Housing Prices and Robustly Optimal Monetary Policy

Klaus Adam — Mannheim University
Michael Woodford — Columbia University

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Asset Prices and Monetary Policy

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  — suffices for policy to be sufficiently sensitive to inflation forecast (Bernanke and Gertler, 1999, 2001)

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- Reconsideration since the recent crisis

  — especially with regard to dangers of housing booms

Asset Prices and Monetary Policy

- Crucial methodological issue: how to model expectations

  — main worry: consequences of policy when housing prices may not reflect rational expectations

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  — hence standard approach (analyze REE implied by different policies) inadequate

- Alternative approach: compare policies under exogenously specified process for expectational errors (as in B + G)

- But will rule optimized for one specification also be desirable if errors are of a different sort?

  — especially difficult issue because matters how expectational errors may change with policy

Robust Policy Analysis

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  - not assume that it knows what PS expectations must be, in case of a particular policy rule
  - might be any beliefs, among those not too different from what CB's model implies ("near-rational expectations")
  - choose the policy that is least vulnerable to deviation of PS expectations from model-consistency

    — as in theories of "ambiguity aversion," "robust control"

Robust Policy: Defining the Problem

- Suppose that policy commitment must be chosen from set C.

- The set of feasible commitments C is such that for any $c \in C$, and for any belief distortion m in the set of feasible belief distortions M, there exists a well-defined eq'm outcome x

  - don't allow policymaker to constrain the set of belief distortions through choice of policy commitment

  - thus must exist an outcome function

$$\mathcal{O} : C \times M \to X$$

  - if a given policy commitment allows multiple eq'a, even for given belief distortions, we may suppose that $\mathcal{O}$ selects the worst such eq'm

Robust Policy: Defining the Problem

- Let there be a welfare measure $U(x)$ associated with any outcome x, and a penalty $V(m)$ for any belief distortion m

  — The form of penalty function $V(m)$ reflects our conception of "near-rational expectations."

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- Then robust policy problem can be written

$$\max_{c \in C} \left\{ \min_{m \in M} U(\mathcal{O}(c, m)) \text{ s.t. } V(m) \leq \bar{V} \right\}$$

where $\bar{V}$ measures the degree of concern for robustness.

Robust Policy: Alternative Formulation

Let c^* be the robustly optimal policy commitment, and m^* the associated worst-case beliefs (solution to inner problem, given c^*).

- Suppose that there exists a Lagrange multiplier $\theta \geq 0$ such that m^* also solves

$$\min_{m \in M} U(\mathcal{O}(c, m)) + \theta V(m),$$

and such that

$$\theta[V(m) - \bar{V}] = 0.$$

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- Then c^* also solves the alternative problem

$$\max_{c \in C} \min_{m \in M} U(\mathcal{O}(c, m)) + \theta V(m)$$

where θ now parameterizes concern for robustness

Robust Policy: General Approach

- A “brute force” approach would first solve the “inner problem”

$$\min_{m \in M} U(\mathcal{O}(c, m)) + \theta V(m)$$

for an arbitrary policy commitment c; thus obtain a lower bound $\underline{U}(c)$ for any c, then seek to find a c that max's this

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  - problem: generally hard to characterize $\underline{U}(c)$ except for special classes of policy commitments (e.g., the linear state-contingent inflation targets considered in Woodford, 2010)
  - but is the best policy within such a special class really the best one can do?

Robust Policy: General Approach

- Our approach instead allows us to find a robustly optimal policy, without any a priori restriction to a particular simple class of policies
    - idea: establish an upper bound for welfare, that is independent of class of policy rules
    - if can find rule that achieves this upper bound, it is robustly optimal policy

Robust Policy: General Approach

- Let the requirements for eq'm (with distorted expectations) be a system of the form

$$F(x, m) = 0$$

— by hypothesis,

$$F(\mathcal{O}(c, m), m) = 0 \qquad \forall c \in C, m \in M$$

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- Again let there be a welfare measure $U(x)$ associated with any outcome x, and a penalty $\theta V(m)$ for any belief distortion m

- Then robust policy problem can be written

$$\max_{c \in C} \min_{m \in M} W(c, m)$$

where

$$W(c, m) \equiv U(\mathcal{O}(c, m)) + \theta V(m)$$

Robust Policy: General Approach

- Then one can show

$$
\begin{aligned}
\max_{c \in C} \min_{m \in M} W(c, m) &\leq \min_{m \in M} \max_{c \in C} W(c, m) \\
&\leq \min_{m \in M} \max_{x \in X} [U(x) + \theta V(m)] \\
&\qquad s.t.\ F(x, m) = 0
\end{aligned}
$$

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$$\leq \min_{m \in M} \max_{x \in X} [U(x) + \theta V(m)]$$

$$s.t.\ F(x, m) = 0$$

- This upper bound on what can robustly be achieved can be computed without any assumption about class of policy rules C
- Then if find a policy that achieves this bound, know that no more general class of rules need be considered

Near-Rational Expectations

- Maintained assumption: PS beliefs must be absolutely continuous wrt truth [over any finite time interval]

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- $\Rightarrow$ there exists a process $\{m_{t+1}\}$ with

$$m_{t+1} \geq 0 \text{ a.s.}, \qquad E_t[m_{t+1}] = 1.$$

such that

$$\hat{E}_t[X_{t+1}] = E_t[m_{t+1} X_{t+1}]$$

for any random X_{t+1}

Near-Rational Expectations

- Degree of distortion of PS beliefs can then be measured by relative entropy

$$E_0 \sum_{t=0}^{\infty} \beta^t m_{t+1} \log m_{t+1}$$

following Hansen-Sargent treatment of robust policy

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  - a positive-valued, convex function of distorted prob. measure, uniquely minimized ($= 0$) when $m_{t+1} = 1$ a.s. [case of RE]
  - a measure of how easily the distorted beliefs should be disconfirmed by data [according to CB beliefs]
  - discounting at rate β means CB concern with potential PS misunderstanding doesn't vanish asymptotically

A New Keynesian Model with a Housing Sector

- Representative household seeks to max

$$\hat{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\tilde{u}(C_t; \xi_t) - \int_0^1 \tilde{v}(H_t(j); \xi_t) dj + \tilde{w}(D_t; \xi_t) \right]$$

where C_t is a Dixit-Stiglitz aggregate

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \qquad (\eta > 1)$$

$H_t(j)$ is labor supplied to sector j, D_t is stock of durable goods (housing), and ξ_t is a vector of aggregate disturbances

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- Here $\widehat{E}_t[\cdot]$ denotes conditional expectation under subjective probability beliefs common to all households

A New Keynesian Model with a Housing Sector

- Flow budget constraint:

$$P_t C_t + B_t + \left(D_t + (1-\delta) D_{t-1}\right) q_t P_t + k_t P_t$$
$$\leq (1+s^d) \tilde{d}(k_t; \xi_t) q_t P_t + \int_0^1 w_t(j) P_t H_t(j) dj + B_{t-1}(1+i_{t-1})$$
$$+ \Sigma_t + T_t,$$

where

$$\begin{aligned} \delta &= \text{depreciation rate for housing} \\ q_t &= \text{real price of housing} \\ k_t &= \text{real resources used to produce housing} \\ \tilde{d}(k;\xi) &= \text{production function for housing} \\ s^d &= \text{net housing subsidy} \end{aligned}$$

A New Keynesian Model with a Housing Sector

- Convenient to also assume isoelastic functional forms

$$\tilde{u}(C_t; \xi_t) \equiv \frac{C_t^{1-\tilde{\sigma}^{-1}} \bar{C}_t^{\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}},$$

$$\tilde{v}(H_t; \xi_t) \equiv \frac{\lambda}{1+\nu} H_t^{1+\nu} \bar{H}_t^{-\nu},$$

$$\tilde{w}(D_t; \xi_t) \equiv \xi_t^d D_t$$

where $\tilde{\sigma}, \nu > 0$, and $\{\bar{C}_t, \bar{H}_t, \xi_t^d\}$ are bounded exogenous disturbance processes (among those included in the vector ξ_t)

A New Keynesian Model with a Housing Sector

- Common production function for each differentiated good i

$$y_t(i) = A_t h_t(i)^{1/\phi}$$

and the production function for housing

$$\tilde{d}(k_t; \xi_t) = \frac{A_t^d}{\tilde{\alpha}} k_t^{\tilde{\alpha}}$$

are also isoelastic; where $\phi > 1$, $0 < \tilde{\alpha} < 1$, and $\{A_t, A_t^d\}$ are additional bounded exogenous disturbances

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- Note two new shocks: "housing demand" shock ξ_t^d and "housing supply" shock A_t^d

Equilibrium Asset Pricing: Short Nominal Rate

- Optimization by rep hh requires that

$$\left(\frac{C_t}{\bar{C}_t}\right)^{-1/\tilde{\sigma}} = \beta(1+i_t)\ \hat{E}_t\left[\Pi_{t+1}^{-1}\left(\frac{C_{t+1}}{\bar{C}_{t+1}}\right)^{-1/\tilde{\sigma}}\right]$$

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  — equations describing how CB is able to control the policy rate not necessary for analysis of how it is desirable to adjust it

- same as in standard NK models, but allowing for distorted expectations

Equilibrium Asset Pricing: Housing

- Optimization similarly requires

$$q_t \left(\frac{C_t}{\bar{C}_t} \right)^{-1/\tilde{\sigma}} = \xi_t^d \; + \; \beta(1-\delta) \; \hat{E}_t \left[q_{t+1} \left(\frac{C_{t+1}}{\bar{C}_{t+1}} \right)^{-1/\tilde{\sigma}} \right]$$

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- Can be written more simply as

$$q_t^u = \xi_t^d + \beta(1-\delta)\,\hat{E}_t q_{t+1}^u$$

where

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is housing price in marginal-utility units

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— esp. relevant because distortions in expected value of q_t^u (not q_t) matter for eq'm

Policy, Belief Distortions, and Housing Prices

- Under RE: the equilibrium evolution of q_t^u would be uniquely determined by the exogenous fundamentals $\{\xi_t^d\}$

  — monetary policy would then have no effect on q_t^u, but could affect q_t, by varying the eq'm marginal utility of income (C_t process influenced by i_t, as indicated by above Euler equation)

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- If instead allow for belief distortions ($m_{t+1} \neq 1$): asset-pricing equation implies that q_t^u can depart from RE value (determined purely by fundamentals)

  — monetary policy can affect the degree to which the belief distortions affect the real price of housing q_t

NK Model with Housing: Price-Setting

- Calvo-Yun staggered price adjustment: fraction $0 < \alpha < 1$ of prices remain unchanged each period, others (randomly selected) re-optimized

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- firm that re-optimizes chooses price to max

$$\hat{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi(p_t(i), P_T^j, P_T;\ Y_T, q_T^u, \xi_T)$$

using stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{\lambda(Y_T, q_T^u; \xi_T)}{\lambda(Y_t, q_t^u; \xi_t)} \frac{P_t}{P_T}$$

NK Model with Housing: Price-Setting

- This results in an equation for the relative price chosen by firms that re-optimize (and hence, the rate of inflation) that is a function of a vector of two forward-looking variables Z_t, which is determined by relations of the form

$$Z_t = z(Y_t, q_t^u; \xi_t) + \alpha\beta\hat{E}_t[\Phi(Z_{t+1})] \qquad (*)$$

— inflation then given by an equation

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- Note that equation (*) defines the Phillips-curve trade-off for this economy

Welfare Objective

- Dixit-Stiglitz monopolistic competition, and iso-elastic functional forms for disutility of work, production function, allow us to write rep hh's disutility of work as

$$\int_0^1 \tilde{v}(H_t(j); \xi_t)dj = v(Y_t; \xi_t)\Delta_t$$

where $v', v'' > 0$ and

$$\Delta_t \equiv \int_0^1 (p_t(i)/P_t)^{-\eta(1+\omega)} di \geq 1$$

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is an index of price dispersion

- Calvo-Yun price adjustment implies law of motion for price dispersion of the form

$$\Delta_t = h(\Delta_{t-1}, Z_t)$$

Welfare Objective

- Assumed linearity of $\tilde{w}(D; \xi)$ allows us to write

$$E_0 \sum_{t=0}^{\infty} \beta^t \, \tilde{w}(D_t; \xi_t) = E_0 \sum_{t=0}^{\infty} \beta^t \, \bar{\xi}_t^d \tilde{d}(k_t; \xi_t) \; + \; \text{exog.},$$

neglecting exogenous terms, where

$$\bar{\xi}_t^d \equiv \sum_{T=t}^{\infty} [\beta(1-\delta)]^{T-t} \, E_t \xi_T^d$$

is the fundamental value of q_t^u (RE eq'm value)

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- Substituting optimal solution for k_t, one can write

$$E_0 \sum_{t=0}^{\infty} \beta^t \, \tilde{w}(D_t; \xi_t) = E_0 \sum_{t=0}^{\infty} \beta^t \, w(Y_t, q_t^u; \xi_t) \; + \; \text{exog.}$$

Welfare Objective

- Hence expected utility of rep hh (evaluated using policymaker's beliefs) can be written as

$$E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, q_t^u, \Delta_t; \xi_t)$$

where

$$U(Y, q^u, \Delta; \xi) \equiv \tilde{u}(C(Y, q^u; \xi); \xi) - v(Y; \xi)\Delta + w(Y, q^u; \xi)$$

Model with Housing: Complete Eq'm Conditions

- A distorted expectations equilibrium is then a collection of processes $\{Y_t, Z_t, \Delta_t, q_t^u, i_t, m_{t+1}\}$ satisfying
  - Euler equation for i_t
  - Euler equation for q_t^u
  - law of motion for Δ_t
  - equations (*) for Z_t

  in addition to the requirement that $E_t m_{t+1} = 1$ at all times

Establishing an Upper Bound for Robust Policy

- Recall strategy:
  - for given distortion process $\{m_{t+1}\}$, compute the policymaker's best response
  - find "worst-case" belief distortions that make this equilibrium as bad as possible

Establishing an Upper Bound for Robust Policy

- Recall strategy:
  - for given distortion process $\{m_{t+1}\}$, compute the policymaker's best response
  - find "worst-case" belief distortions that make this equilibrium as bad as possible

- Note that in first step,
  - need only consider possible equilibrium evolutions consistent with $\{m_{t+1}\}$; need not specify type of policy rule to implement each outcome
  - if ZLB never binds (true for small enough shocks and belief distortions), can ignore Euler equation as constraint on possible evolution of output, inflation, price dispersion and housing prices

Establishing an Upper Bound for Robust Policy

Problem:

$$\min_{\{m_{t+1}\}_{t=0}^{\infty}} \max_{\{Y_t, Z_t, \Delta_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[U(Y_t, q_t^u, \Delta_t; \xi_t) + \theta \beta m_{t+1} \log m_{t+1} \right]$$

where inner max is subject to constraint that $\{Y_t, Z_t, q_t^u, \Delta_t\}_{t=0}^{\infty}$ be a DEE consistent with $\{m_{t+1}\}_{t=0}^{\infty}$ and initial condition Δ_{-1}

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where inner max is subject to constraint that $\{Y_t, Z_t, q_t^u, \Delta_t\}_{t=0}^{\infty}$ be a DEE consistent with $\{m_{t+1}\}_{t=0}^{\infty}$ and initial condition Δ_{-1}

- no need to specify $\{i_t\}$ (find to satisfy Euler eq)
- don't need to max over $\{q_t^u\}$, as it is completely determined by exog. disturbance processes and belief distortions

Establishing an Upper Bound for Robust Policy

- Constraints and associated Lagrangian multipliers, for policymaker's best-response problem:

$$\gamma_t : \quad \Delta_t = h(\Delta_{t-1}, Z_t)$$

$$\Gamma_t : \quad Z_t = z(Y_t, q_t^u; \xi_t) + \alpha\beta E_t[m_{t+1}\Phi(Z_{t+1})]$$

$$\Psi_t : \quad q_t^u = \xi_t^d + \beta(1-\delta)\, E_t[m_{t+1} q_{t+1}^u]$$

Establishing an Upper Bound for Robust Policy

- FOCs for policymaker best response: 5 conditions per period
  - 4 are like RE analysis in Woodford (2011), but with belief distortions and effects of $\{q_t^u\}$ variations
  - additional FOC w.r.t. q_t^u allows solution for Ψ_t, shadow value of relaxing eq'm asset pricing relation

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  - additional FOC w.r.t. q_t^u allows solution for Ψ_t, shadow value of relaxing eq'm asset pricing relation

- FOC for worst-case belief distortions yields

$$m_{t+1} = \frac{\exp\left\{-\theta^{-1}\left[\alpha\Gamma_t'\Phi(Z_{t+1}) + (1-\delta)\Psi_t q_{t+1}^u\right]\right\}}{E_t\left[\exp\left\{-\theta^{-1}\left[\alpha\Gamma_t'\Phi(Z_{t+1}) + (1-\delta)\Psi_t q_{t+1}^u\right]\right\}\right]}$$

where Γ_t' are multipliers indicating value to policymaker of shifting expectation terms in AS constraints (*), Ψ_t is multiplier indicating value of shifting expectation term in house pricing equation

Establishing an Upper Bound for Robust Policy

- Characterizing upper-bound worst-case dynamics: we now have a system of 10 conditions per period
  - 4 structural equations
  - 5 policymaker best-response FOCs
  - 1 eq for worst-case belief distortions

  to solve for 10 variables per period
  - 5 endogenous variables $\{Y_t, Z_t, \Delta_t\ q_t^u\}$
  - 4 Lagrange multipliers $\{\gamma_t, \Gamma_t\ \Psi_t\}$
  - belief distortions $\{m_{t+1}\}$

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  to solve for 10 variables per period
  - 5 endogenous variables $\{Y_t, Z_t, \Delta_t\ q_t^u\}$
  - 4 Lagrange multipliers $\{\gamma_t, \Gamma_t\ \Psi_t\}$
  - belief distortions $\{m_{t+1}\}$

- Establishing that this upper bound is attainable: need to exhibit a policy rule that is
  - consistent with the upper-bound dynamics, and such that
  - given solution for $\{m_{t+1}\}$, policy rule + 4 structural eqs uniquely determine the upper-bound dynamics $\{Y_t, Z_t, \Delta_t, q_t^u\}$

Linearized Upper-Bound Dynamics

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- Since in this case, $m_{t+1} = 1$ at all times, upper-bound steady state is same as optimal steady state under RE analysis
- Hence it is the zero inflation steady state (Benigno and Woodford, 2005)
  - robustly optimal policy rule will have to be consistent with this steady state

Linearized Upper-Bound Dynamics

- Log-linearization of 4 structural equations:
  - To first order, dynamics of price dispersion simply imply $\hat{\Delta}_t = 0$ at all times, and variable can be ignored
  - And asset pricing eq implies $\hat{q}_t^u = \xi_t^d$ (exogenous fundamental) to first order, so can replace q_t^u by exog term

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  - Two remaining log-linear eqs can be combined to yield

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \qquad (\text{“}NKPC\text{''})$$

where $\kappa > 0$, $x_t \equiv \log(Y_t / Y^*_t)$, and Y^*_t, u_t are composite exog. terms

— note that belief distortions do not matter, to first order

Linearized Upper-Bound Dynamics

- Log-linearization of FOCs for policymaker best response: Lagrange multipliers can be eliminated to yield

$$\pi_t + \phi_x(x_t - x_{t-1}) + \phi_m \log m_t = 0$$

where in the (standard) case that flexible-price steady-state output is inefficiently low, but distortion not too large,

$$\phi_x, \ \phi_m > 0$$

— optimal to reduce growth/inflation in states that are over-weighted under subjective expectations

Linearized Upper-Bound Dynamics

- Log-linearization of FOC for worst-case beliefs: Lagrange multipliers can be eliminated to yield

$$\log m_{t+1} = \lambda_m \left(\pi_{t+1} - E_t \pi_{t+1} \right) + \lambda_q \left(\hat{q}^u_{t+1} - E_t \hat{q}^u_{t+1} \right)$$

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— worst-case beliefs increase probability of states with largest inflation surprise or largest surprise increase in housing prices

Robustly Optimal Policy

- Eliminating m_t from equation characterizing policymaker best response, one shows that upper-bound dynamics must satisfy

$$\pi_t + \phi_s(\pi_t - E_{t-1}\pi_t) + \phi_x(x_t - x_{t-1}) \\ + \phi_q(\hat{q}^u_{t+1} - E_t\hat{q}^u_{t+1}) = 0$$

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- Consider a policy rule under which CB commits to adjust path of interest rates so that projected paths of inflation and output gap fulfill above criterion at all times

Robustly Optimal Policy

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- Can show that given such a policy rule, worst-case beliefs are the ones characterized above
- Hence commitment to such a target criterion achieves the upper bound

Robustly Optimal Policy

- The robustly optimal target criterion:

$$\pi_t + \phi_s(\pi_t - E_{t-1}\pi_t) + \phi_x(x_t - x_{t-1}) + \phi_q(\hat{q}^u_{t+1} - E_t\hat{q}^u_{t+1}) = 0$$

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- Note that if assume RE, both terms in red vanish

— and one recovers same form of optimal target criterion as for NK model w/o housing sector

Implications for Policy

- Thus conventional analysis of optimal policy commitment under RE would conclude that monitoring inflation and output gap suffice to tell if policy is on track, even in model with housing market (subject to tax distortions)

  — can state optimal target criterion without reference to behavior of housing prices (or other housing variables, except to extent that they affect definition of target output Y_t^*)

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- If instead one wants policy to be robust to possible departures from RE, one should commit to a target criterion that also involves housing price

Implications for Policy

- If $s^d > 0$ (realistic case), $\phi_q = \phi_m \lambda_q > 0 \quad \Rightarrow \quad$ CB should "lean against the wind": target lower inflation and/or output gap when housing price (in m.u. units) is unexpectedly high

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- Would have required raising interest rates more/sooner in mid-2000s?