

Speculation, trading and bubbles

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Plan I

- 1 Discuss some stylized facts concerning bubbles.
- 2 Present a very simple model for bubbles and argue that it fits these facts.
 - Distillation of some previous work with Hong, Xiong.
- 3 Make a remark on leverage.
- 4 Present some additional evidence

Stylized facts a theory of bubbles should accommodate I

- ① Asset price bubbles coincide with increases in trading volume.
- ② Asset price bubble implosions often coincide with increases in asset supply.
 - shorting
- Asset price bubbles often coincide with financial or technological innovations.

Bubbles and trading volume: South Sea Bubble I

Introduction

Stylized Facts

Model

Remark on
leverage

Additional
evidence

References

- Extraordinary rise and fall of price of South Sea Company shares and other similar joint-stock companies in 1720.
- $\sim 2,000$ transactions per year in Bank of England stock 1717-1719, 6,846 transactions (100% of stocks outstanding) in 1720.
- East India Company and Royal African Company turned over 150% of stock outstanding in 1720.
- Carlos et al. (2006)

Bubbles and trading volume: Roaring Twenties I

Introduction

Stylized Facts

Model

Remark on
leverage

Additional
evidence

References

- Accounts of stock-market boom of late 1920s emphasize overtrading in 28-29.
- Annual turnover at NYSE climbs from 100% per annum in 1925-27 to over 140% in 1928 and 1929. Davis et al. (2005)
- All-time daily records of share trading volume were reached 10 times in 1928 and 3 times in 1929. New record not set until April 1, 1968, when LBJ announced he would not seek re-election (Hong and Stein (2007))

Bubbles and trading volume: Internet... I

Introduction

Stylized Facts

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Remark on
leverage

Additional
evidence

References

- During the DotCom bubble internet stocks had 3 times the turnover of other similar stocks.
- Lamont and Thaler (2003): 6 cases of spinoffs average 38% daily turnover.
 - Typical NYSE stock turnover of 100% per year.
- Cochrane (2002) documents cross sectional correlation between a stock's market/book and that stock's turnover.
- China's A& B shares - Correlation between higher A-B premium and volume in panel. (Mei et al. (2009))

Bubble implosion and supply I

- In 1720, new issues by the South Sea Company doubled the amount of shares outstanding.
- Outstanding shares of the Royal African Company more than tripled.
- Numerous other joint-stock companies started (Bubbles).
- Bubble Act of 1720: Parliament banned joint-stock companies not authorized by Royal Charter or the extension of corporate charters into new ventures.
 - “the [Bubble Act] was a special-interest legislation for the [South-Sea Company], which controlled its framing and its passage” (Harris (1994))
 - Bubble act used by South Sea Company to sue old chartered companies that had changed activities and where attracting speculators.

Bubble implosion and supply II

Introduction

Stylized Facts

Model

Remark on
leverage

Additional
evidence

References

- Extraordinary number of lock up expirations for DotCom companies in H1 2000. (Ofek and Richardson (2003))
- Venture capital firms that had distributed 3.9 billion to limited partners in third quarter of 1999, distributed 21 billion in 2000 Q1. (Janeway (2012))
- Regulatory innovation (CDS on CDOs), and financial engineering (ABX index, synthetic Collateralized Debt Obligation (CDO)) increased supply of “safe” securities and led to the implosion of the credit bubble.

Bubbles: definition(s) I

- ① Asset prices exceed an asset's fundamental value
- ② Asset prices exceed fundamentals because owners believe they can resell the asset for a higher price in the future. (Brunnermeier (2008))

Bubbles: Theories I

- Rational Bubbles (Tirole (1985), Santos and Woodford (1997))
 - Prices exceed fundamental value because they are expected to exceed fundamental value by even more tomorrow.
 - Difficulty dealing with finite-lived assets.
 - Do not generate correlation with trading volume.
- A positive shock is amplified by extrapolation of past returns (Shiller (2006))
 - Getting association with trading volume requires a link between extrapolation and disagreement.
- **Limited arbitrage**
 - Asymmetry between costs of going short vs. long.
 - Heterogeneous beliefs (Miller, 1977; **Harrison and Kreps (1978).**)
 - Absence of common knowledge that bubble exists (Allen et al. (1993); Abreu and Brunnermeier (2003).)

Principal assumptions I

Introduction

Stylized Facts

Model

Remark on
leverage

Additional
evidence

References

- Costly shorting
- Heterogeneous beliefs from overconfidence, the tendency of people to overestimate the precision of their knowledge.
 - Disciplining device
- Certainly not the only mechanisms that cause bubbles in reality.
- Far from being standard in economics
 - Economic models typically assume symmetric costs between going long and going short
 - Results showing that rational investors with common priors cannot agree to disagree.
 - No trade theorems (Milgrom, Stokey, Tirole): Unless some traders trade for “irrational” reasons, there is no trade.

Evidence for costly short-sale I

- Some obvious cases
 - Housing
 - CDO's before the introduction of ABX and synthetic CDO's.
- Shorting mechanisms for stocks (D'Avolio (2002))
- Stocks with higher dispersion of earnings forecasts have lower future returns (Diether et al. (2002))
 - It is easier for optimists to express their beliefs in markets.

Evidence of overconfidence I

Introduction

Stylized Facts

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leverage

Additional
evidence

References

- Alpert and Raiffa (1982).
- Documented among: Engineers (Kidd (1970)),
Entrepreneurs (Cooper et al. (1988))...
- Ben-David et al. (2010) on CFO predictions of S&P
returns.
 - Realized returns are within executives [10%,90%] intervals
33% of the time.

A sketch of a model I

- With Hong, Xiong
- Very simplified version
- Investors in model estimate the “state” of the system using signals they believe are related to that state.
 - Filtering.
- Investors have heterogeneous beliefs
 - Some investors attribute excessive informativeness to certain (volatile) signals. Others may be rational
 - Group A is “rational” but group B thinks that opinion of a business commentator correlates well with future dividends.
 - Overconfidence (miscalibration): Some investors overestimate how much they know .
 - No learning about overconfidence (horizon).
 - Investors know relative opinions fluctuate.

A sketch of a model I

- Buyers know that in the future optimists may be willing to pay more than their own reservation value.
- Short sales are costly
 - Optimists have an easier time expressing their opinions.
- Bubble = value of resale option. (*cf.* definition)
 - Conservative measure

Consequences I

- A higher degree of overconfidence leads to higher prices and a higher value for the resale option.
- Also leads to more volatile *relative* opinions and thus **higher trading volume**.
- Lower borrowing costs make resale option more valuable.
- Shorter horizon implies fewer opportunities to resell, thus smaller bubble.
- When investors have limited capacity to bear risk, an increase in the supply of the asset is absorbed by less optimistic buyers.
- Valuation that marginal buyer has of the future payoffs declines as supply increases.
 - Lower discounted fundamental value of the asset.

Consequences II

- Buyer also knows that because the larger supply needs to be absorbed, future marginal buyers are likely to be less optimistic and thus the value of the resale option declines.
- **Increase in asset supply diminishes (deflates) the bubble.**
- Shorting
- Leverage.

A simple model I

- 4 periods $t = 1, 2, 3, 4$, a single good, and single risky asset in finite supply S .
- Risk-free technology. An investment of δ units of the good in any period t yields one unit in period $t + 1$.
- Large number of risk-neutral investors, that only value consumption in the final period $t = 4$.
- Risky asset produces dividend θ_t at $t = 2, 3, 4$.
- $\theta_t \in \{\theta_\ell, \theta_h\}$ with $\theta_h > \theta_\ell$, is independent of the past and future dividends and $\text{Prob}[\theta_t = \theta_\ell] = .5$

•

$$\bar{\theta} = E(\theta_t) = .5\theta_\ell + .5\theta_h.$$

- No short-sales, no borrowing.
- Assets traded at $t = 1, 2, 3, 4$, **ex-dividend**.
- $p_4 = 0$.

A simple model: Signals I

- Signal s_t at $t = 1, 2, 3$ observed *before* trading occurs at t , but *after* the dividend at t (if $t > 1$) is observed.
- $s_t \in \{0, 1, 2\}$, has no predictive power for future dividends.
- Two sets of investors, A and B , each with many investors.
- Agents in A are rational.
- Agents in B actually believe that s_t predicts θ_{t+1} :

$$\text{Prob}[\theta_{t+1} = \theta_h | s_t] = .5 + g(s_t - 1), \quad 0 < g < .5.$$

- A and B agree s_t is *i.i.d* and does not predict θ_{t+j} , $j \geq 2$
- $\text{Prob}[s_t = 0] = \text{Prob}[s_t = 2] = q \leq .5$
- *Ex-ante* no optimism or pessimism.
- Precision inverse of variance. B 's have exaggerated view of the precision of their beliefs when $s_t \in \{0, 2\}$.

A simple model: no capital constraints I

- v_t^C willingness to pay of member of group $C \in \{A, B\}$ for an infinitesimal amount of the risky asset.

-

$$v_t^C = \delta \left[E^C(\theta_{t+1} | s_t) + E(p_{t+1}) \right]$$

A simple model: no capital constraints II

s_t	Buyer	p_3	p_2
0	A	$\delta\bar{\theta}$	$\delta(\bar{\theta} + \mathbb{E}p_3)$
1	A, B	$\delta\bar{\theta}$	$\delta(\bar{\theta} + \mathbb{E}p_3)$
2	B	$\delta[\bar{\theta} + g(\theta_h - \theta_\ell)]$	$\delta(\bar{\theta} + g(\theta_h - \theta_\ell) + \mathbb{E}p_3)$
$\mathbb{E}p$	—	$\delta[\bar{\theta} + qg(\theta_h - \theta_\ell)]$	$(\delta + \delta^2)[\bar{\theta} + qg(\theta_h - \theta_\ell)]$
Bubble	—	0	$\delta^2 qg(\theta_h - \theta_\ell)$

s_t	Buyer	p_1
0	A	$\delta(\bar{\theta} + \mathbb{E}p_2)$
1	A, B	$\delta(\bar{\theta} + \mathbb{E}p_2)$
2	B	$\delta(\bar{\theta} + g(\theta_h - \theta_\ell) + \mathbb{E}p_2)$
$\mathbb{E}p$	—	...
Bubble	—	$(\delta^2 + \delta^3)qg(\theta_h - \theta_\ell)$

Volume of trade I



$$EV_1 = \frac{1}{2} \times 2q \times S = qS.$$

- Trade in period 2 if history (2,0) or (0,2) occurs. Also (1,2) and (1,0) but only half the volume.

$$EV_2 = 2q^2S + (1 - 2q)2q \times \frac{S}{2} = qS.$$

- Similarly $EV_3 = qS.$

Summary of results I

Proposition

In the presence of fluctuating differences in beliefs and short-sale constraints, bubble exists - investors are willing to pay for an asset in excess of their own valuation of future dividends. In addition,

- (i) Bubble increases when the probability of disagreement increases.*
- (ii) Volume of trade increases with the probability of disagreement.*
- (iii) Size of the bubble decreases with the risk-free interest rate.*
- (iv) The bubble declines as the time to maturity of the asset approaches, because there are fewer opportunities to trade.*

Limited Capital I

- Group A has enough capital to acquire full supply at their reservation price v_t^A . Group B has limited capital.
- The introduction of agents with limited capital can only lower v_t^C
- S_t for the total supply of the asset, K_t^C the capital available for purchases for agents in group C , at t .
- Assume

$$\frac{K_t^A}{S_t - S_{t-1}^A} > v_t^A \quad (1)$$

- If $v_t^A \geq v_t^B$ then the price of the asset $p_t = v_t^A$.
- If $\frac{K_t^B}{(S_t - S_{t-1}^B)} \geq v_t^B > v_t^A$ then $p_t = v_t^B$.

Limited Capital II

- If $v_t^A \leq \frac{K_t^B}{(S_t - S_{t-1}^B)} < v_t^B$, then

$$p_t = \frac{K_t^B}{(S_t - S_{t-1}^B)}.$$

- Cash-in-the-market-pricing, Allen and Gale (2002).
- If $v_t^A > \frac{K_t^B}{(S_t - S_{t-1}^B)}$ then $p_t = v_t^A$.

-

$$p_t = \max \left\{ v_t^A, \min \left\{ v_t^B, \frac{K_t^B}{(S_t - S_{t-1}^B)} \right\} \right\}$$

- Other things equal, this price decreases as S_t increases.
- If float is large enough, then even when B agents are optimists some of the asset supply ends up in the hands of A agents, because of the limited capital of group B agents.

Limited Capital III

- Larger float lowers the price and the turnover of the asset.
- If the supply of risky asset remains constant through time, a larger float is associated with smaller turnover.

Change in Float I

- Buyer's of the stock of the South Sea Company or buyers of Internet stocks in the late 90's did not know with certainty the future supply of these assets.
- $S_0 = S_1 = S$ and $S_3 = S_2 = S$ with probability π and $S + \Delta S > S$ with probability $1 - \pi$.
- Realization of the supply of the risky asset is independent of the realization of $(\theta_1, \theta_2, s_1, s_2)$, and observed in period 2 *before* the signal s_2 is observed.
- Investors know the supply when they trade in period 2.
- Increase in supply will come from sales of the asset by "insiders", as in Hong et al. (2006).
- To simplify matters assume that insiders only wish to sell.

Bubble implosion I

- Will show that for certain parameter values, bubble exists for $t < 2$ and persists if $S_2 = S$ but deflates when the supply increases.
- Start by imposing bounds on the aggregate portfolio of group B agents that will insure that the bubble persists when the supply of the asset is unchanged and the bubble deflates when the supply increases
- Later show that these bounds would hold when the initial supply S is sufficiently small and the potential increase in the supply in period 2, ΔS , is sufficiently large.
- Assume

$$\frac{K_3^B}{S - S_2^B} \geq \delta[\bar{\theta} + g(\theta_h - \theta_\ell)], \quad (2)$$

$$\frac{K_3^B}{S + \Delta S - S_2^B} < \delta\bar{\theta}. \quad (3)$$

Bubble implosion II

- Equivalently could assume reservation price of insiders $> v_2^B$, with probability π , and $< \delta\bar{\theta}$, otherwise.
- Given (2) and (3) insiders sell no shares with probability π and sell all their shares with probability $1 - \pi$.
- When $S_2 = S$, p_3 matches the price with no capital constraints whereas when $S_2 = S + \Delta S$, $p_3 = v_3^A$.
- If $S_2 = S + \Delta S$, no period 2 bubble, since it is known that period 3 prices are independent of signal s_3 .
- if $S_2 = S$

$$E p_3 = \delta[\bar{\theta} + qg(\theta_h - \theta_\ell)],$$

since B agents acquire the whole float if $s_3 = 2$.

- To insure that if $S_2 = S$, $p_2 =$ price no constraints,

$$\frac{K_2^B}{S - S_1^B} \geq (\delta + \delta^2)[\bar{\theta} + qg(\theta_h - \theta_\ell)]. \quad (4)$$

Bubble implosion III

- To insure that when $S_2 = S + \Delta S$, the marginal buyer of the asset in period 2 always belongs to A , it suffices that

$$\frac{K_2^B}{S + \Delta S - S_1^B} \leq (\delta + \delta^2)\bar{\theta}. \quad (5)$$

- If inequalities (2) - (5) hold then before S_2 (and hence s_2) is observed

$$Ep_2 = (\delta + \delta^2)[\bar{\theta} + q\pi g(\theta_h - \theta_\ell)]$$

- If inequalities (2), (3) and (4) hold, rational buyers in period 1 will always be willing to pay in excess of their own valuations of future dividends of the risky asset, because if supply does not increase they may have an opportunity to sell the asset to over-optimistic B agents in the future.

Bubble implosion IV

- Since A agents have sufficient capital to buy the total supply of the asset at these higher prices, the price of the asset in period 1 exceeds the expected discounted dividends independently of the realized signal in period 1 (s_1) and the capital constraints of group B agents.

- When $s_1 = 2$,

$$v_1^B(2) = \delta (\bar{\theta} + qg(\theta_h - \theta_\ell)) + (\delta + \delta^2)[\bar{\theta} + q\pi g(\theta_h - \theta_\ell)],$$

which exceeds the present value of dividends expected by group B agents by $(\delta + \delta^2)[q\pi g(\theta_h - \theta_\ell)]$.

- To guarantee that when $s_1 = 2$, $p_1 = v_1^B(2)$ one must assume that:

$$\frac{K_1^B}{S - S_0^B} \geq v_1^B(2) \quad (6)$$

Bubble implosion V

- In this case $b_1 = (\delta^2 + \delta^3)\pi qg(\theta_h - \theta_\ell)$,
- Smaller than the bubble that obtains when liquidity constraints are not binding - reflecting the possibility that future supplies may increase.
- Inequalities (2), (4) and (6) hold provided S is small enough (relative to wealth of group B agents)
- Inequalities (3) and (5) hold provided ΔS is large enough (relative to initial wealth of group B agents.)
- Showing this amounts to examining evolution of wealth in equilibrium. (see Appendix)
- Bubble arises if initial supply is small relative to optimists capital and there is a chance that supply will not increase and implodes if there is a realization of a large supply.
- Supply

Bubble implosion VI

- Shorting
- Leverage (how much wealth is available to B's).

Leverage I

- Loans from pessimists to optimists, Geanakoplos (2010)
- For tax or regulatory reasons, not all optimists are the adequate holders of certain risky assets.
 - Although not the most appropriate direct investors in houses, optimistic banks could make loans charging more than prime rates to subprime buyers that would be capable of repaying their loans in the “likely” event that house prices continued to behave as in the previous ten years.
 - Foote et al. (2012)
 - A money market fund that was willing to finance 98.4% of the purchase-price of a AAA mortgage security to an investor in 2006 probably thought that these securities were actually nearly risk-free, warranting a leverage of 60.
 - Landesbanks

Leverage II

- Two periods $t = 0, 1$.
- Two non-storable consumption goods.
- Single asset that pays in period 1, $x \in \{\kappa, K\}$ units of the period 1 good, with $0 < \kappa \ll K$.
- Three groups of same size A, B, N , of risk-neutral agents with utility function

$$c_0 + \delta \left[\pi^C c_1(K) + (1 - \pi^C) c_1(\kappa) \right]$$

for $C \in (A, B, N)$.

- $\pi^N \geq \pi^B > \pi^A$.
- Agents in group N are not allowed to purchase asset.
- Asset in inelastic supply =1 from agents outside model.
- Agent in group C has endowment W^C of the good in period 0 and endowment 0 of the period 1 good.

Leverage III

- Collateralized loans. Loan of s units of 0-good in exchange for promise (y_1, y_2) collateralized by 1 unit of the asset. To be credible, $y_1 \leq \kappa$ and $y_2 \leq K$. Contract $C = (s, y_1, y_2)$.
- High leverage requires $y_1 > y_2$ and since $\pi^A < \pi^B$ leverage by A 's is expensive for B 's.
- Suppose A 's lend to B 's using a contract \bar{C} . If $\bar{y}_2 > \bar{y}_1$, N 's value payoff more than A 's. In equilibrium no loans from A 's, unless N 's exhaust their capital.
- Thus if N 's have enough capital, any contract involving A 's must have $y_2 \leq y_1 \leq \kappa$. (riskless, low leverage)
- In this example, if N 's have sufficient capital, p can exceed the valuation of B 's.
- Leverage as a result of (successful) attempt to circumvent regulation.

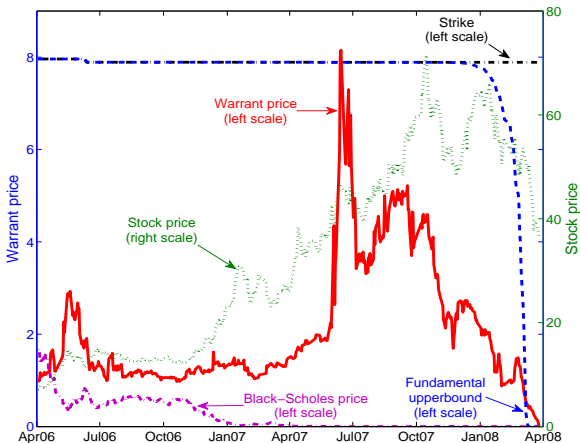
Additional evidence I

- China's A and B stocks (Mei et al. (2009))
- Chinese put warrants (Xiong and Yu (2011))
- Panel of prices, trading volume etc... of 18 put warrants trading in 2005-2007.
 - Extraordinary rise of prices in Chinese stocks between 2005 and 2007 made it almost certain that these warrants would expire without being exercised
- Black-Scholes option-pricing formula implied that close to their expiration date, these warrants often were worth less than .05 hundredth of a yuan.
- Average price of warrants in days when B-S price below .05 hundredth of yuan was .948 yuan
- Warrants with a value of less than .05 hundredth of a yuan had an average turnover rate of 328 percent.

Additional evidence II

- On last trading day, virtually worthless, warrants turned, on average, over 100% of their float every 20 minutes!
- Larger float of a warrant associated with smaller bubble.
- Put warrant on the stock of WuLiangYe Corporation, a liquor producer
 - Warrant issued on April 3, 2006 in-the-money with an exercise price of 7.95 yuan while WuLiangYe's stock traded at 7.11 yuan.
 - In October 15th 2007, stock reached peak of 71.56 yuan and then drifted down to close at 26 yuan at expiration.
 - Calculations by Xiong and Yu (2011) is that after July 07, the Black-Scholes price of this put was below .05 hundredth of a yuan, but the warrant traded for a few yuans, and only dropped below its initial price of .99 yuan in the last few trading days.
 - Warrant on the stock of WuLiangYe Corporation volume of trade on the last trading day was 1,841%.

WuLiangYe Corporation



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Evolution of Wealth I

- Agents in group B start with $0 < S_0^B < S$ units of the risky asset and $0 < K_0^B$ units of the good, and $S_{t-1}^B \leq S$ and K_t^B for $t = 2, 3$ are consequences of their actions, realizations of the random shocks and equilibrium prices in period 1 and 2.
- Given $K_0^B > 0$ inequalities (2), (4) and (6) hold, whenever S is small enough.
- Proof: If S is small enough, even if B agents acquire the full supply of the risky asset and have no possibility of borrowing, they would have a minimum amount left over to invest in the risk-free technology. This delivers a lower bound on K_1^B , K_2^B and K_3^B , the amounts available to agents in group B to acquire additional shares in periods 1 to 3. By assuming an even smaller value for S , if necessary, we can thus guarantee that inequalities (2), (4) and (6) hold.

Evolution of Wealth II

- To examine (3) and (5) must study the dynamics of the evolution of the aggregate wealth of B agents. Given K_0^B and S_0^B , the evolution of the aggregate wealth of group B in equilibrium depends on the realizations of the dividends, signals and supply and on the way assets are allocated between the two groups when their valuations are identical.
 - Assume that when the two groups have identical valuations for the risky asset, group B agents get all the shares they want.
- Write $W_t^B(K_0^B, S_0^B)$ for the maximum wealth that agents in group B can have after dividend payments in period t , where the maximum is taken over all possible realizations of signals, dividends and all portfolio choices.

Evolution of Wealth III

- Increase in ΔS cannot increase W_1^B or W_2^B because the price of the risky asset can only decrease with an increase in ΔS .
- $p_2 \geq (\delta + \delta^2)\bar{\theta}$, the expected (by A agents) discounted dividends of the asset.
- If ΔS is large enough,

$$W_2^B < (\delta + \delta^2) \bar{\theta} \frac{S + \Delta S}{2} \leq p_2 \frac{S + \Delta S}{2}.$$

- Even if B agents use all their wealth in period 2 to buy the asset they cannot acquire more than half the total (larger) supply, and thus inequality (5) holds and,

$$S_2^B < \frac{S + \Delta S}{2}. \quad (7)$$

Evolution of Wealth IV

- Marginal buyer in period 2 when supply increases always belongs to group A .
- *Ex-post* rate of return of the risky asset between periods 2 and 3 depend on (p_2, p_3, θ_3) .
- Since $p_3 \leq$ the expected (by B agents) discounted dividends of the asset when $s_3 = 2$, and the dividend paid $\leq \theta_h$, the rate of return is at most

$$\bar{R} := \frac{\theta_h + \delta (\bar{\theta} + .25(\theta_h - \theta_\ell))}{(\delta + \delta^2)\bar{\theta}}$$

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- This bound exceeds $1 + r$ and thus is also a bound for the growth in wealth. Hence,

$$W_3^B \leq \bar{R}W_2^B$$

and by choosing if necessary a larger ΔS we can insure that

$$K_3^B \leq W_3^B \leq \bar{R}W_2^B \leq \delta\bar{\theta}\frac{\Delta S}{2} < \delta\bar{\theta}\left(S + \Delta S - S_2^B\right),$$

where the last inequality follows from equation (7). Hence inequality (3) holds.

- Bubble arises in period 1 provided that “irrational” agents have enough initial wealth relative to the initial supply of the risky asset and the bubble implodes in period 2 if and only if the supply of the risky asset increases by a sufficiently large amount in period 2.