Housing Prices and Robustly Optimal Monetary Policy

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To what extent should movements of asset prices be taken into account in the conduct of monetary policy (in addition to measures of, and projections for, inflation and output gap)?
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Conventional wisdom at Fed (and other CBs) a decade ago:

— no need for concern with asset prices, except as one of many variables with implications for future inflation

— suffices for policy to be sufficiently sensitive to inflation forecast (Bernanke and Gertler, 1999, 2001)
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Reconsideration since the recent crisis

— especially with regard to dangers of housing booms
Asset Prices and Monetary Policy

Crucial methodological issue: how to model expectations

— main worry: consequences of policy when housing prices may not reflect rational expectations

— hence standard approach (analyze REE implied by different policies) inadequate
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Alternative approach: compare policies under exogenously specified process for expectational errors (as in B + G)
Crucial methodological issue: how to model expectations

— main worry: consequences of policy when housing prices may not reflect rational expectations

— hence standard approach (analyze REE implied by different policies) inadequate

Alternative approach: compare policies under exogenously specified process for expectational errors (as in B + G)

But will rule optimized for one specification also be desirable if errors are of a different sort?

— especially difficult issue because matters how expectational errors may change with policy
Robust Policy Analysis

Alternative: let CB recognize that PS expectations may \textit{differ} from the probabilities implied by its own model

\begin{itemize}
\item \textbf{not} assume that it \textit{knows} what PS expectations must be, in case of a particular policy rule
\end{itemize}
Robust Policy Analysis

Alternative: let CB recognize that PS expectations may differ from the probabilities implied by its own model

- not assume that it knows what PS expectations must be, in case of a particular policy rule

- might be any beliefs, among those not too different from what CB’s model implies (‘near-rational expectations’)
Robust Policy Analysis

Alternative: let CB recognize that PS expectations may differ from the probabilities implied by its own model

not assume that it knows what PS expectations must be, in case of a particular policy rule

might be any beliefs, among those not too different from what CB’s model implies (“near-rational expectations”)

choose the policy that is least vulnerable to deviation of PS expectations from model-consistency

— as in theories of “ambiguity aversion,” “robust control”
Robust Policy: Defining the Problem

- Suppose that policy commitment must be chosen from set $C$.

- The set of feasible commitments $C$ is such that for any $c \in C$, and for any belief distortion $m$ in the set of feasible belief distortions $M$, there exists a well-defined eq’m outcome $x$

  - don’t allow policymaker to constrain the set of belief distortions through choice of policy commitment

  - thus must exist an outcome function

    $$O : C \times M \rightarrow X$$

- if a given policy commitment allows multiple eq’a, even for given belief distortions, we may suppose that $O$ selects the worst such eq’m
Let there be a \textit{welfare} measure $U(x)$ associated with any outcome $x$, and a \textit{penalty} $V(m)$ for any belief distortion $m$.

— The form of penalty function $V(m)$ reflects our conception of “near-rational expectations.”
Robust Policy: Defining the Problem

Let there be a welfare measure $U(x)$ associated with any outcome $x$, and a penalty $V(m)$ for any belief distortion $m$.

— The form of penalty function $V(m)$ reflects our conception of “near-rational expectations.”

Then robust policy problem can be written

$$\max_{c \in C} \left\{ \min_{m \in M} U(\mathcal{O}(c, m)) \text{ s.t. } V(m) \leq \bar{V} \right\}$$

where $\bar{V}$ measures the degree of concern for robustness.
Let $c^*$ be the robustly optimal policy commitment, and $m^*$ the associated worst-case beliefs (solution to inner problem, given $c^*$).

- Suppose that there exists a Lagrange multiplier $\theta \geq 0$ such that $m^*$ also solves

$$\min_{m \in M} U(O(c, m)) + \theta V(m),$$

and such that

$$\theta [V(m) - \bar{V}] = 0.$$
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and such that

$$\theta [V(m) - \bar{V}] = 0.$$

- Then $c^*$ also solves the alternative problem

$$\max_{c \in C} \min_{m \in M} U(\mathcal{O}(c, m)) + \theta V(m)$$

where $\theta$ now parameterizes concern for robustness.
A “brute force” approach would first solve the “inner problem”

$$\min_{m \in M} U(O(c, m)) + \theta V(m)$$

for an arbitrary policy commitment $c$; thus obtain a lower bound $U(c)$ for any $c$, then seek to find a $c$ that max's this
A “brute force” approach would first solve the “inner problem”

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for an arbitrary policy commitment $c$; thus obtain a lower bound $U(c)$ for any $c$, then seek to find a $c$ that max’s this problem: generally hard to characterize $U(c)$ except for special classes of policy commitments (e.g., the linear state-contingent inflation targets considered in Woodford, 2010)

but is the best policy within such a special class really the best one can do?
Robust Policy: General Approach

Our approach instead allows us to find a robustly optimal policy, without any a priori restriction to a particular simple class of policies.

- idea: establish an upper bound for welfare, that is independent of class of policy rules
- if can find rule that achieves this upper bound, it is robustly optimal policy
Robust Policy: General Approach

- Let the requirements for eq’m (with distorted expectations) be a system of the form
  \[ F(x, m) = 0 \]
  — by hypothesis,
  \[ F(\mathcal{O}(c, m), m) = 0 \quad \forall c \in C, m \in M \]
Robust Policy: General Approach

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\[ F(x, m) = 0 \]

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Again let there be a welfare measure \( U(x) \) associated with any outcome \( x \), and a penalty \( \theta V(m) \) for any belief distortion \( m \)

Then robust policy problem can be written

\[ \max_{c \in C} \min_{m \in M} W(c, m) \]

where

\[ W(c, m) \equiv U(O(c, m)) + \theta V(m) \]
Then one can show

$$\max_{c \in C} \min_{m \in M} W(c, m) \leq \min_{m \in M} \max_{c \in C} W(c, m)$$

$$\leq \min_{m \in M} \max_{x \in X} \left[U(x) + \theta V(m)\right]$$

$$s.t. F(x, m) = 0$$
Robust Policy: General Approach

Then one can show

\[
\max_{c \in C} \min_{m \in M} W(c, m) \leq \min_{m \in M} \max_{c \in C} W(c, m) \\
\leq \min_{m \in M} \max_{x \in X} \left[ U(x) + \theta V(m) \right] \\
s.t. \ F(x, m) = 0
\]

This upper bound on what can robustly be achieved can be computed without any assumption about class of policy rules \( C \).

Then if find a policy that achieves this bound, know that no more general class of rules need be considered.
Near-Rational Expectations

- Maintained assumption: PS beliefs must be absolutely continuous wrt truth [over any finite time interval]
Near-Rational Expectations

- **Maintained assumption:** PS beliefs must be **absolutely continuous** wrt truth [over any finite time interval]

  \[ \Rightarrow \text{there exists a process } \{m_{t+1}\} \text{ with} \]

  \[ m_{t+1} \geq 0 \text{ a.s., } E_t[m_{t+1}] = 1. \]

  such that

  \[ \hat{E}_t[X_{t+1}] = E_t[m_{t+1}X_{t+1}] \]

  for any random \( X_{t+1} \)
Degree of distortion of PS beliefs can then be measured by relative entropy

\[
E_0 \sum_{t=0}^{\infty} \beta^t m_{t+1} \log m_{t+1}
\]

following Hansen-Sargent treatment of robust policy
Near-Rational Expectations

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following Hansen-Sargent treatment of robust policy

- a positive-valued, convex function of distorted prob. measure, uniquely minimized (= 0) when \( m_{t+1} = 1 \) a.s. [case of RE]

- a measure of how easily the distorted beliefs should be disconfirmed by data [according to CB beliefs]

- discounting at rate \( \beta \) means CB concern with potential PS misunderstanding doesn’t vanish asymptotically
A New Keynesian Model with a Housing Sector

- Representative household seeks to max

\[ \hat{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \tilde{u}(C_t; \xi_t) - \int_0^1 \tilde{v}(H_t(j); \xi_t) dj + \tilde{w}(D_t; \xi_t) \right] \]

where \( C_t \) is a Dixit-Stiglitz aggregate

\[ C_t \equiv \left[ \int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} \frac{\eta}{\eta-1} \right] \]

\( H_t(j) \) is labor supplied to sector \( j \), \( D_t \) is stock of durable goods (housing), and \( \xi_t \) is a vector of aggregate disturbances.
Representative household seeks to max

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where \( C_t \) is a Dixit-Stiglitz aggregate

\[
C_t \equiv \left[ \int_0^1 c_t(i) \frac{\eta^{-1}}{\eta^{\eta-1}} \, di \right]^{\eta^{-1}}, \quad (\eta > 1)
\]

\( H_t(j) \) is labor supplied to sector \( j \), \( D_t \) is stock of durable goods (housing), and \( \xi_t \) is a vector of aggregate disturbances

Here \( \hat{E}_t[\cdot] \) denotes conditional expectation under subjective probability beliefs common to all households.
A New Keynesian Model with a Housing Sector

- Flow budget constraint:

\[ P_t C_t + B_t + (D_t + (1 - \delta)D_{t-1}) q_t P_t + k_t P_t \]

\[ \leq (1 + s^d) \tilde{d}(k_t; \xi_t) q_t P_t + \int_0^1 w_t(j) P_t H_t(j) dj + B_{t-1}(1 + i_{t-1}) \]

\[ + \Sigma_t + T_t, \]

where

- \( \delta \) = depreciation rate for housing
- \( q_t \) = real price of housing
- \( k_t \) = real resources used to produce housing
- \( \tilde{d}(k; \xi) \) = production function for housing
- \( s^d \) = net housing subsidy
A New Keynesian Model with a Housing Sector

Convenient to also assume isoelastic functional forms

\[ \tilde{u}(C_t; \zeta_t) \equiv \frac{C_t^{1-\tilde{\sigma}^{-1}} \tilde{C}_t^{\tilde{\sigma}^{-1}}}{1 - \tilde{\sigma}^{-1}}, \]

\[ \tilde{v}(H_t; \zeta_t) \equiv \frac{\lambda}{1 + \nu} H_t^{1+\nu} \tilde{H}_t^{-\nu}, \]

\[ \tilde{w}(D_t; \zeta_t) \equiv \tilde{\zeta}^d_t D_t \]

where \( \tilde{\sigma}, \nu > 0 \), and \( \{ \tilde{C}_t, \tilde{H}_t, \tilde{\zeta}^d_t \} \) are bounded exogenous disturbance processes (among those included in the vector \( \zeta_t \))
**A New Keynesian Model with a Housing Sector**

- Common production function for each differentiated good $i$

$$y_t(i) = A_t h_t(i)^{1/\phi}$$

and the production function for housing

$$\tilde{d}(k_t; \tilde{\zeta}_t) = \frac{A^d_t}{\tilde{\alpha}} k_t^{\tilde{\alpha}}$$

are also isoelastic; where $\phi > 1$, $0 < \tilde{\alpha} < 1$, and $\{A_t, A^d_t\}$ are additional bounded exogenous disturbances.
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- Note two new shocks: “housing demand” shock $\xi_t^d$ and “housing supply” shock $A_t^d$
Optimization by rep hh requires that

\[
\left( \frac{C_t}{\bar{C}_t} \right)^{-1/\tilde{\sigma}} = \beta (1 + i_t) \hat{E}_t \left[ \prod_{t+1}^{t+1} \left( \frac{C_{t+1}}{\bar{C}_{t+1}} \right)^{-1/\tilde{\sigma}} \right]
\]

where \( i_t \) is riskless one-period nominal interest rate
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where \(i_t\) is riskless one-period nominal interest rate

— indicates how CB control of short rate affects aggregate demand

— equations describing how CB is able to control the policy rate not necessary for analysis of how it is desirable to adjust it
Equilibrium Asset Pricing: Short Nominal Rate

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where \( i_t \) is riskless one-period nominal interest rate

- indicates how CB control of short rate affects aggregate demand

- equations describing how CB is able to control the policy rate not necessary for analysis of how it is desirable to adjust it

- same as in standard NK models, but allowing for distorted expectations
Optimization similarly requires

\[ q_t \left( \frac{C_t}{\bar{C}_t} \right)^{-1/\bar{\sigma}} = \zeta^d_t + \beta (1 - \delta) \hat{E}_t \left[ q_{t+1} \left( \frac{C_{t+1}}{\bar{C}_{t+1}} \right)^{-1/\bar{\sigma}} \right] \]
Equilibrium Asset Pricing: Housing

- Optimization similarly requires

\[ q_t \left( \frac{C_t}{\bar{C}_t} \right)^{-1/\tilde{\sigma}} = \zeta_t^d + \beta(1 - \delta) \hat{E}_t \left[ q_{t+1} \left( \frac{C_{t+1}}{\bar{C}_{t+1}} \right)^{-1/\tilde{\sigma}} \right] \]

- Can be written more simply as

\[ q^u_t = \zeta_t^d + \beta(1 - \delta) \hat{E}_t q^u_{t+1} \]

where

\[ q^u_t \equiv q_t \left( \frac{C_t}{\bar{C}_t} \right)^{-1/\tilde{\sigma}} \]

is housing price in marginal-utility units.
Equilibrium Asset Pricing: Housing

- Optimization similarly requires

\[ q_t \left( \frac{C_t}{\bar{C}_t} \right)^{-1/\bar{\sigma}} = \zeta_t^d + \beta(1 - \delta) \hat{E}_t \left[ q_{t+1} \left( \frac{C_{t+1}}{\bar{C}_{t+1}} \right)^{-1/\bar{\sigma}} \right] \]

- Can be written more simply as

\[ q_t^u = \zeta_t^d + \beta(1 - \delta) \hat{E}_t q_{t+1}^u \]

where

\[ q_t^u \equiv q_t \left( \frac{C_t}{\bar{C}_t} \right)^{-1/\bar{\sigma}} \]

is housing price in marginal-utility units

— esp. relevant because distortions in expected value of \( q_t^u \) (not \( q_t \)) matter for eq’m
Policy, Belief Distortions, and Housing Prices

Under RE: the equilibrium evolution of $q_t^u$ would be uniquely determined by the exogenous fundamentals $\{\xi_t^d\}$ — monetary policy would then have no effect on $q_t^u$, but could affect $q_t$, by varying the eq’m marginal utility of income ($C_t$ process influenced by $i_t$, as indicated by above Euler equation).
Under RE: the equilibrium evolution of $q_t^u$ would be uniquely determined by the exogenous fundamentals $\{\zeta_t^d\}$ — monetary policy would then have no effect on $q_t^u$, but could affect $q_t$, by varying the eq’m marginal utility of income ($C_t$ process influenced by $i_t$, as indicated by above Euler equation).

If instead allow for belief distortions ($m_{t+1} \neq 1$): asset-pricing equation implies that $q_t^u$ can depart from RE value (determined purely by fundamentals) — monetary policy can affect the degree to which the belief distortions affect the real price of housing $q_t$. 
NK Model with Housing: Price-Setting

- Calvo-Yun **staggered price adjustment**: fraction $0 < \alpha < 1$ of prices remain unchanged each period, others (randomly selected) re-optimized
NK Model with Housing: Price-Setting

- Calvo-Yun **staggered price adjustment**: fraction $0 < \alpha < 1$ of prices remain unchanged each period, others (randomly selected) re-optimized

- firm that re-optimizes chooses price to max

$$
\hat{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi(p_t(i), P_T^j, P_T; Y_T, q_T^u, \xi_T)
$$

using **stochastic discount factor**

$$
Q_{t,T} = \beta^{T-t} \frac{\lambda(Y_T, q_T^u; \xi_T)}{\lambda(Y_t, q_t^u; \xi_t)} \frac{P_t}{P_T}
$$
This results in an equation for the relative price chosen by firms that re-optimize (and hence, the rate of inflation) that is a function of a vector of two forward-looking variables $Z_t$, which is determined by relations of the form

$$Z_t = z(Y_t, q_t^u; \xi_t) + \alpha \beta \hat{E}_t[\Phi(Z_{t+1})]$$  \hspace{1cm} (\ast)

— inflation then given by an equation

$$\Pi_t \equiv \frac{P_t}{P_{t-1}} = \Pi(Z_t)$$
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— inflation then given by an equation

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Note that equation (*) defines the Phillips-curve trade-off for this economy.
Welfare Objective

- **Dixit-Stiglitz** monopolistic competition, and **iso-elastic** functional forms for disutility of work, production function, allow us to write rep hh’s disutility of work as

\[
\int_0^1 \tilde{v}(H_t(j); \xi_t) \, dj = v(Y_t; \xi_t) \Delta_t
\]

where \( v' > 0 \) and \( v'' > 0 \) and

\[
\Delta_t \equiv \int_0^1 (p_t(i) / P_t)^{-\eta(1+\omega)} \, di \geq 1
\]

is an index of **price dispersion**
Welfare Objective

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\int_0^1 \tilde{v}(H_t(j); \xi_t) dj = v(Y_t; \xi_t) \Delta_t
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where \(v', v'' > 0\) and

\[
\Delta_t \equiv \int_0^1 \left( \frac{p_t(i)}{P_t} \right)^{-\eta(1+\omega)} di \geq 1
\]

is an index of price dispersion

- Calvo-Yun price adjustment implies law of motion for price dispersion of the form

\[
\Delta_t = h(\Delta_{t-1}, Z_t)
\]
Welfare Objective

- Assumed linearity of $\tilde{w}(D; \xi)$ allows us to write

  $$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{w}(D_t; \xi_t) = E_0 \sum_{t=0}^{\infty} \beta^t \bar{\xi}_t \tilde{d}(k_t; \xi_t) + \text{exog.},$$

  neglecting exogenous terms, where

  $$\bar{\xi}_t \equiv \sum_{T=t}^{\infty} [\beta(1 - \delta)]^{T-t} E_t \xi_T^{d}$$

  is the fundamental value of $q_t^u$ (RE eq’m value)
Welfare Objective

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$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{w}(D_t; \xi_t) = E_0 \sum_{t=0}^{\infty} \beta^t \overline{\xi}_t \tilde{d}(k_t; \xi_t) + \text{exog.},$$

neglecting exogenous terms, where

$$\overline{\xi}_t \equiv \sum_{T=t}^{\infty} [\beta(1 - \delta)]^{T-t} E_t \xi^d_T$$

is the fundamental value of $q^u_t$ (RE eq’m value)

Substituting optimal solution for $k_t$, one can write

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{w}(Y_t; q^u_t; \xi_t) = E_0 \sum_{t=0}^{\infty} \beta^t w(Y_t, q^u_t; \xi_t) + \text{exog.}.$$
Welfare Objective

Hence expected utility of rep hh (evaluated using policymaker’s beliefs) can be written as

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, q_t^u, \Delta_t; \zeta_t) \]

where

\[ U(Y, q^u, \Delta; \zeta) \equiv \tilde{u}(C(Y, q^u; \zeta); \zeta) - v(Y; \zeta) \Delta + w(Y, q^u; \zeta) \]
A distorted expectations equilibrium is then a collection of processes \( \{ Y_t, Z_t, \Delta_t, q^u_t, i_t, m_{t+1} \} \) satisfying

- Euler equation for \( i_t \)
- Euler equation for \( q^u_t \)
- law of motion for \( \Delta_t \)
- equations (*) for \( Z_t \)

in addition to the requirement that \( E_t m_{t+1} = 1 \) at all times
Establishing an Upper Bound for Robust Policy

• Recall strategy:
  • for given distortion process \( \{ m_{t+1} \} \), compute the policymaker’s best response
  • find “worst-case” belief distortions that make this equilibrium as bad as possible
Establishing an Upper Bound for Robust Policy

- Recall strategy:
  - for given distortion process \( \{m_{t+1}\} \), compute the policymaker’s best response
  - find “worst-case” belief distortions that make this equilibrium as bad as possible

- Note that in first step,
  - need only consider possible equilibrium evolutions consistent with \( \{m_{t+1}\} \); need not specify type of policy rule to implement each outcome
  - if ZLB never binds (true for small enough shocks and belief distortions), can ignore Euler equation as constraint on possible evolution of output, inflation, price dispersion and housing prices
Establishing an Upper Bound for Robust Policy

Problem:

$$\min_{\{m_{t+1}\}_{t=0}^{\infty}} \max_{\{Y_t, Z_t, \Delta_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ U(Y_t, q_t^u, \Delta_t; \zeta_t) + \theta \beta m_{t+1} \log m_{t+1} \right]$$

where inner max is subject to constraint that $\{Y_t, Z_t, q_t^u, \Delta_t\}_{t=0}^{\infty}$ be a DEE consistent with $\{m_{t+1}\}_{t=0}^{\infty}$ and initial condition $\Delta_{-1}$
Establishing an Upper Bound for Robust Policy

Problem:

\[
\min_{\{m_{t+1}\}_{t=0}^{\infty}} \max_{\{Y_t, Z_t, \Delta_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ U(Y_t, q^u_t, \Delta_t; \xi_t) + \theta \beta m_{t+1} \log m_{t+1} \right]
\]

where inner max is subject to constraint that \( \{Y_t, Z_t, q^u_t, \Delta_t\}_{t=0}^{\infty} \) be a DEE consistent with \( \{m_{t+1}\}_{t=0}^{\infty} \) and initial condition \( \Delta_{-1} \)

- no need to specify \( \{i_t\} \) (find to satisfy Euler eq)
- don’t need to max over \( \{q^u_t\} \), as it is completely determined by exog. disturbance processes and belief distortions
Establishing an Upper Bound for Robust Policy

- Constraints and associated Lagrangian multipliers, for policymaker’s best-response problem:

\[
\gamma_t : \quad \Delta_t = h(\Delta_{t-1}, Z_t)
\]

\[
\Gamma_t : \quad Z_t = z(Y_t, q^u_t; \xi_t) + \alpha \beta E_t[m_{t+1} \Phi(Z_{t+1})]
\]

\[
\Psi_t : \quad q^u_t = \xi^d_t + \beta(1 - \delta) E_t[m_{t+1} q^u_{t+1}]
\]
FOCs for **policymaker best response**: 5 conditions per period

- 4 are like RE analysis in Woodford (2011), but with belief distortions and effects of \( \{ q^u_t \} \) variations
- additional FOC w.r.t. \( q^u_t \) allows solution for \( \Psi_t \), shadow value of relaxing eq’m asset pricing relation

\[
\begin{align*}
mt + 1 &= \exp\left\{- \theta - 1 \left[ \alpha \Gamma_t' \Phi \left( Z_t + 1 \right) + \left( 1 - \delta \right) \Psi_t q^u_t + 1 \right] \right\} \\
\exp\left\{- \theta - 1 \left[ \alpha \Gamma_t' \Phi \left( Z_t + 1 \right) + \left( 1 - \delta \right) \Psi_t q^u_t + 1 \right] \right\} &\text{where} \\
\Gamma_t' &\text{are multipliers indicating value to policymaker of shifting expectation terms in AS constraints} \\
\Psi_t &\text{is multiplier indicating value of shifting expectation term in house pricing equation}
\end{align*}
\]

Adam and Woodford
Establishing an Upper Bound for Robust Policy

- **FOCs for policymaker best response:** 5 conditions per period
  - 4 are like RE analysis in Woodford (2011), but with belief distortions and effects of \( \{q^u_t\} \) variations
  - Additional FOC w.r.t. \( q^u_t \) allows solution for \( \Psi_t \), shadow value of relaxing eq’m asset pricing relation

- **FOC for worst-case belief distortions yields**

\[
    m_{t+1} = \frac{\exp \left\{ -\theta^{-1} \left[ \alpha \Gamma'_t \Phi(Z_{t+1}) + (1 - \delta) \Psi_t q^u_t \right] \right\}}{E_t \left[ \exp \left\{ -\theta^{-1} \left[ \alpha \Gamma'_t \Phi(Z_{t+1}) + (1 - \delta) \Psi_t q^u_t \right] \right\} \right]}
\]

where \( \Gamma'_t \) are multipliers indicating value to policymaker of shifting expectation terms in AS constraints (*), \( \Psi_t \) is multiplier indicating value of shifting expectation term in house pricing equation.
Establishing an Upper Bound for Robust Policy

- Characterizing upper-bound worst-case dynamics: we now have a system of 10 conditions per period
  - 4 structural equations
  - 5 policymaker best-response FOCs
  - 1 eq for worst-case belief distortions
- to solve for 10 variables per period
  - 5 endogenous variables \( \{ Y_t, Z_t, \Delta_t q_t^U \} \)
  - 4 Lagrange multipliers \( \{ \gamma_t, \Gamma_t \Psi_t \} \)
  - belief distortions \( \{ m_{t+1} \} \)
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- Establishing that this upper bound is attainable: need to exhibit a policy rule that is consistent with the upper-bound dynamics, and such that given solution for \( \{m_{t+1}\} \), policy rule + 4 structural eqs uniquely determine the upper-bound dynamics \( \{Y_t, Z_t, \Delta_t, q_t^U\} \)
We linearize around the deterministic steady state solution to the above equations, in the absence of shocks ($\xi_t = \bar{\xi}$ at all times). Since in this case, $m_t + 1 = 1$ at all times, upper-bound steady state is same as optimal steady state under RE analysis. Hence it is the zero inflation steady state (Benigno and Woodford, 2005) robustly optimal policy rule will have to be consistent with this steady state.
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Robustly optimal policy rule will have to be consistent with this steady state.
Log-linearization of 4 structural equations:

- To first order, dynamics of price dispersion simply imply $\dot{\Delta}_t = 0$ at all times, and variable can be ignored.

- And asset pricing eq implies $\dot{q}_t^u = \zeta_t^d$ (exogenous fundamental) to first order, so can replace $q_t^u$ by exog term.
Log-linearization of 4 structural equations:

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- Two remaining log-linear eqs can be combined to yield

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \quad \text{("NKPC")}$$

where $\kappa > 0$, $x_t \equiv \log(Y_t/Y_t^*)$, and $Y_t^*$, $u_t$ are composite exog terms.

--- note that belief distortions do not matter, to first order.
Log-linearization of FOCs for policymaker best response: Lagrange multipliers can be eliminated to yield

\[ \pi_t + \phi_x (x_t - x_{t-1}) + \phi_m \log m_t = 0 \]

where in the (standard) case that flexible-price steady-state output is inefficiently low, but distortion not too large,

\[ \phi_x, \phi_m > 0 \]

— optimal to reduce growth/inflation in states that are over-weighted under subjective expectations
Linearized Upper-Bound Dynamics

Log-linearization of FOC for worst-case beliefs: Lagrange multipliers can be eliminated to yield

\[
\log m_{t+1} = \lambda_m (\pi_{t+1} - E_t \pi_{t+1}) + \lambda_q (\hat{q}^u_{t+1} - E_t \hat{q}^u_{t+1})
\]

where

- in standard case (flex-price output ineff low) \( \lambda_m > 0 \)
Log-linearization of FOC for worst-case beliefs: Lagrange multipliers can be eliminated to yield

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- if in addition $s^d > 0$ (flex-price housing supply ineff large), $\lambda_q > 0$
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— worst-case beliefs increase probability of states with largest inflation surprise or largest surprise increase in housing prices
Eliminating $m_t$ from equation characterizing policymaker best response, one shows that upper-bound dynamics must satisfy

$$\pi_t + \phi_s (\pi_t - E_{t-1} \pi_t) + \phi_x (x_t - x_{t-1}) + \phi_q (\hat{q}_{t+1}^u - E_t \hat{q}_{t+1}^u) = 0$$
Robustly Optimal Policy

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- One can show that this necessary condition is also sufficient: upper-bound dynamics are the unique non-explosive solution to system consisting of structural eq’ns plus the above (as specification of monetary policy)
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- Consider a policy rule under which CB commits to adjust path of interest rates so that projected paths of inflation and output gap fulfill above criterion at all times
Can show that given such a policy rule, worst-case beliefs are the ones characterized above.
Robustly Optimal Policy

- Can show that given such a policy rule, *worst-case beliefs* are the ones characterized above.

- Hence commitment to such a *target criterion* achieves the upper bound.
Robustly Optimal Policy

The robustly optimal target criterion:

$$
\pi_t + \phi_s(\pi_t - E_{t-1}\pi_t) + \phi_x(x_t - x_{t-1})
$$

$$
+ \phi_q(\hat{q}_{t+1}^u - E_t\hat{q}_{t+1}^u) = 0
$$

— a guide to whether policy is on track that requires no reference to the particular shocks hitting the economy

— type of “flexible inflation target”
Robustly Optimal Policy

- The robustly optimal target criterion:

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— a guide to whether policy is on track that requires no reference to the particular shocks hitting the economy

— type of “flexible inflation target”

- Note that if assume RE, both terms in red vanish

— and one recovers same form of optimal target criterion as for NK model w/o housing sector
Implications for Policy

Thus conventional analysis of optimal policy commitment under RE would conclude that monitoring inflation and output gap suffice to tell if policy is on track, even in model with housing market (subject to tax distortions)

— can state optimal target criterion without reference to behavior of housing prices (or other housing variables, except to extent that they affect definition of target output $Y_t^*$)
Implications for Policy

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  — can state optimal target criterion without reference to behavior of housing prices (or other housing variables, except to extent that they affect definition of target output $Y^*_t$)

- If instead one wants policy to be robust to possible departures from RE, one should commit to a target criterion that also involves housing price
Implications for Policy

- If $s^d > 0$ (realistic case), $\phi_q = \phi_m \lambda_q > 0 \implies \text{CB should “lean against the wind”: target lower inflation and/or output gap when housing price (in m.u. units) is unexpectedly high.}$

Note that the target criterion makes no reference to nature of disturbances (except as needed to measure “output gap”) — does not require different response to housing price increases due to expectational errors than to those due to fundamentals!

Would have required raising interest rates more/sooner in mid-2000s?
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Implications for Policy

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