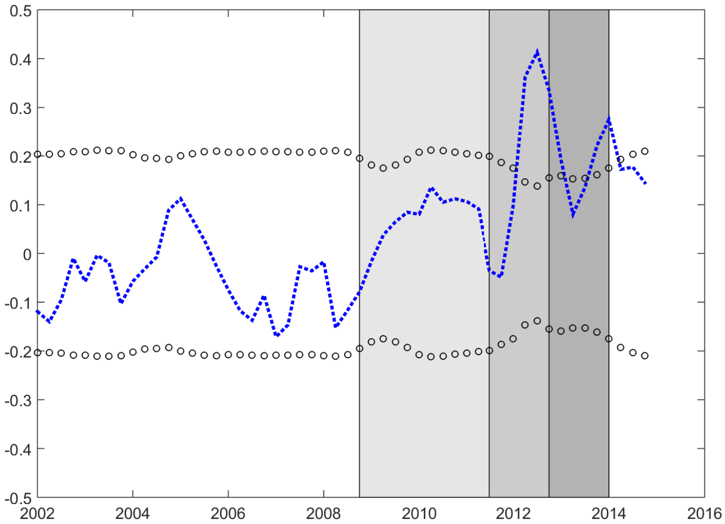


Figure 3: Disagreement about future short-term interest rates.

The chart displays the evolution of a moving average over the last 4 quarters of the 75/25 inter-quantile range in the distribution of 1-quarter (black line), 1-year (red line), and 2-year (blue line) ahead individual mean point forecasts for 3-month T-Bill interest rate. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

(a) Consumption - 2 years ahead



(b) Inflation - 2 years ahead

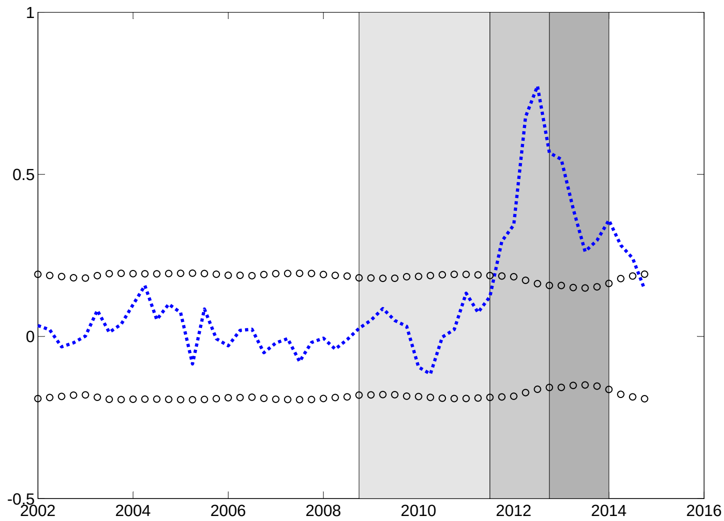


Figure 4: Excess disagreement about future consumption and inflation .

The Figure plots the residuals of a regression of the (log) disagreement on (1-year and 2-year ahead) inflation forecasts on the (log) disagreement on (1-year and 2-year ahead) short-term interest rate and disagreement on 1-quarter ahead inflation forecast. The regression is estimated on a pre-crisis sample (1982Q2-2008Q4). Black circles give the bands of a 95% confidence interval that take into account autocorrelation and heteroskedasticity of the residuals. The shaded areas correspond to the periods of the ZLB and “open-date” forward guidance, “date-based” forward guidance and the “state-contingent” forward guidance.

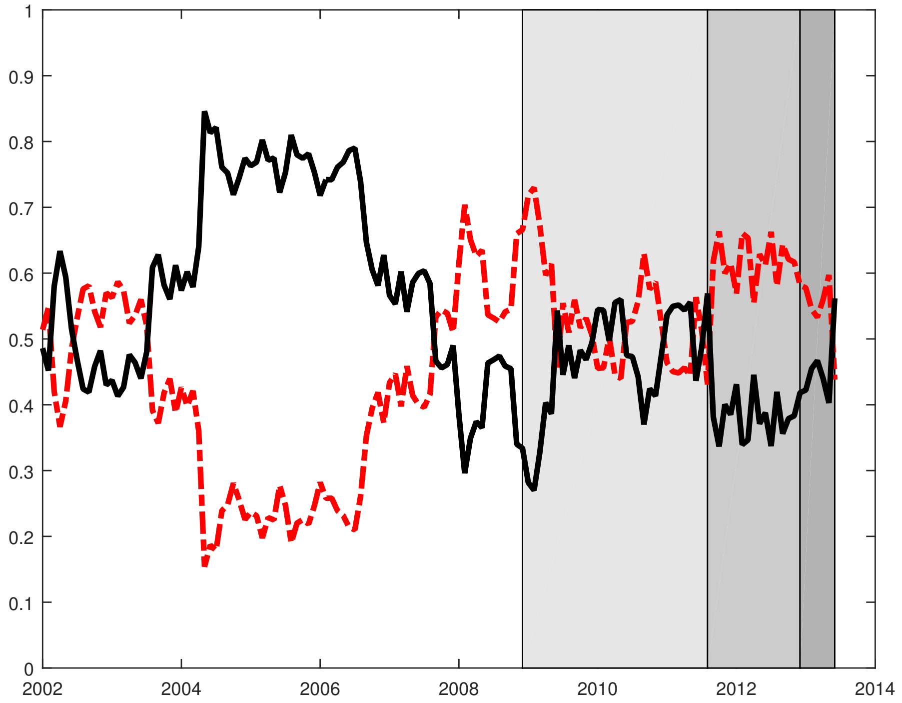


Figure 5: Interest rate expectations in the Michigan survey of households.

The chart displays the evolution of the share of respondents to the survey who thought that over the next 12 months, interest rates will increase ((blue line), stay constant (red line) or decline (dark line).

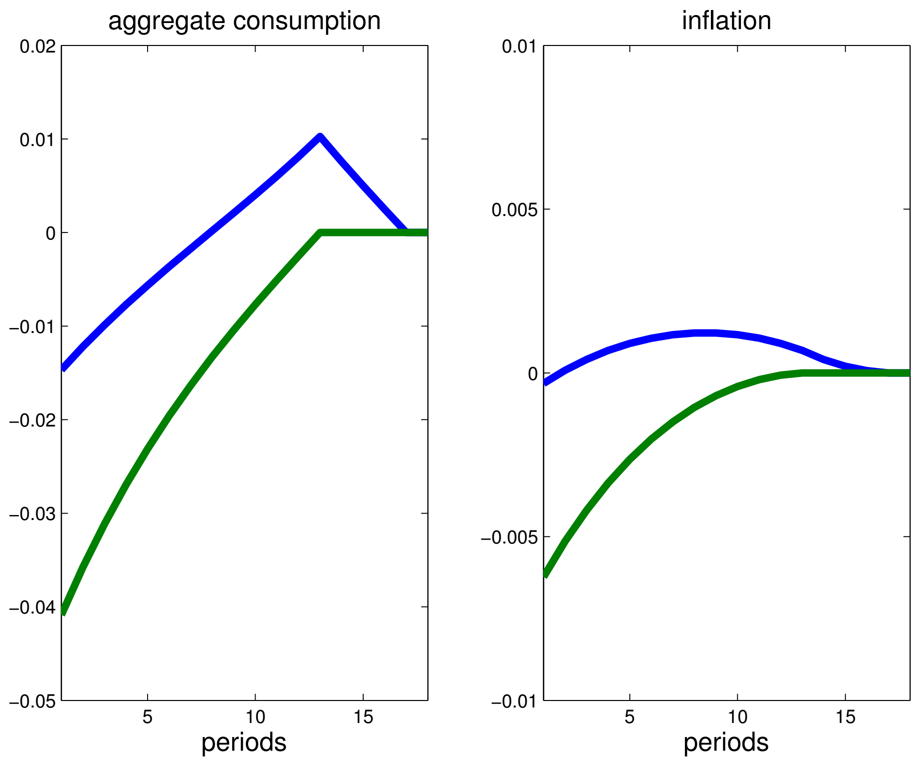


Figure 6: The effect of Delphic (green) and Odyssean (blue) policies.

We consider a shock ($\xi = -0.01$) on the discount rate that lasts 12 quarters and implies a drop of consumption of 4% at impact in the absence of Odyssean forward guidance, which provides for 4 extra quarters of accommodation. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027. We use $\psi = 2$ to compute the optimal policy, so as to have $\gamma = \psi$ as in our benchmark case.

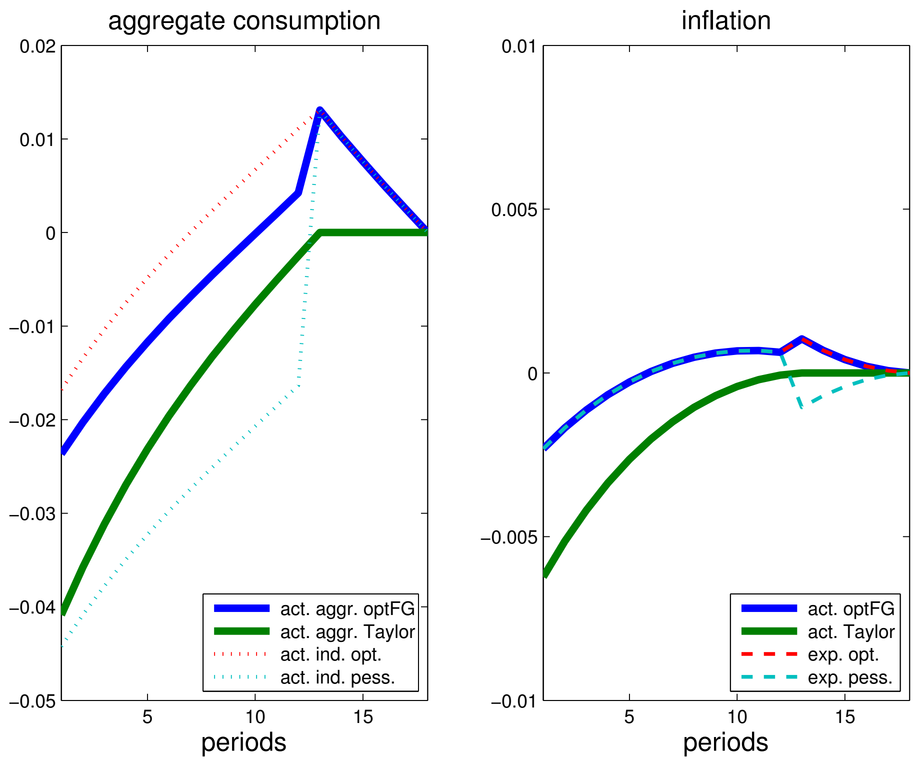
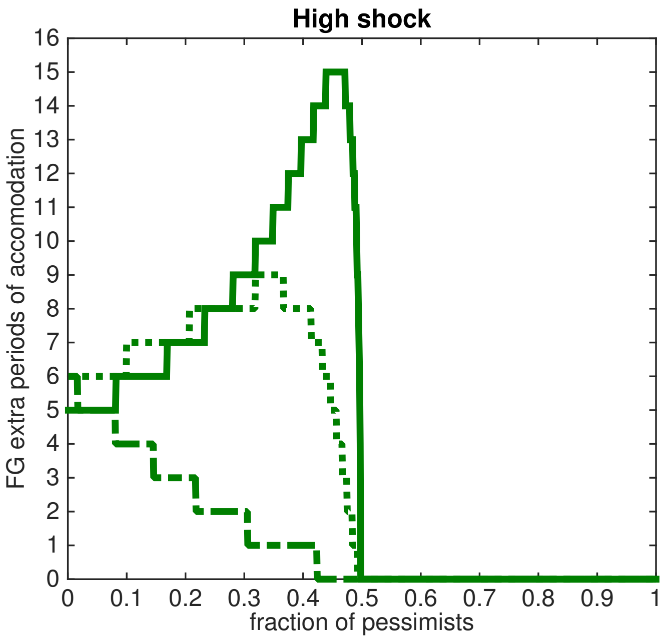


Figure 7: The effect of Odyssean (blue) with a fraction $\alpha = .25$ of pessimists.

We consider a shock ($\xi = -0.01$) on the discount rate that lasts 12 quarters and implies a drop of consumption of 4% at impact in the absence of Odyssean forward guidance, which provides for 4 extra quarters of accommodation. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027. We use $\psi = 2$ to compute the optimal policy, so as to have $\gamma = \psi$ as in our benchmark case.



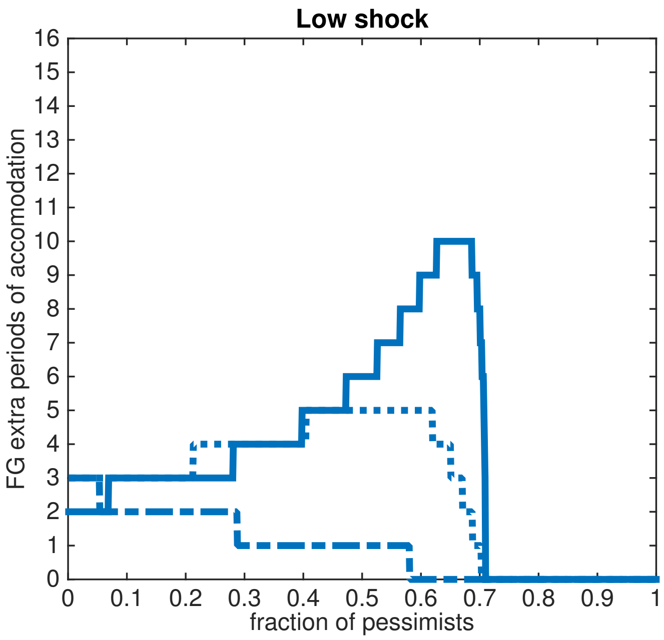


Figure 8: We plot the optimal $T_{cb}(\alpha, 20) - 20$ for $\xi = -.007$ (lower panel) and $\xi = -.01$ (upper panel) for: $\lambda = 0$ with a solid line; $\lambda = 50$ and and $\gamma/\psi = 1$ with a dotted line; $\lambda = 50$ and and $\gamma/\psi = 4$ with a dashed line. λ is the weight on the average volatility of working hours in the loss function of the central bank. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2%, The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.

A Additional evidence: Household survey

Only a few households in the Michigan survey have expectations that are consistent with what New-Keynesian models predict the impact of forward guidance policy should be: they foresee stable or lower interest rates, more inflation, a boom in future activity and therefore want to consume more today. That their number is quite limited is consistent with the evidence in Bachmann et al. (2015). These authors showed using data from the Michigan survey that, on average, when the US economy was at the ZLB, households who expected higher inflation expectations also considered the time as less favorable to consume.

Werning (2012) showed that forward guidance does not require that agents expect higher future inflation (hence lower future real interest rates) to have a positive impact on consumption today. Such policy can be effective if agents only expect a future boom. This suggests to conduct the same analysis than before but with a broader definition of optimistic and pessimistic households that does not depend on their inflation expectations. More specifically we sort households into two categories only: optimists if they expect better future conditions and pessimists otherwise. Table 3 shows the average of macro expectations of these two groups observed at the dates following date-based forward guidance announcements. Three comments can be made. First, two views (an optimistic and a pessimistic ones) about the macroeconomic outlook prevailed within the group of households who foresaw stable or lower interest rates. Second, there is a now substantial number, sometimes a majority, of optimistic households, who have expectations consistent with the effects of forward guidance as emphasized in Werning (2012): after the forward guidance announcements they expected better future economic conditions and were likely to consume more today. Third, and again consistent with Bachmann et al. (2015)'s results aforementioned, households who expected better economic conditions in the future and consumed more after date-based forward guidance, were also expecting lower inflation on average.

	Optimists	Pessimists
Averages observed in 2011m9		
Fraction of respondents	14%	86%
Good times for durable	.64	.21
Inflation	2.95	3.90
Averages observed in 2012m2		
Fraction of respondents	60%	40%
Good times for durable	.56	.19
Inflation	2.45	3.99
Averages observed in 2012m10		
Fraction of respondents	68%	32%
Good times for durable	.48	.17
Inflation	2.81	4.69

Table 3: Average of qualitative forecasts across groups of households.

This table computes the cross-sectional mean for current durable consumption (DUR), expected inflation over next 12 months and expected short-term nominal interest rates (RATEX) over next 12 months when forecasters are sorted according to their expected business conditions and nominal interest rate over next 12 months. Optimistic forecasters had a positive view of the business/financial conditions over the next 12 months. Pessimists had a negative view of the same business/financial conditions. All forecasters considered expect constant or decreasing nominal interest rates over the next 12 months.

B Additional evidence: patterns in various measures of macroeconomic uncertainty

In this paragraph, we investigate whether forward guidance has had effects on another channel than just the first moments namely a reduction in uncertainty. Figure 9 plots three different measures of uncertainty between 2002 and 2016: the CBOE financial market volatility index (VIX), the macroeconomic uncertainty measure developed by Jurado et al. (2015) (JLN), the economic policy uncertainty measure developed by Baker et al. (2016) (EPU). A first observation is that, consistent with e.g. Bianchi and Melosi (2015b), macroeconomic uncertainty increased as the economy hit the ZLB and the usual monetary policy stabilisation instrument has been lost. Yet, when the Fed switched to date-based forward guidance, there is no clear common pattern in the three measures of uncertainty. The index by Jurado et al. (2015) remained almost unaffected, while economic policy uncertainty measure and the VIX both peaked around the time of the first announcement. In sum, this evidence is not consistent with a systematic reduction of uncertainty due to date-based forward guidance announcements.[32]

We also checked that the fact that date-based forward guidance is associated with an increase in disagreement about medium-run forecasts of consumption growth and inflation, illustrated in Figure 4, does not primarily result from variations in macroeconomic uncertainty.

We regressed the disagreement about 2-year ahead forecasts of consumption (resp. inflation) on the disagreement about 2-year ahead forecasts of short-term nominal interest rates estimated on a pre-crisis sample, controlling for the disagreement about 1-quarter ahead consumption and inflation forecasts as previously, well as for the JLN measure of macroeconomic uncertainty or the EPU measure of economic policy uncertainty.

Figure 10 displays the residuals from these regressions. It shows that the beginning of the date-based forward guidance policy is again a striking outlier: controlling for fundamental uncertainty, disagreement about future inflation should have been significantly lower given how much agents agreed on future short-term interest rates. So, changes in uncertainty are not the main explanation for why the normal time correlation between disagreement about future interest rates and disagreement about future fundamentals disappears at the time of forward guidance.

[32] Note, in contrast, that state-contingent forward guidance has had a large negative impact on the economic policy uncertainty measure by Baker et al. (2016). As a result, the reduction in uncertainty channel seems to be more relevant for state-contingent than for date-based forward guidance.

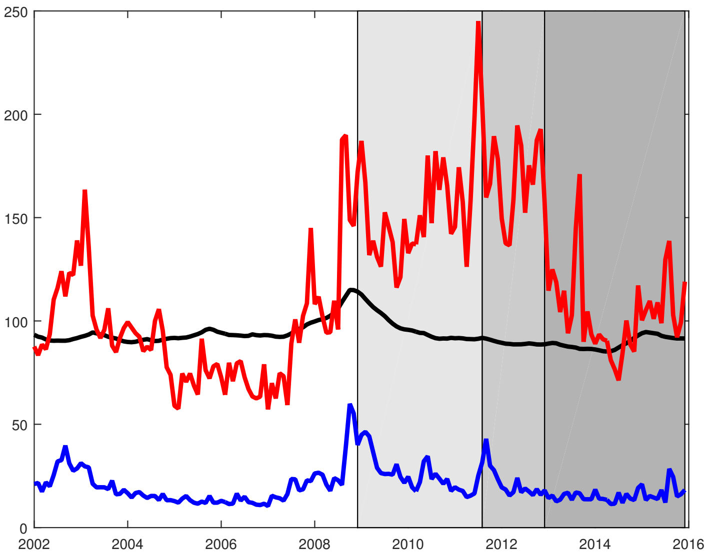
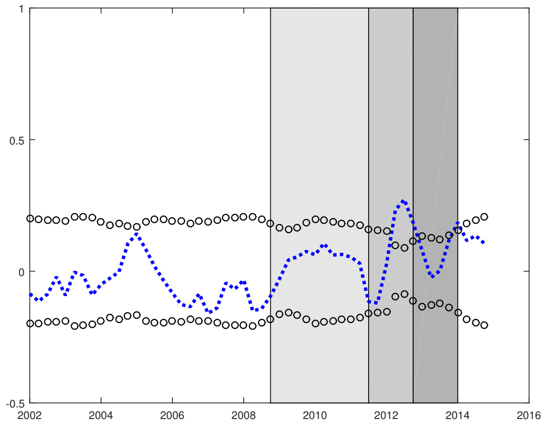


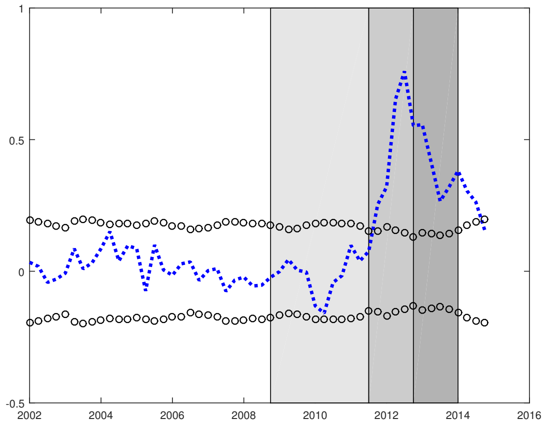
Figure 9: Measures of uncertainty.

The chart displays the evolution of 3 different measures of uncertainty: the CBOE financial market volatility index (VIX, blue line), the macroeconomic uncertainty measure developed by Jurado et al. (2015) (JLN, dark line), the economic policy uncertainty measure developed by Baker et al. (2016) (EPU, red line).

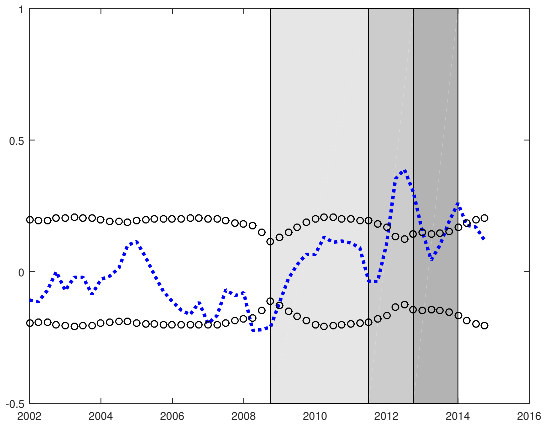
(a) Consumption - 2 years ahead - EPU



(b) Inflation - 2 years ahead - EPU



(c) Consumption - 2 years ahead - JLN



(d) Inflation - 2 years ahead - JLN

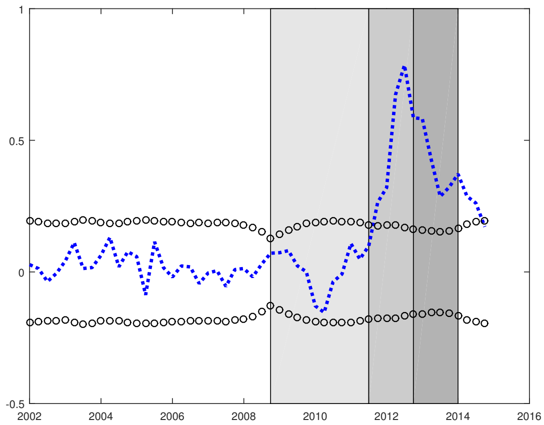


Figure 10: Excess disagreement about future consumption and inflation, controlled by uncertainty.

The Figure plot the residuals of a regression of the (log) disagreement on (1-year and 2-year ahead) inflation and consumption forecasts on the (log) disagreement on (1-year and 2-year ahead) short-term interest rate and disagreement on 1-quarter ahead inflation forecast and the uncertainty measure (JLN : Jurado et al. (2015) and EPU : economic policy uncertainty measure developed by Baker et al. (2016)). The regression is estimated on a pre-crisis sample (1982Q2-2008Q4). Black circles give the bands of a 95% confidence interval that take into account autocorrelation and heteroskedasticity of the residuals. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

C Detailed model derivation

In this appendix, we micro-found the linear model that we use in the core of this paper.

C.1 Environnment

The economy is populated by a continuum of households, firms and the central bank. Time is discrete and indexed by $t \in \{0, ...\infty\}$.

Household. The household family is constituted by a continuum of agents of mass one indexed by $i \in [0, 1]$. Each agent decides how much to work, consume and save in order to maximally contribute to the household welfare:

$$U = \int_0^1 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) di, \tag{10}$$

where $C_{i,t}$ and $L_{i,t}$ are respectively consumption and labor supply of agent i in period t. The parameter $\beta \in (0, 1)$ is a discount factor, the parameter $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution, and the parameter $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply. The variable ξ_t is a preference shock discussed below.

Each agent manages a portfolio representing a fraction of the household wealth. Between periods t and $t + 1$, agent i deals with the following flow budget constraint:

$$B_{i,t} = R_{t-1} B_{i,t-1} + W_t L_{i,t} + D_t - P_t C_{i,t} + Z_{i,t}, \tag{11}$$

where $B_{i,t}$ are bond holdings of the agent between periods $t-1$ and t, R_{t-1} is the gross nominal interest rate on bond holdings between periods $t - 1$ and t, W_t is the nominal wage rate in period t, D_t is the difference between nominal profits received and nominal lump-sum taxes paid, by each agent in period t (we assume here diffuse ownership), and P_t is the price of the final good in period t. The agent can borrow (formally, bond holdings can be negative), but the household is not allowed to run a Ponzi scheme. Finally, the term $Z_{i,t}$ denotes a nominal intra-household transfer by agent i.

Intra-Household risk sharing. Each period is divided into three stages. In the first stage, current shocks hit and agents observe them. At this stage agents form their beliefs on the state of the world. In the second stage of each period, agents can implement a feasible transfer plan in which each agent $i \in [0, 1]$ at date t contributes by an amount $Z_{i,t}$ and such that:

$$\int_0^1 Z_{i,t} di = 0. \tag{12}$$

only if every agent agrees on it. Without loss of generality,[33] we assume that when no unanimity is reached, then no transfers are made; in such a case each agent owns the wealth resulting from her own portfolio management. Let us therefore introduce the following formal definition.

Definition 3. *An implementable transfer plan at time t is a feasible transfer plan $\{\hat{Z}_{i,t}\}_{i=0}^{1}$ such that*

$$E_{t,i}[U_t|\{\hat{Z}_{i,t}\}_{i=0}^{1}] \geq E_{t,i}[U_t|\{Z_{i,t}\}_{i=0}^{1}],$$

for each $i \in [0,1]$ and each feasible transfer plan $\{Z_{i,t}\}_{i=0}^{1}$.

In the last stage, once intra-household wealth transfers are carried out, each agent decides on her own labor supply and consumption, based on their own individual beliefs and taking other agents' decisions as given. The crucial assumption we are making here is that agents cannot commit on future transfers: each period they decide under discretion. We also assume that the whole mechanism is common knowledge.

Firms. Production is implemented in the context of a standard monopolistic competition environment. The final good is produced by competitive firms using the technology: $Y_t = (\int Y_{j,t}^{(\theta-1)/\theta} dj)^{\theta/(\theta-1)}$. Y_t denotes output of the final good and $Y_{j,t}$ denotes input of intermediate good j. The parameter θ is the elasticity of substitution between intermediate goods. Final good firms have perfect information and fully flexible prices. Profit maximization of firms producing final goods implies the following demand function for intermediate good j:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{\theta} Y_t, \tag{13}$$

where $P_{j,t}$ is the price of intermediate good j and P_t is the price of the final good. Furthermore, the zero profit condition of firms producing final goods implies $P_t = (\int P_{j,t}^{1-\theta} dj)^{1/(1-\theta)}$. Each intermediate good j is produced by a monopolist using the linear technology:

$$Y_{j,t} = L_{j,t}, \tag{14}$$

where $Y_{j,t}$ is output and $L_{j,t}$ is labor input of this monopolist.

Monopolists producing intermediate goods are subject to a price-setting friction as in Calvo (1983). Each monopolist can optimize its price with probability $1-\chi$ in any given period. Finally, we assume that firms' stocks are held by households in equal shares.

[33] To explain why is without loss of generality, we need to introduce a bit more structure. See footnote 34 below.

Monetary Policy. Monetary policy is specified as in the main text.

Equilibrium. Given the detailed description of the model, we are now ready to define a (perfect bayesian) equilibrium:

Definition 4. *For a given sequence of shocks $\{\xi_\tau\}_{\tau=0}^{\infty}$, an equilibrium at time $t = 0$ is defined by the set of following conditions:*

i) given T_{cb} and a set of beliefs about the end of the trap and the type of the authority $\{E_{i,0}[T], E_{i,0}[\varrho]\}_{i\in[0,1]}$,

$$\{C_{i,t}, L_{i,t}, B_{i,t}, D_t, R_t, W_t, Z_{i,t}, P_t\}_{i\in[0,1],t\geq 0}$$

solves household's and firms' problems, satisfies the monetary policy rule (4) and so that markets clear;

ii) given a type of the central bank and given agents' optimal reaction, T_{zlb} solves the central bank's problem;

iii) agents beliefs $\{E_{i,0}[T], E_{i,0}[\varrho]\}_{i\in[0,1]}$ are updated at each period t using all the information observed by agents. They may follow Bayes' law whenever possible.

In this definition, we require that agents' actions maximize expected utility conditional on agents' beliefs about the length of the trap which have to be consistent with the observed current allocation, the size of the discount shock (not the length) and the optimal monetary policy given the disagreement in the economy. The equilibrium is defined in terms of time 0 posteriors of agents' beliefs and the actual allocation. Agents of each type anticipate, consistently with their own beliefs, that the other type will update once the truth unfolds.

Condition (iii) establishes that agents' beliefs must be rational expectations in the sense that any available observable produced in equilibrium will be used by agents to restrict their beliefs about the length of the trap. In this respect, *we do not assume any informational friction or ad-hoc asymmetry*. As said, the only two elements that are not directly observable to agents are the length of the trap and the commitment-type of the authority.

In general, the set of equilibria satisfying this definition is large. In the following, we will identify a subset of these equilibria that feature similar patterns as what is observed in the data, starting with the agreement on the path of interest rates.

In addition, we are always focusing on equilibria as in Eggertsson and Woodford (2003) or Werning (2012), where the trap only results from the discount factor shock, and so we rule out either permanent expectation-driven liquidity traps as in Benhabib et al. (2001) or transitory self-fulfilling ones as in Mertens and Ravn (2014).

C.2 Intra-household risk-sharing

In this subsection, we derive our result on endogenous risk sharing. Disagreement has major consequences for the dynamics of intra-family transfers. At the second stage of each period, agents need to decide on the wealth transfers. In the absence of disagreement, this would optimally result in an even distribution of wealth. Yet, the type of the central bank will be revealed only once date $E_{o,t}[T]$ is reached. Before that date, agents have different beliefs on the future course of the economy and so on which transfer plan maximizes family welfare; this prevents transfers from happening before the truth unfolds. In any case, all agents anticipate that they will share their wealth in the future as soon as they have evidence on which they cannot disagree any longer. This implies that no transfer plans can be implemented before date $E_{o,t}[T]$.

The following proposition states this formally.

Proposition 3. *Consider the case of heterogeneous beliefs about the end of the trap, namely $E_{o,0}[T] < E_{p,0}[T]$, then the only equilibrium sequence of implementable plans of transfers $\{\{Z^*_{i,t}\}^1_0\}^\infty_{t=0}$ is the one providing for $\{Z^*_{i,t}\}^1_0 = 0$ at each $t \neq E_{o,0}[T]$ and $\{Z^*_{i,E_{o,0}[T]}\}^1_0$ such that*

$$U_c(C_{i,T}) = U_c(C_{j,T}) \text{ for } t = E_{o,0}[T], \tag{15}$$

namely, the marginal utility of consumption is equal between types at the time where the truth unfolds, which implies $B_{i,t} = B_{j,t}, \forall (i, j)$ for $t \geq E_{o,0}[T]$.

Proof. See Appendix D.1. □

As no transfers are made during the period of the trap, the two types of agents then consume according to their beliefs, managing the share of wealth that they hold at the beginning of the trap.

It is worth to remark that proposition 3 relies on the assumption that households cannot commit to future transfers. As a consequence, agents of each type anticipate that, whatever their financial position, intra-household wealth will be equalized at a future date, when the truth will eventually unfold. Before that date, intra-family transfers, even if they were implemented,[34] cannot change agents' perceptions of their permanent income, and so cannot

[34] With different opinions about which plan achieves the first best, agents cannot implement any transfer plan. However, this rule has the mere role of selecting a unique feasible plan when agents disagree. That is, another backup rule would not change the results. For example, we could have equally assumed that, when agents disagree, a dictator decides on their transfers. Given that the dictator cannot enforce future transfers (no commitment), agents commonly know that, from some future date onward, they will agree again, and so, their wealth will be equalized. In this case, the dictator's transfers cannot affect the perceived permanent income of an agent, and so cannot change agents consumption-saving plans.

affect current consumption-saving choices. In other words, as they expect wealth to be equalized in the future – even though not at the same level – but anticipate different paths of real interest rates, pessimists and optimists select different paths of consumption. If different transfers are implemented, pessimists and optimists both modify their portfolio choices, keeping consumption paths unmodified and anticipating future transfers.

Finally, once we obtain 3, we can log-linearize our model around the unique steady state where the ZLB is not binding.

C.3 Aggregate Behavior and the New-Keynesian Phillips Curve.

Following standard steps, we can write down the log-linearized versions of optimality conditions as:

$$c_{i,t} = -\tfrac{1}{\gamma}\left(E_{i,t}\xi_{t+1} - \xi_t + r_t - E_{i,t}\pi_{t+1}\right) + E_{i,t}c_{i,t+1}, \tag{16}$$

$$\gamma c_{i,t} + \psi l_{i,t} = w_t - p_t \tag{17}$$

Notice that that $\xi_t < 0$ in the trap and $\xi_t = 0$ out of the trap. This means that an exit form the trap, say at time $t+1$, implies $\xi = E_{i,t}\xi_{t+1} - \xi_t > 0$. So, the term $\xi = E_{i,t}\xi_{t+1} - \xi_t$ is positive at the time of reverting to normal times and equals 0 otherwise. As a result, the Euler equation (16) implies that consumption decreases at the beginning of the liquidity trap before it gradually increases during the trap.

Aggregate behavior. Assuming that ξ can be anticipated a period in advance and by solving forward, we obtain that individual consumption equals:

$$c_{i,t} \quad = \quad -\frac{1}{\gamma}E_{i,t}\left[\sum_{\tau=t}^{\infty}(r_\tau - \pi_{\tau+1} + \xi_{\tau+1} - \xi_\tau)\right]$$

and aggregate consumption equals:

$$c_t \quad = \quad -\frac{1}{\gamma}E_t\left[\sum_{\tau=t}^{\infty}(r_\tau - \pi_{\tau+1})_{\tau+1} + \xi_{\tau+1} - \xi_\tau\right]$$

Notice that as long as agents do not disagree on the size of the shock (this is the case as they observe it), but only on the future date on which it will unfold, it enters as a fix wedge in the IS curve. This wedge will disappear only at the optimistic date when agents will discover the truth.

New-Keynesian Phillips Curve. The optimal price setting for producer j is given by:

$$x_{j,t} = (1-\chi\beta) E_t^j \left[\sum_{\tau=t}^{\infty} (\chi\beta)^{\tau-t} w_\tau \right]$$

as standard in the sticky price literature. Aggregating over producers yields:

$$x_t = (1-\chi\beta) w_t + \chi\beta \int E_{i,t} x_{i,t+1} \mathrm{di},$$

which is a standard realtion. We obtain the New-keynesian Phillips Curve in the presence of heterogeneous beliefs as follows. By defining $\Delta_t \equiv \int E_{i,t} x_{i,t+1} \mathrm{di} - E_t x_{t+1}$, we can write x_t recursively as:

$$x_t = (1-\chi\beta) w_t + \chi\beta E_t x_{t+1} + \chi\beta\Delta_t$$

At the same time, $x_t = \frac{p_t - \chi p_{t-1}}{1-\chi}$ and so, we can write

$$p_t - \chi p_{t-1} = (1-\chi)(1-\chi\beta) w_t + \chi\beta E_t (p_{t+1} - \chi p_t) + (1-\chi)\chi\beta\Delta_t$$

Thus, by noticing that $\pi_t = p_t - p_{t-1}$, we obtain:

$$\pi_t = \frac{(1-\chi)(1-\chi\beta)}{\alpha} (w_t - p_t) + \beta E_t \pi_{t+1} + (1-\chi)\beta\Delta_t$$

By definition, $\Delta_t \equiv \int E_{i,t} x_{i,t+1} \mathrm{di} - E_t x_{t+1}$ and $x_{i,t}$ is a function of current and future wages (w_τs). As a result, we can rewrite Δ_t as follows:

$$\Delta_t = (1-\chi\beta) \sum_{\tau=0}^{\infty} (\chi\beta)^\tau \int E_{i,t} \left(w_{t+\tau+1} - \int E_{t+1}^i [w_{t+\tau+1}] \,\mathrm{di} \right) \mathrm{di}$$

which equals 0 in this case, yielding the New Keynesian Phillips Curve

$$\pi_t = \frac{(1-\chi)(1-\chi\beta)}{\chi} (w_t - p_t) + \beta E_t \pi_{t+1}, \tag{18}$$

which is identical to the one under homogeneous beliefs. This result crucially relies on the assumption that producers observe all current variables, wage included, and that there is a unique labor market. As a result, it is common knowledge that there will be no aggregate forecast error on the wage neither at present nor at a future date, which makes Δ nil.

C.4 Forward Guidance Puzzle

In our model, disagreement on aggregate consumption and inflation arises if and only if the share of pessimists is sufficiently low ($\alpha \in (0, \bar{\alpha})$), in which case the central bank implements an Odyssean forward guidance. As already mentioned, such a presence of heterogeneity can help to explain why the empirical effect of Odyssean forward guidance is usually found to be much lower than what the theory under homogeneous beliefs predicts (see Carlstrom et al., 2012; Del Negro et al., 2015, among others).

Here, we give further insights on our explanation to the puzzle. Similarly to McKay et al. (2015) with discounting, our mechanism limits the effects of future shocks on current aggregate consumption via the Euler equation. To highlight the mechanism, suppose the simplest case of a trap of one period and with two periods of Odyssean forward guidance, i.e. $T_{cb} = 2$, before monetary policy fully stabilizes inflation and output starting in period $t = 2$: $c_{o,2} = c_{p,2} = \pi_2 = 0$.

In period $t = 1$, individual consumption and inflation equal:

$$c_{o,1} = \gamma^{-1}(\log R) > 0 \text{ and } c_{p,1} = \gamma^{-1}(\log R - r)$$
$$\pi_{o,1} = \kappa\gamma^{-1}(\log R) > 0 \text{ and } \pi_{p,1} = \kappa\gamma^{-1}(\log R - r)$$

Using the Euler equation for each household, we obtain date-0 consumption levels:

$$c_{o,0} = c_{o,1} - \gamma^{-1}(\log R - r - \pi_{o,1})$$
$$c_{p,0} = c_{p,1} - \gamma^{-1}(\log R - r - \pi_{p,1})$$

This allows us to write the aggregate Euler equation.

$$c_0 = (1 - \alpha)c_{o,1} - \gamma^{-1}(\log R - r - \pi_{o,1}) + \alpha(c_{p,1} + \gamma^{-1}(\pi_{p,1} - \pi_{o,1})).$$

Finally, using the fact that $c_{p,1}$ and $\pi_{p,1}$ are expected to be negative by pessimists, aggregate consumption c_0 satisfies:

$$c_0 \leq (1 - \alpha)c_{o,1} - \gamma^{-1}(\log R - r - (1 - \alpha)\pi_{o,1}).$$

$(1-\alpha)$ appears in the aggregate Euler equation as the limit of the effect of forward guidance, as measured by optimists' beliefs. Therefore, the presence of pessimists affects the effectiveness of Odyssean forward guidance by reducing the aggregate impact of the future boom on current aggregate consumption.

C.5 Alternative assumptions on the NKPC

In this appendix, we investigate the transmission channels of our mechanism in the NK model and more precisely the precise role played by the NKPC. We have already consider a case where firms' shares are not traded and another one where prices are fully rigid. We now allow firms' shares to be traded. As optimists have a higher valuation of future firms' profits, they are the only stockholders in the economy, thus implying that firms share the same views as the optimists about future economic conditions. We show that even in this situation our main results still hold, as the pessimists still weight on current consumption. This can be observed in Figure 11b. This happens even though pessimists are forecasting inflation in the short run taking as given firms' optimistic views on future outcomes: pessimists' expectation of a recession is sufficient to drive down their current consumption and, then, to dampen the effects of forward guidance.

C.6 The welfare function.

To determine optimal policy, the central bank's problem is to maximize the expected utility of agents:

$$U = \int_0^1 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) di, \tag{19}$$

where $C_{i,t}$ and $L_{i,t}$ are respectively consumption and labor supply of agent i in period t. The parameter $\beta \in (0,1)$ is a discount factor, the parameter $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution, and the parameter $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply. The variable ξ_t is a preference shock as discussed above.

We show that proceeding similarly to Gali (2008) (pag.87), we can approximate the per period utility of each agent around a steady state as:

$$\mathbb{W}_i \equiv \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t U_{i,t} - U\right] \simeq \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t U_c C \left(c_{i,t} + \frac{1-\gamma}{2} c_{i,t}^2\right) + U_l L \left(l_{i,t} + \frac{1+\psi}{2} l_{i,t}^2\right)\right]. \tag{20}$$

The next step is to use the fact $L_t = Y_t \int (P_{j,t}/P_t)^{-\theta} di$ to derive

$$(1-\alpha) l_{o,t} + \alpha l_{p,t} = (1-\alpha) c_{o,t} + \alpha c_{p,t} + d_t \tag{21}$$

where the last price dispersion term is derived from as a direct implication of the Calvo assumption (for a proof see Woodford (2003) p.399) as being proportional to the square of inflation π_t^2. Given the first order condition on labor supply, and in particular because of the assumption of homogeneous labor market (17), we have that $\gamma c_{o,t} + \psi l_{o,t} = \gamma c_{p,t} + \psi l_{p,t}$ that is:

$$l_{p,t} - l_{o,t} = -\frac{\gamma}{\psi} \left(c_{p,t} - c_{o,t}\right). \tag{22}$$

Therefore we can rewrite

$$\begin{aligned} l_{o,t} + \alpha(l_{p,t} - l_{o,t}) &= c_{o,t} + \alpha(c_{p,t} - c_{o,t}) + d_t, \\ l_{p,t} + (1-\alpha)(l_{o,t} - l_{p,t}) &= c_{p,t} + (1-\alpha)(c_{o,t} - c_{p,t}) + d_t, \end{aligned}$$

or

$$
\begin{aligned}
l_{o,t} &= c_{o,t} + \alpha \left(1 + \frac{\gamma}{\psi}\right)(c_{p,t} - c_{o,t}) + d_t, \\
l_{p,t} &= c_{p,t} + (1-\alpha)\left(1 + \frac{\gamma}{\psi}\right)(c_{o,t} - c_{p,t}) + d_t.
\end{aligned}
$$

In the special case $\gamma = \psi$ we can show that

$$
\alpha l_{p,t}^2 + (1-\alpha) l_{o,t}^2 = \alpha c_{p,t}^2 + (1-\alpha) c_{o,t}^2
$$

since

$$
\alpha \left(c_{p,t} + 2\left(1-\alpha\right)\left(c_{o,t} - c_{p,t}\right)\right)^2 + \left(1-\alpha\right)\left(c_{o,t} + 2\alpha(c_{p,t} - c_{o,t})\right)^2 = \left(1-\alpha\right) c_{o,t}^2 + \alpha c_{p,t}^2.
$$

Therefore, as in Gali (2008), we get

$$
\mathbb{W}_i = -\varpi \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left((1+\psi) l_{i,t}^2 - (1-\gamma) c_{i,t}^2 + \theta \pi_t^2\right)\right] \tag{23}
$$

where ϖ is a positive constant, so that finally social welfare can be approximated by $\mathbb{W} = \int_0^1 \mathbb{W}_i di$. In the special case, $\gamma = \psi$, (23) becomes

$$
\tilde{\mathbb{W}}_i = -\varpi \theta^{-1} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\lambda c_{i,t}^2 + \pi_t^2\right)\right] \tag{24}
$$

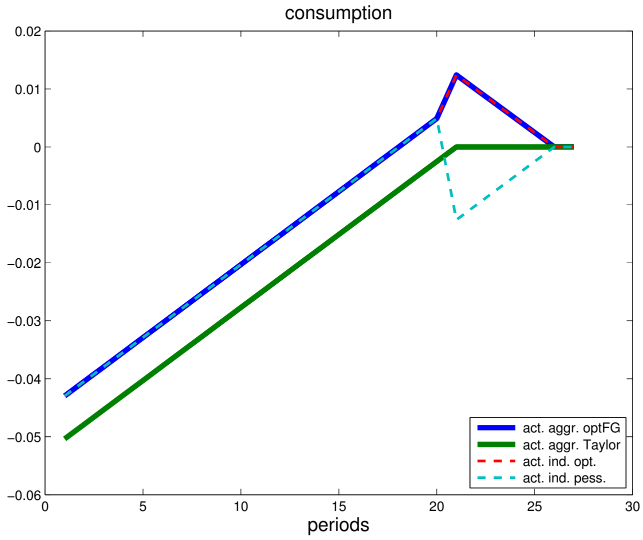
with $\lambda = 2\gamma/\theta$, which is identical to the case with homogeneous agents.

C.7 Identifying the welfare effects.

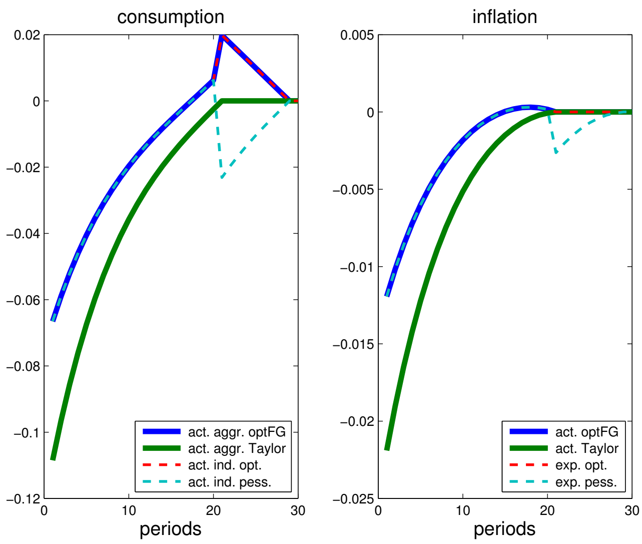
In this paragraph, we further investigate what drives our results on the hump-shaped optimal policy as well as what are the relevant effect in terms of welfare. In particular, there are two additional channels can also affect the central bank's optimal policy. First, once disagreement vanishes the boom engineered by the central bank will concern all agents, even though the *ex-ante* benefits from the future boom are dampened by the presence of heterogeneous beliefs; we refer to this effect as a "costly boom" channel. Second, the monetary authority's objective function also includes additional cross-product terms which relate to the inequality generated by disagreement; we refer to this effect as a "inequality" channel. These additional channels do not qualitatively alter the effects of the "bad news" channel as their effects are small.

This is shown by Figure 12a that reports the optimal period of extra-accommodation for the benchmark model as well as for the case where the boom is not "paid in full". The latter means that, to turn off the “costly boom” channel, we exogenously assume that pessimists do not catch up with optimists once the truth unfolds but go directly to the steady state. This implies that the boom negatively weights on welfare only to the extend that it was anticipated by optimists. We obtain in both cases a similar pattern for the optimal policy. Intuitively, when the boom is not paid in full, the monetary authority can afford a longer period of accommodation, as this can be observed in the graph. In the end, only the “bad news” channel, which is present in both cases, is necessary to obtain qualitatively our results. We do not report the effect of the “inequality” channel that was negligible in all calibrations that we have explored.

Where the hump-shaped pattern is then coming from? To explore this issue, we turn off the NK Phillips curve by assuming that prices are pre-determined for all periods at date -1, as in Werning (2012). Figure 12b plots the resulting optimal policy and compares it to our benchmark. Importantly, there is no hump-shaped pattern anymore. The main intuition for this observation is that the NK Phillips curve introduces non-linear effects of the extra-accommodation, as future monetary easing not only increases future consumption but also future inflation, thus having a double effects on current consumption, through both lower real interest rate and the expectation of higher consumption.



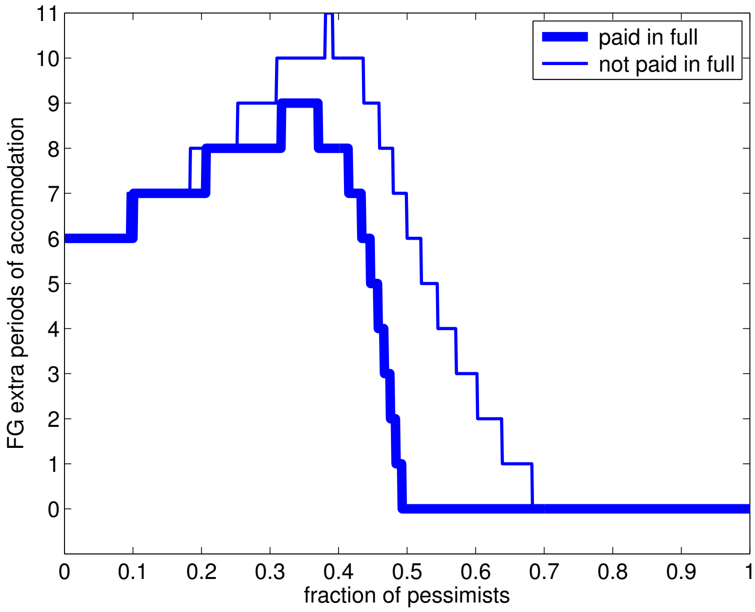
(a) Fully predetermined prices.



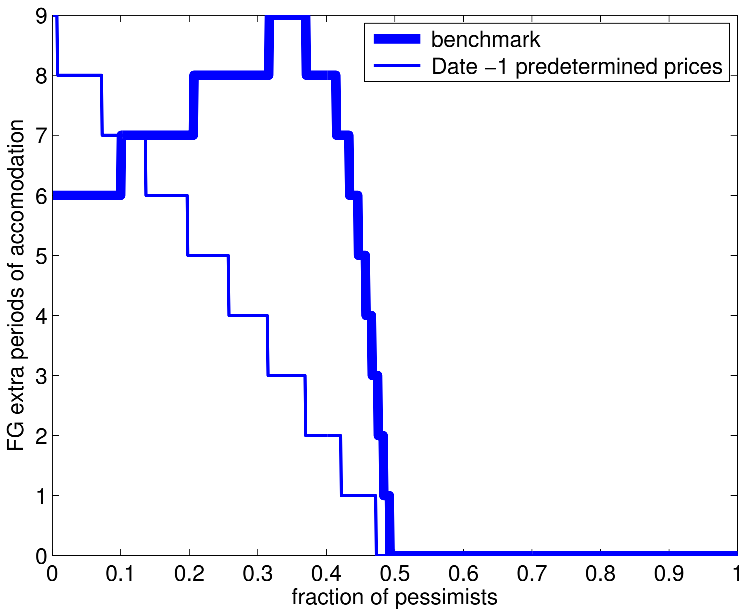
(b) Endogenous stockholdings.

Figure 11: The effect of Odyssean (blue) with a fraction $\alpha = .25$ of pessimists with alternative NK Phillips curve specifications.

We consider a shock ($\xi = -0.01$) on the discount rate that lasts 12 quarters and implies a drop of consumption of 4% at impact in the absence of Odyssean forward guidance, which provides for 4 extra quarters of accommodation. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.



(a) Optimal policy when the boom is not "paid in full".



(b) Optimal policy with pre-determined prices at date -1.

Figure 12: We plot the optimal $T_{cb}(\alpha, 20) - 20$ for $\xi = -.01$ with $\gamma/\psi = 1$;We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2%, The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.

D Proofs

D.1 Proof of Proposition 3

The proof is organized in five steps. *First step.* Consider an economy with homogeneous agents at the date $T_{cb}+1$ just after the end of the zero-rate period, so that the steady state can be restored. Because of Ricardian equivalence holds, the present value of their life-utility is the same irrespective of the stock of bonds they hold at that time, which is a legacy of the realized states of the words. Therefore, because of the permanent income hypothesis, the level of homogenous individual consumption $C_{T_{cb}+1} = \bar{C}$ is pin down only by the forward evolution of the economy that will remain at steady state. *Second step.* At time T, as soon as agents become homogeneous, they would agree on a plan of transfers $\{Z^*_{i,t}\}^1_0$ such that $B_{o,t} = B_{p,t}$, that is, their stock bonds is equalized. In fact, as a consequence, consumption is equalized and so $U_{C_{o,T}} = U_{C_{p,T}}$, that is, social welfare is maximized. After that period, irrespective of whether or not the economy is already at steady state (preference shock does not hit), individual consumption will converge to $C_{T_{cb}+1} = \bar{C}$ because of what argued in the first step. *Third Step.* Consider now the sequence of transfers $\{\{Z^*_{i,t}\}^1_0\}^\infty_{t=0}$, then since step two and three are common knowledge, there is only one equilibrium consumption path associated to each state of the word as described in the proposition. *Fourth step.* Different transfers plans, which modify the path of consumption of the two types, imply, because of the permanent income hypothesis, different level of consumption at steady state. Given that agents anticipate step 2, no plan of this kind can be implemented. In other words, agents anticipate that at time t they will agree to equalize their wealth so that $\bar{C}$ will be their steady state consumption that in turn determines the unique consumption path described at step three. *Fifth step.* Among all the transfer plans that can engineer an equalization in the stock of bonds at time T onwards, $\{Z^*_{i,t}\}^1_0$ is the only one that is implementable because before time T agents disagree on the actual transfer that will equalize bonds holding at time T as they expect different real interest rates paths, after time T they agree on no transfers.

D.2 Proof of Proposition 2

To enlighten the main intuition behind the proof, we firstly only consider a one-period trap that hits at time 0, in the case $\lambda = 0$. Let us then denote by $FG(k) = \sum_{t>0} \beta^t \pi_t^2$ when there is k of periods of Odyssean forward guidance. $FG(k)$ is increasing in k and does not depend on α. The last two properties are general all the periods after the end of the trap, irrespective of its length and the value λ. The reson is that for $t > T$ agents will not disagree and anticipate that at time 0.

Inflation and consumption at time 0 are given by

$$\begin{aligned}
\pi_0 &= \left(\alpha\left(\beta\pi_{p,1}+\kappa c_{p,0}\right)+(1-\alpha)\left(\beta\pi_{o,1}+\kappa c_{o,0}\right)\right),\\
c_0 &= \alpha c_{p,0}+(1-\alpha)c_{o,0}\\
c_{o,0} &= c_{o,1}-\gamma^{-1}\left(\rho_l-\pi_{o,1}\right)\\
c_{p,0} &= c_{p,1}-\gamma^{-1}\left(\rho_l-\pi_{p,1}\right)
\end{aligned}$$

where $\pi_{i,1}$ is a short notation for the expectation of agent i about inflation at time 1 and $\rho_l = -\log R - \xi < 0$. Bear in mind that $c_{i,1}$ and $\pi_{i,1}$ do not depend on α as agents consistently expect homogeneous beliefs are restored after that date.

Let us investigate the conditions for which for $k > k'$ we can have $\pi_0^2(k) + FG(k) \leq \pi_0^2(k') + FG(k')$, i.e. forward guidance for k period is not less efficient of a forward guidance for k' periods. First note that

$$\frac{\partial c_0\left(k\right)}{\partial\alpha} = c_{p,0}\left(k\right)-c_{o,0}\left(k\right) = \gamma^{-1}(\pi_{p,1}(k')-\pi_{o,1}(k'))+c_{p,1}(k')-c_{o,1}(k'))$$

as $\partial c_{p,0}/\partial\alpha = \partial c_{o,0}/\partial\alpha = 0$, and so

$$\begin{aligned}
\frac{\partial c_0\left(k\right)}{\partial\alpha} &< \frac{\partial c_0\left(k'\right)}{\partial\alpha}\\
\frac{\partial \pi_0\left(k\right)}{\partial\alpha} &= \beta\left(\pi_{p,1}\left(k\right)-\pi_{o,1}\left(k\right)\right)+\kappa\left(c_{p,0}\left(k\right)-c_{o,0}\left(k\right)\right) < \frac{\partial \pi_0\left(k'\right)}{\partial\alpha} < 0.
\end{aligned}$$

given the facts:

i) $\pi_{p,1}(k) < \pi_{p,1}(k') < \pi_{p,1}(0) = 0$, $\pi_{o,1}(k) > \pi_{o,1}(k') > \pi_{o,1}(0)$,

ii) $c_{p,1}(k) < c_{p,1}(k') < c_{p,1}(0) = 0$, $c_{o,1}(k) > c_{o,1}(k') > c_{o,1}(0)$.

The derivative of $\Pi\left(k,k',\alpha\right) = \pi_0^2(k) - \pi_0^2(k')$ with respect to α is:

$$\frac{\partial \Pi\left(k,k',\alpha\right)}{\partial\alpha} = 2\left(\pi_0(k)\frac{\pi_0(k)}{\partial\alpha} - \pi_0(k')\frac{\pi_0(k')}{\partial\alpha}\right) \tag{25}$$

whereas, $\Phi\left(k,k'\right) = FG(k') - FG(k) < 0$. By substitution we get:

$$\begin{aligned}
\pi_0\frac{\pi_0}{\partial\alpha} &= \alpha\left(\left(\beta+\kappa\gamma^{-1}\right)\left(\pi_{p,1}-\pi_{o,1}\right)+\kappa(c_{p,1}-c_{o,1})\right)^2+\\
&+\left(\left(\beta+\kappa\gamma^{-1}\right)\pi_{o,1}+\kappa c_{o,1}-\gamma^{-1}\rho_l\right)\left(\left(\beta+\kappa\gamma^{-1}\right)\left(\pi_{p,1}-\pi_{o,1}\right)+\kappa(c_{p,1}-c_{o,1})\right)
\end{aligned}$$

where the term

$$\left(\left(\beta+\kappa\gamma^{-1}\right)\left(\pi_{p,1}(k)-\pi_{o,1}(k)\right)+\kappa(c_{p,1}(k)-c_{o,1}(k))\right),$$

is smaller than

$$\left(\left(\beta+\kappa\gamma^{-1}\right)\left(\pi_{p,1}(k')-\pi_{o,1}(k')\right)+\kappa(c_{p,1}(k')-c_{o,1}(k'))\right),$$

for the facts i) and ii) above. As a result, when $\alpha=0$, the derivative $\partial\Pi\left(k,k'\right)/\partial\alpha$ is negative. In addition, $\partial\Pi\left(k,k'\right)/\partial\alpha$ is a linear and increasing function of α.

Therefore, let us consider a situation in which $\Pi\left(k,k',\alpha\right)>\Phi\left(k,k'\right)$ - i.e. forward guidance for k' is preferred to k, with $k>k'$, in the absence of pessimists. As α increases in the range $(0,1)$, the inequality can switch sign either never or twice, given that by construction $\Pi\left(k,k',1\right)>0$ (all agents are Delphic). In particular, the upper threshold $\bar{\alpha}$ is such that $\Pi\left(1,0,\bar{\alpha}\right)=\Phi\left(1,0\right)$.

Let us go back now to the case $\lambda>0$. In this case the relevant inequality becomes

$$(1-\alpha)\left(c_{o,0}^{2}\left(k\right)-c_{o,0}^{2}\left(k'\right)\right)+\alpha\left(c_{p,0}^{2}\left(k\right)-c_{p,0}^{2}\left(k'\right)\right)+\Pi\left(k,k',\alpha\right)\leq\hat{\Phi}(k,k')$$

where $\hat{\Phi}(k,k')$, which preserves the properties of $\Phi(k,k')$, has been extended accordingly. As before with $k>k'$, we have facts i) and ii). To show that the additional term

$$c_{o,0}^{2}\left(k\right)-c_{o,0}^{2}\left(k'\right)+\alpha\left(c_{p,0}^{2}\left(k\right)-c_{o,0}^{2}\left(k\right)\right)-\alpha\left(c_{p,0}^{2}\left(k'\right)-c_{o,0}^{2}\left(k'\right)\right)$$

is also increasing in α, notice that $0>c_{p,0}\left(k'\right)>c_{p,0}\left(k\right)$ and $c_{o,0}\left(k\right)>c_{o,0}\left(k'\right)>0$ implies

$$c_{o,0}\left(k\right)>c_{p,0}\left(k'\right)+c_{o,0}\left(k'\right)$$

so that

$$\left(c_{p,0}\left(k\right)+c_{o,0}\left(k\right)\right)\left(c_{p,0}\left(k\right)-c_{o,0}\left(k\right)\right)>\left(c_{p,0}\left(k'\right)+c_{o,0}\left(k'\right)\right)\left(c_{p,0}\left(k'\right)-c_{o,0}\left(k'\right)\right)$$

can be easily shown given that $c_{p,0}\left(k\right)-c_{o,0}\left(k\right)>c_{p,0}\left(k'\right)-c_{o,0}\left(k'\right)$ from facts ii). Nevertheless, the additional term is positive at $\alpha=0$. This implies that whereas all the qualitative feature of our analysis equally hold considering $\lambda>0$, a longer forward guidance are ceteris paribus more efficient at low α.

Let us look at how the reasoning can be extended to multiple periods in the liquidity trap. Without loss of generality, let us go back to the simple case $\lambda=0$. We add a period $t=-1$ that takes place just before period 0, then the reasoning can be extended recursively. We have then to compare:

$$1/\beta\pi_{-1}^{2}\left(k\right)-1/\beta\pi_{-1}^{2}\left(k'\right)+\Pi\left(k,k',\alpha\right)\leq\Phi(k,k').$$

Notice that the additional term is typically positive, so ceteris paribus, with a longer trap a longer forward is needed for low α. The derivative with respect to α of the additional terms $\Pi_{-1}\left(k,k',\alpha\right)$ is:

$$\frac{\partial\Pi_{-1}\left(k,k',\alpha\right)}{\partial\alpha}=\beta^{-1}\left(2\pi_{-1}(k)\frac{\partial\pi_{-1}(k)}{\partial\alpha}-2\pi_{-1}(k')\frac{\partial\pi_{-1}(k')}{\partial\alpha}\right),$$

which has the same structure than (25) and can be expressed similarly as a linear combination of future actual aggregate consumption and inflation. In particular, we can show

$$
\begin{aligned}
\frac{\partial c_{-1}(k)}{\partial \alpha} &= \frac{\partial c_0(k)}{\partial \alpha} + \gamma^{-1}\frac{\partial \pi_0(k)}{\partial \alpha} < \frac{\partial c_{-1}(k')}{\partial \alpha} < 0 \\
\frac{\partial \pi_{-1}(k)}{\partial \alpha} &= \kappa\frac{\partial c_{-1}(k)}{\partial \alpha} + \beta\frac{\partial \pi_0(k)}{\partial \alpha} < \frac{\partial \pi_{-1}(k')}{\partial \alpha} < 0
\end{aligned}
$$

using previous relations. Therefore, $\partial \pi^2_{-1}(k)/\partial \alpha$ is a linear downward sloping function of α. Given this result, we can then extend recursively the analysis to an arbitrarily number of periods.

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