A Model of Monetary Exchange in Over-the-Counter Markets

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Money in financial over-the-counter markets

Broad question:

Effects of monetary policy on OTC markets

What we do:

Build a model where fiat money is used as a medium of exchange in financial OTC markets

How we do it:

Embed the OTC market structure and gains from trade in financial assets of Duffie, Gârleanu and Pedersen (2005) into the monetary framework of Lagos and Wright (2005)

Connection to Duffie, Gârleanu and Pedersen (2005)

- DGP has two compelling features:
  - realistic OTC marketstructure
  - gains from trading financial assets

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Connection to Lagos and Wright (2005)

- Standard LW: money (and sometimes other assets) facilitates exchange between consumers and producers in goods markets
- This model: money facilitates the allocation of financial assets among dealers and investors with heterogeneous valuations

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Recent related work on money and asset prices

Inflation and asset prices in LW environments

Lagos (2010, 2011), Lester, Postlewaite and Wright (2012), Nosal and Rocheteau (2013), He, Wright and Zhu (2013), Li and Li (2013)

Blend of DGP and Shi-Trejos-Wright

Trejos and Wright (2013)

Blend of DGP and LW

Geromichalos and Herrenbrueck (2013)

Environment

- *Time.* Discrete, infinite horizon, two subperiods per period
- *Population.* $[0, 1]$ investors, $[0, v]$ dealers (both infinitely lived)
- *Commodities.* Two divisible, nonstorable consumption goods:
  - *dividend good*
  - *general good*

Preferences

Investors: $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\varepsilon_{ti} y_{ti} + c_{ti} - h_{ti}\right)$

Dealers: $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_{td} - h_{td}\right)$

- $\beta \in (0,1)$: discount factor
- ε_{ti} : preference shock, i.i.d. over time, cdf $G(\cdot)$ on $[\varepsilon_L, \varepsilon_H]$
- y_{ti} : consumption of dividend good
- c_{td}, c_{ti} : consumption of general good
- h_{td}, h_{ti} : effort to produce general good

Endowments and production technology

First subperiod

A^s productive units (*trees*)

- Each unit yields y dividend goods *at the end of the first subperiod*
- Each unit permanently "fails" with probability $1 - \pi$ at the beginning of the period
- Failed units immediately replaced by new units (allocated uniformly to investors)

Second subperiod

- Linear technology allows dealers and investors to transform effort into general goods

Assets

Equity shares

- A^s equity shares
- At the beginning of period t:
  - $(1-\pi)\,A^s$ shares of failed trees disappear
  - $(1-\pi)\,A^s$ shares of new trees allocated uniformly to investors

Fiat money

- Money supply: A_t^m dollars
- Monetary policy: $A_{t+1}^m = \gamma A_t^m$, $\gamma \in \mathbb{R}_{++}$
  (implemented with lump-sum injections/withdrawals)

Market structure

First subperiod: OTC market

- money, equity (*cum dividend*)
- dealer-investor and investor-investor pairwise trade
- Walrasian trade between all dealers

Second subperiod: centralized market

- general good, money, equity (*ex dividend*)
- Walrasian trade between all dealers and investors

"Anonymity" $\Rightarrow$ *quid pro quo* trade
$\Rightarrow$ money serves as means of payment

OTC market structure

Investors

- Contact another investor with probability α
- Contact a dealer with probability δ

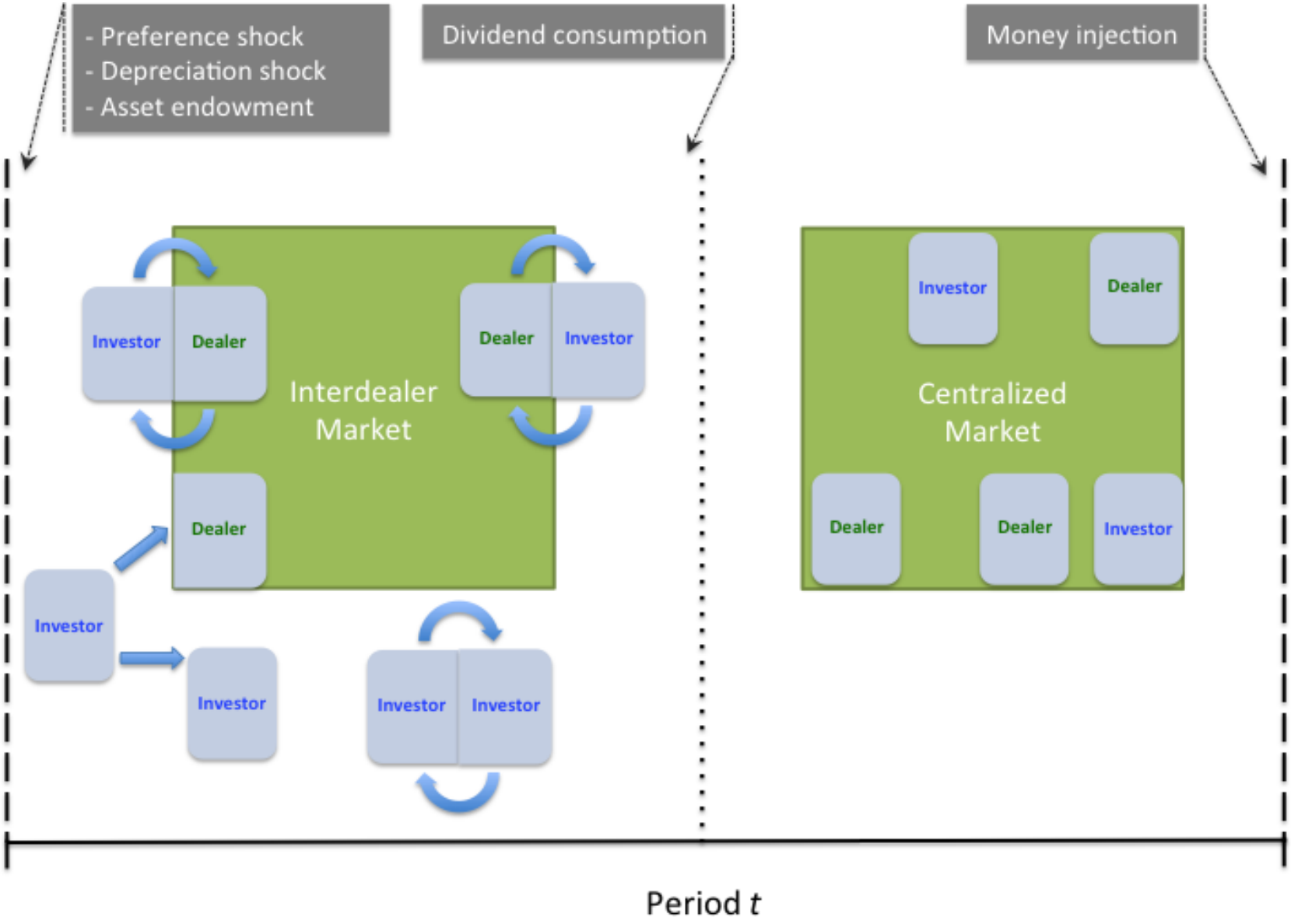
Dealers

- Contact an investor with probability $\kappa \equiv \delta / \nu$
- Access to a competitive interdealer market

OTC terms of trade

- Investor-dealer meetings:
  - investor makes offer with probability θ
  - dealer makes offer with probability $1 - \theta$
- Investor-investor meetings:
  - higher-valuation investor makes offer with probability η
  - lower-valuation investor makes offer with probability $1 - \eta$

Timeline and marketstructure



Social planner

Proposition

The efficient allocation is characterized by three properties:

(i) Only dealers carry equity overnight.

(ii) In direct bilateral trades between investors, all equity is allocated to the investor with higher valuation.

(iii) Among those investors who have a trading opportunity with a dealer, only those with the highest preference type hold equity shares at the end of the OTC round of trade.

Value functions

Dealers

$$W_t^D(\mathbf{a}_t) \quad : \quad \text{value of entering CM with } \mathbf{a}_t \equiv (a_t^m, a_t^s)$$

$$\hat{W}_t^D(\mathbf{a}_t) \quad : \quad \text{value of rebalancing portfolio } \mathbf{a}_t \text{ in OTCM}$$

$$V_t^D(\mathbf{a}_t) \quad : \quad \text{value of entering OTCM}$$

Investors

$$W_t^I(\mathbf{a}_t) \quad : \quad \text{value of entering CM}$$

$$V_t^I(\mathbf{a}_t) \quad : \quad \text{value of entering OTCM}$$

Centralized market

Dealers

$$W_t^D(\mathbf{a}_t) = \max_{c_t, h_t, \tilde{\mathbf{a}}_{t+1}} \left[c_t - h_t + \beta V_{t+1}^D(\mathbf{a}_{t+1}) \right]$$

$$c_t + \boldsymbol{\phi}_t \tilde{\mathbf{a}}_{t+1} \leq h_t + \boldsymbol{\phi}_t \mathbf{a}_t$$

$$\mathbf{a}_{t+1} = (\tilde{a}_{t+1}^m, \pi \tilde{a}_{t+1}^s)$$

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Investors

$$W_t^I(\mathbf{a}_t) = \max_{c_t, h_t, \tilde{\mathbf{a}}_{t+1}} \left[c_t - h_t + \beta \int V_{t+1}^I(\mathbf{a}_{t+1}, \varepsilon') \, dG(\varepsilon') \right]$$

$$c_t + \boldsymbol{\phi}_t \tilde{\mathbf{a}}_{t+1} \leq h_t + \boldsymbol{\phi}_t \mathbf{a}_t + T_t$$

$$\mathbf{a}_{t+1} = (\tilde{a}_{t+1}^m, \pi \tilde{a}_{t+1}^s + (1-\pi) A^s)$$

Dealer problem in OTCM

$$
\begin{aligned}
V_t^D\left(\mathbf{a}_{td}\right) &= \kappa\theta \int \hat{W}_t^D\left(\overline{a}_{td}^m, \overline{a}_{td}^s\right) dH_t\left(\mathbf{a}_{ti}, \varepsilon\right) \\
&\quad + \kappa\left(1-\theta\right) \int \hat{W}_t^D\left(\overline{a}_{td^*}^m, \overline{a}_{td^*}^s\right) dH_t\left(\mathbf{a}_{ti}, \varepsilon\right) \\
&\quad + \left(1-\kappa\right) \hat{W}_t^D\left(\mathbf{a}_{td}\right)
\end{aligned}
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&\quad + (1-\kappa)\, \hat{W}_t^D(\mathbf{a}_{td})
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{W}_t^D(\mathbf{a}_t) &= \max_{\hat{a}_t^m, \hat{a}_t^s} W_t^D(\hat{a}_t^m, \hat{a}_t^s) \\
\hat{a}_t^m + p_t \hat{a}_t^s &\leq a_t^m + p_t a_t^s
\end{aligned}
$$

p_t : nominal equity price in the OTC interdealer market

Investor problem in OTCM

$$
\begin{aligned}
V_t^I\left(\mathbf{a}_{ti},\varepsilon_i\right) \;=\; & \delta\theta\int\left[\varepsilon_i y\overline{a}_{ti*}^s + W_t^I\left(\overline{a}_{ti*}^m,\overline{a}_{ti*}^s\right)\right]dF_t^D\left(\mathbf{a}_{td}\right)\\
& +\delta\left(1-\theta\right)\int\left[\varepsilon_i y\overline{a}_{ti}^s + W_t^I\left(\overline{a}_{ti}^m,\overline{a}_{ti}^s\right)\right]dF_t^D\left(\mathbf{a}_{td}\right)\\
& +\alpha\int\tilde{\eta}\left(\varepsilon_i,\varepsilon_j\right)\left[\varepsilon_i y\underline{a}_{ti*}^s + W_t^I\left(\underline{a}_{ti*}^m,\underline{a}_{ti*}^s\right)\right]dH_t\left(\mathbf{a}_{tj},\varepsilon_j\right)\\
& +\alpha\int\left[1-\tilde{\eta}\left(\varepsilon_i,\varepsilon_j\right)\right]\left[\varepsilon_i y\underline{a}_{ti}^s + W_t^I\left(\underline{a}_{ti}^m,\underline{a}_{ti}^s\right)\right]dH_t\left(\mathbf{a}_{tj},\varepsilon_j\right)\\
& +\left(1-\alpha-\delta\right)\left[\varepsilon_i y a_{ti}^s + W_t^I\left(\mathbf{a}_{ti}\right)\right]
\end{aligned}
$$

...

Trading situations in OTCM

1. Dealer with interdealer market
2. Dealer-investor trade
   - investor offers w.p. θ
   - dealer offers w.p. $1 - \theta$
3. Investor-investor trade
   - investor i offers investor j w.p. η if $\varepsilon_j < \varepsilon_i$

Dealer with interdealer market

Dealer with $\mathbf{a}_t = (a_t^m, a_t^s)$ chooses $(\hat{a}_{td}^m, \hat{a}_{td}^s)$

$$\hat{a}_{td}^m = \begin{cases} 0 & \text{if } p_t \phi_t^m < \phi_t^s \\ a_t^m + p_t a_t^s & \text{if } \phi_t^s < p_t \phi_t^m \end{cases}$$

$$\hat{a}_{td}^s = \begin{cases} a_t^s + \frac{1}{p_t} a_t^m & \text{if } p_t \phi_t^m < \phi_t^s \\ 0 & \text{if } \phi_t^s < p_t \phi_t^m \end{cases}$$

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$$\hat{W}_t^D(\mathbf{a}_t) = (a_t^m + p_t a_t^s)\bar{\phi}_t + C$$

$$\bar{\phi}_t \equiv \max(\phi_t^m, \phi_t^s / p_t)$$

Dealer-investor trade: formulation

Investor with type ε and $\left(a_{ti}^{m}, a_{ti}^{s}\right)$ contacts dealer with $\left(a_{td}^{m}, a_{td}^{s}\right)$

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- w.p. θ investor offers $\left\langle\left(\overline{a}_{ti*}^{m}, \overline{a}_{ti*}^{s}\right), \left(\overline{a}_{td}^{m}, \overline{a}_{td}^{s}\right)\right\rangle$, solves:

$$\max_{\overline{a}_{ti*}^{m}, \overline{a}_{ti*}^{s}, \overline{a}_{td}^{m}, \overline{a}_{td}^{s}} \left[\varepsilon y \overline{a}_{ti*}^{s} + W_t^{I}\left(\overline{a}_{ti*}^{m}, \overline{a}_{ti*}^{s}\right)\right]$$

$$\overline{a}_{ti*}^{m} + \overline{a}_{td}^{m} + p_t\left(\overline{a}_{ti*}^{s} + \overline{a}_{td}^{s}\right) \leq a_{ti}^{m} + a_{td}^{m} + p_t\left(a_{ti}^{s} + a_{td}^{s}\right)$$

$$\hat{W}_t^{D}\left(\overline{a}_{td}^{m}, \overline{a}_{td}^{s}\right) \geq \hat{W}_t^{D}\left(a_{td}^{m}, a_{td}^{s}\right)$$

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$$\hat{W}_t^D\left(\overline{a}_{td}^m, \overline{a}_{td}^s\right) \geq \hat{W}_t^D\left(a_{td}^m, a_{td}^s\right)$$

- w.p. $1-\theta$ dealer offers $\left\langle \left(\overline{a}_{ti}^m, \overline{a}_{ti}^s\right), \left(\overline{a}_{td*}^m, \overline{a}_{td*}^s\right)\right\rangle$, solves:

$$\max_{\overline{a}_{ti}^m, \overline{a}_{ti}^s, \overline{a}_{td*}^m, \overline{a}_{td*}^s} \hat{W}_t^D\left(\overline{a}_{td*}^m, \overline{a}_{td*}^s\right)$$

$$\overline{a}_{ti}^m + \overline{a}_{td*}^m + p_t\left(\overline{a}_{ti}^s + \overline{a}_{td*}^s\right) \leq a_{ti}^m + a_{td}^m + p_t\left(a_{ti}^s + a_{td}^s\right)$$
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Dealer-investor trade: solution when investor offers

$$\overline{a}^m_{ti*} = \begin{cases} 0 & \text{if } \varepsilon^*_t < \varepsilon \\ a^m_{ti} + p_t a^s_{ti} & \text{if } \varepsilon < \varepsilon^*_t \end{cases}$$

$$\overline{a}^s_{ti*} = \begin{cases} a^s_{ti} + \frac{1}{p_t} a^m_{ti} & \text{if } \varepsilon^*_t < \varepsilon \\ 0 & \text{if } \varepsilon < \varepsilon^*_t \end{cases}$$

where

$$\varepsilon^*_t \equiv \frac{p_t \phi^m_t - \phi^s_t}{y}$$

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$$\overline{a}_{ti}^{m} = \begin{cases} 0 & \text{if } \varepsilon_t^* < \varepsilon \\ a_{ti}^{m} + p_t^o(\varepsilon)\, a_{ti}^{s} & \text{if } \varepsilon < \varepsilon_t^* \end{cases}$$

$$\overline{a}_{ti}^{s} = \begin{cases} a_{ti}^{s} + \frac{1}{p_t^o(\varepsilon)} a_{ti}^{m} & \text{if } \varepsilon_t^* < \varepsilon \\ 0 & \text{if } \varepsilon < \varepsilon_t^* \end{cases}$$

where

$$p_t^o(\varepsilon) \equiv \left(\frac{\varepsilon y + \phi_t^s}{\varepsilon_t^* y + \phi_t^s} \right) p_t$$

Dealer-investor trade: dollar value of post-trade portfolios

- If investor offers:

$$\overline{a}^m_{td} + p_t \overline{a}^s_{td} = a^m_{td} + p_t a^s_{td}$$

$$\overline{a}^m_{ti^*} + p_t \overline{a}^s_{ti^*} = a^m_{ti} + p_t a^s_{ti}$$

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$$\overline{a}^m_{ti^*} + p_t \overline{a}^s_{ti^*} = a^m_{ti} + p_t a^s_{ti}$$

- If dealer offers:

$$\overline{a}^m_{td^*} + p_t \overline{a}^s_{td^*} = \begin{cases} a^m_{td} + p_t a^s_{td} + \left[p^o_t(\varepsilon) - p_t\right] \frac{a^m_{ti}}{p^o_t(\varepsilon)} & \text{if } \varepsilon^*_t \leq \varepsilon \\ a^m_{td} + p_t a^s_{td} + \left[p_t - p^o_t(\varepsilon)\right] a^s_{ti} & \text{if } \varepsilon < \varepsilon^*_t \end{cases}$$

$$\overline{a}^m_{ti} + p_t \overline{a}^s_{ti} = \begin{cases} a^m_{ti} + p_t a^s_{ti} - \left[p^o_t(\varepsilon) - p_t\right] \frac{a^m_{ti}}{p^o_t(\varepsilon)} & \text{if } \varepsilon^*_t \leq \varepsilon \\ a^m_{ti} + p_t a^s_{ti} - \left[p_t - p^o_t(\varepsilon)\right] a^s_{ti} & \text{if } \varepsilon < \varepsilon^*_t \end{cases}$$

Investor-investor trade: formulation

Investor type ε_i and $\left(a_{ti}^m, a_{ti}^s\right)$ contacts investor type ε_j and $\left(a_{tj}^m, a_{tj}^s\right)$

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If investor i offers, he chooses $\left\langle \left(\overline{a}_{ti*}^m, \overline{a}_{ti*}^s\right), \left(\overline{a}_{tj}^m, \overline{a}_{tj}^s\right) \right\rangle$ by solving

$$\max_{\underline{a}_{ti*}^m, \underline{a}_{ti*}^s, \underline{a}_{tj}^m, \underline{a}_{tj}^s} \left[\varepsilon_i y \underline{a}_{ti*}^s + W_t^I\left(\underline{a}_{ti*}^m, \underline{a}_{ti*}^s\right)\right]$$

$$\underline{a}_{ti*}^m + \underline{a}_{tj}^m \leq a_{ti}^m + a_{tj}^m$$

$$\underline{a}_{ti*}^s + \underline{a}_{tj}^s \leq a_{ti}^s + a_{tj}^s$$

$$\varepsilon_j y \underline{a}_{tj}^s + W_t^I\left(\underline{a}_{tj}^m, \underline{a}_{tj}^s\right) \geq \varepsilon_j y a_{tj}^s + W_t^I\left(a_{tj}^m, a_{tj}^s\right)$$

Investor-investor trade: solution

Investor type ε_i and (a^m_{ti}, a^s_{ti}) offers investor type ε_j and (a^m_{tj}, a^s_{tj}):

$$\underline{a}^m_{ti*} = \begin{cases} a^m_{ti} - \min\left[p^o_t(\varepsilon_j)a^s_{tj}, a^m_{ti}\right] & \text{if } \varepsilon_j < \varepsilon_i \\ a^m_{ti} + \min\left[p^o_t(\varepsilon_j)a^s_{ti}, a^m_{tj}\right] & \text{if } \varepsilon_i < \varepsilon_j \end{cases}$$

$$\underline{a}^s_{ti*} = \begin{cases} a^s_{ti} + \min\left[\frac{a^m_{ti}}{p^o_t(\varepsilon_j)}, a^s_{tj}\right] & \text{if } \varepsilon_j < \varepsilon_i \\ a^s_{ti} - \min\left[\frac{a^m_{tj}}{p^o_t(\varepsilon_j)}, a^s_{ti}\right] & \text{if } \varepsilon_i < \varepsilon_j \end{cases}$$

$$\underline{a}^m_{tj} = a^m_{tj} + a^m_{ti} - \underline{a}^m_{ti*}$$

$$\underline{a}^s_{tj} = a^s_{tj} + a^s_{ti} - \underline{a}^s_{ti*}$$

Euler equations: dealers

$$\phi_t^m \geq \beta \max\left(\phi_{t+1}^m, \phi_{t+1}^s / p_{t+1}\right)$$

$$\phi_t^s \geq \beta \pi \max\left(p_{t+1}\phi_{t+1}^m, \phi_{t+1}^s\right)$$

Euler equations: investors

$$\phi_t^m \geq \beta \left[\phi_{t+1}^m + \delta\theta \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} \left(\frac{\varepsilon_i y + \phi_{t+1}^s}{p_{t+1}} - \phi_{t+1}^m \right) dG\left(\varepsilon_i\right) \right.$$

$$\left. + \alpha\eta \int_{\varepsilon_{t+1}^c}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \left(\frac{\varepsilon_i y + \phi_{t+1}^s}{p_{t+1}^o\left(\varepsilon_j\right)} - \phi_{t+1}^m \right) dG\left(\varepsilon_i\right) dG\left(\varepsilon_j\right) \right]$$

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$$\phi_t^s \geq \beta\pi \left[\bar{\varepsilon} y + \phi_{t+1}^s + \delta\theta \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} \left[p_{t+1}\phi_{t+1}^m - \left(\varepsilon_i y + \phi_{t+1}^s\right) \right] dG\left(\varepsilon_i\right) \right.$$

$$\left. + \alpha\left(1-\eta\right) \int_{\varepsilon_L}^{\varepsilon_{t+1}^c} \int_{\varepsilon_L}^{\varepsilon_j} \left[p_{t+1}^o\left(\varepsilon_j\right)\phi_{t+1}^m - \left(\varepsilon_i y + \phi_{t+1}^s\right) \right] dG\left(\varepsilon_i\right) dG\left(\varepsilon_j\right) \right]$$

Euler equations: investors

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$$\left. + \alpha\eta \int_{\varepsilon_{t+1}^c}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \left(\frac{\varepsilon_i y + \phi_{t+1}^s}{p_{t+1}^o\left(\varepsilon_j\right)} - \phi_{t+1}^m \right) dG\left(\varepsilon_i\right) dG\left(\varepsilon_j\right) \right]$$

$$\phi_t^s \geq \beta\pi \left[\bar{\varepsilon} y + \phi_{t+1}^s + \delta\theta \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} \left[p_{t+1}\phi_{t+1}^m - \left(\varepsilon_i y + \phi_{t+1}^s\right) \right] dG\left(\varepsilon_i\right) \right.$$

$$\left. + \alpha\left(1-\eta\right) \int_{\varepsilon_L}^{\varepsilon_{t+1}^c} \int_{\varepsilon_L}^{\varepsilon_j} \left[p_{t+1}^o\left(\varepsilon_j\right)\phi_{t+1}^m - \left(\varepsilon_i y + \phi_{t+1}^s\right) \right] dG\left(\varepsilon_i\right) dG\left(\varepsilon_j\right) \right]$$

$$\varepsilon_{t+1}^c \text{ satisfies } \phi_{t+1}^m A_{It+1}^m = \left(\varepsilon_{t+1}^c y + \phi_{t+1}^s\right) A^s$$

Definition of equilibrium

Definition

An equilibrium is a sequence of:

(a) bilateral terms of trade in the OTCM

(b) asset holdings

(c) prices

such that:

(i) terms of trade solve bargaining problems

(ii) asset holdings solve individual problems in the CM

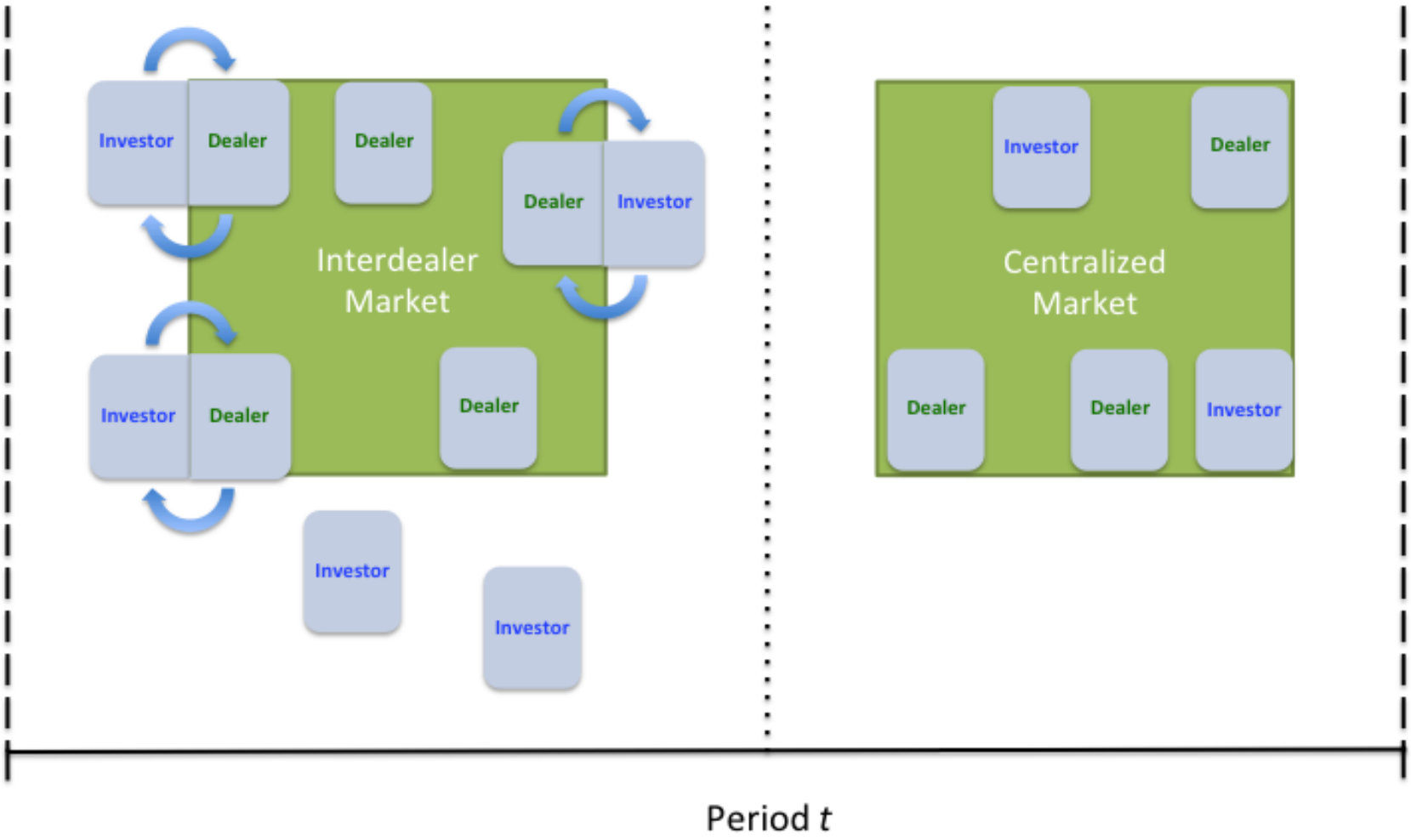
(iii) all Walrasian markets clear

Friedman rule

Proposition

The allocation implemented by the stationary monetary equilibrium converges to the symmetric efficient allocation as $\gamma \to \beta$.

Pure-dealer OTCM



Pure-dealer OTCM: nonmonetary equilibrium

Proposition

(i) A nonmonetary equilibrium exists for any parametrization.

(ii) In the nonmonetary equilibrium:

- *there is no trade in the OTC market*
- $A_I^s = A^s - A_D^s = A^s$ *(only investors hold equity shares)*
- *the equity price is:*

$$\phi^s = \frac{\beta\pi}{1-\beta\pi}\bar{\varepsilon}y.$$

Pure-dealer OTCM: nonmonetary equilibrium

Proposition

(i) *If* $\gamma \in (\beta, \bar{\gamma})$, *there is one stationary monetary equilibrium.*

(ii) *For any* $\gamma \in (\beta, \bar{\gamma})$, $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ *is the unique solution to*

$$\frac{(1-\beta\pi)\int_{\varepsilon^*}^{\varepsilon_H}\left[1-G(\varepsilon)\right]d\varepsilon}{\varepsilon^* + \beta\pi\left[\bar{\varepsilon} - \varepsilon^* + \delta\theta\int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\,d\varepsilon\right]\mathbb{I}_{\{\hat{\gamma}<\gamma\}}} - \frac{\gamma-\beta}{\beta\delta\theta} = 0.$$

(iii) *As* $\gamma \to \bar{\gamma}$, $\varepsilon^* \to \varepsilon_L$ *and* $\phi^s \to \frac{\beta\pi}{1-\beta\pi}\bar{\varepsilon}y$.

(iv) *As* $\gamma \to \beta$, $\varepsilon^* \to \varepsilon_H$ *and* $\phi^s \to \frac{\beta\pi}{1-\beta\pi}\varepsilon_H y$.

Pure-dealer OTCM: stationary monetary equilibrium

Low Inflation	High Inflation
$A_D^s = \pi A^s$	$A_D^s = 0$
$A_I^s = (1-\pi)A^s$	$A_I^s = A^s$
$A_{It}^m = A_t^m$	$A_{It}^m = A_t^m$

β $\quad \hat{\gamma} \quad \bar{\gamma} \quad \gamma$

$$\phi^s = \frac{\beta\pi}{1-\beta\pi}\varepsilon^* y \quad > \quad \phi^s = \frac{\beta\pi}{1-\beta\pi}\left(\bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)d\varepsilon\right)$$

$$Z = \frac{A_D^s + \delta G(\varepsilon^*)A_I^s}{\delta\theta\left[[1-G(\varepsilon^*)]\frac{1}{\varepsilon^* y + \phi^s} + \delta(1-\theta)\int_{\varepsilon^*}^{\varepsilon_H}\frac{1}{\varepsilon y + \phi^s}dG(\varepsilon)\right]}$$

Pure-dealer OTCM: dealers' liquidity provision

$$R_D^s\left(\varepsilon^*\right) \equiv \frac{\varepsilon^* y + \phi^s}{\phi^s}$$

$$R_I^s\left(\varepsilon^*\right) \equiv \frac{\bar{\varepsilon} y + \delta\theta G\left(\varepsilon^*\right)\left(\varepsilon^* - \bar{\varepsilon}^{*I}\right) y + \phi^s}{\phi^s}$$

Pure-dealer OTCM: dealers' liquidity provision

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$$R_I^s\left(\varepsilon^*\right) \equiv \frac{\bar{\varepsilon} y + \delta\theta G\left(\varepsilon^*\right)\left(\varepsilon^* - \bar{\varepsilon}^{*I}\right) y + \phi^s}{\phi^s}$$

$$\frac{\partial R_I^s\left(\varepsilon^*\right)}{\partial \varepsilon^*} = \delta\theta G\left(\varepsilon^*\right)\frac{y}{\phi^s} < \frac{y}{\phi^s} = \frac{\partial R_D^s\left(\varepsilon^*\right)}{\partial \varepsilon^*}$$

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$$R_D^s\left(\varepsilon_L\right) = \frac{\varepsilon_L y + \phi^s}{\phi^s} < \frac{\bar{\varepsilon} y + \phi^s}{\phi^s} = R_I^s\left(\varepsilon_L\right)$$

Pure-dealer OTCM: dealers' liquidity provision

$$R_D^s(\varepsilon^*) \equiv \frac{\varepsilon^* y + \phi^s}{\phi^s}$$

$$R_I^s(\varepsilon^*) \equiv \frac{\bar{\varepsilon} y + \delta\theta G(\varepsilon^*)(\varepsilon^* - \bar{\varepsilon}^{*I}) y + \phi^s}{\phi^s}$$

$$\frac{\partial R_I^s(\varepsilon^*)}{\partial \varepsilon^*} = \delta\theta G(\varepsilon^*) \frac{y}{\phi^s} < \frac{y}{\phi^s} = \frac{\partial R_D^s(\varepsilon^*)}{\partial \varepsilon^*}$$

$$R_D^s(\varepsilon_L) = \frac{\varepsilon_L y + \phi^s}{\phi^s} < \frac{\bar{\varepsilon} y + \phi^s}{\phi^s} = R_I^s(\varepsilon_L)$$

$$R_I^s(\varepsilon_H) = \frac{[\delta\theta\varepsilon_H + (1-\delta\theta)\bar{\varepsilon}] y + \phi^s}{\phi^s} < \frac{\varepsilon_H y + \phi^s}{\phi^s} = R_D^s(\varepsilon_H)$$

Asset prices and inflation

Proposition (Pure-dealer market)

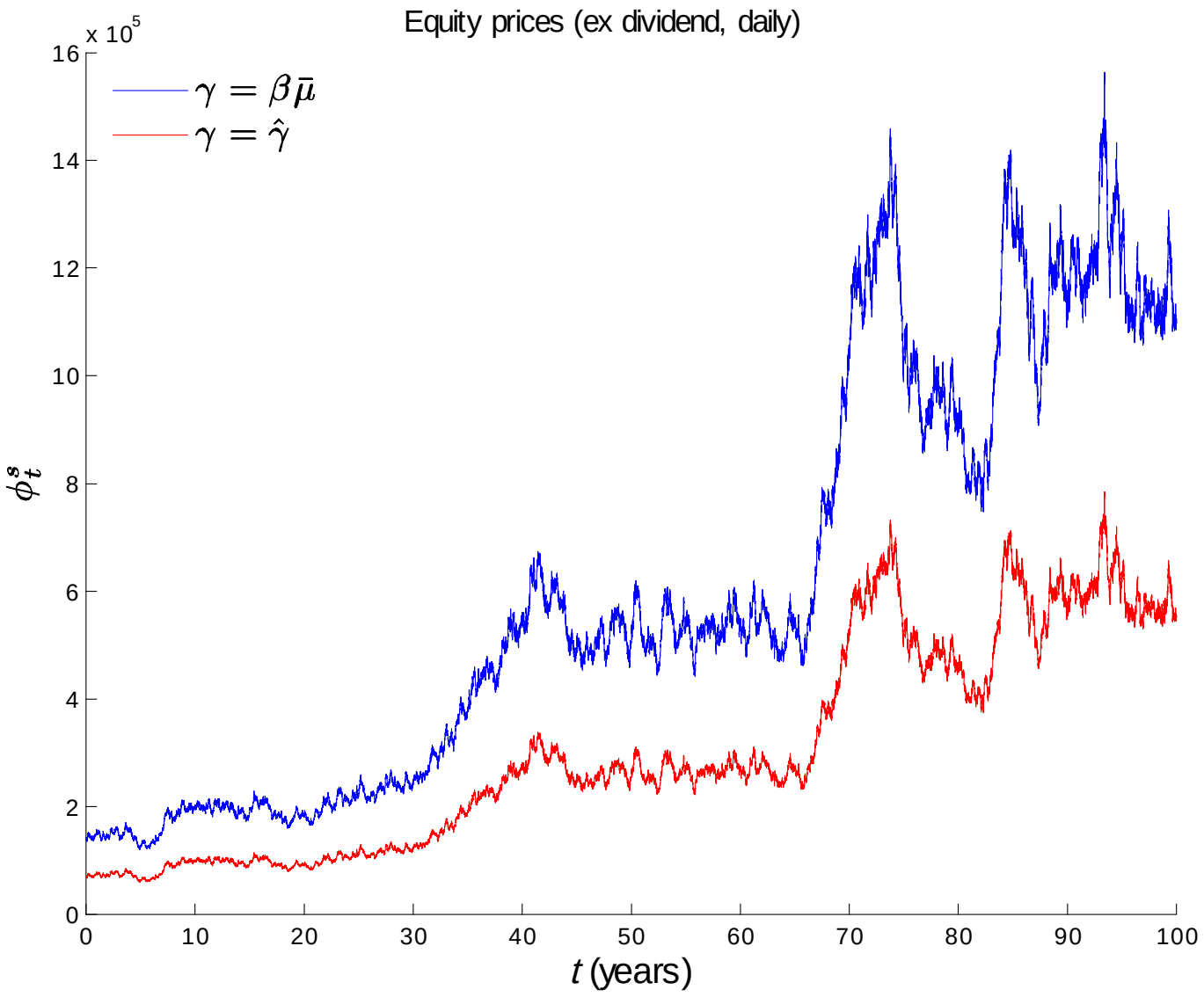
In the stationary monetary equilibrium of the model with $\alpha = 0$:

(i) $\partial\phi^s / \partial\gamma < 0$

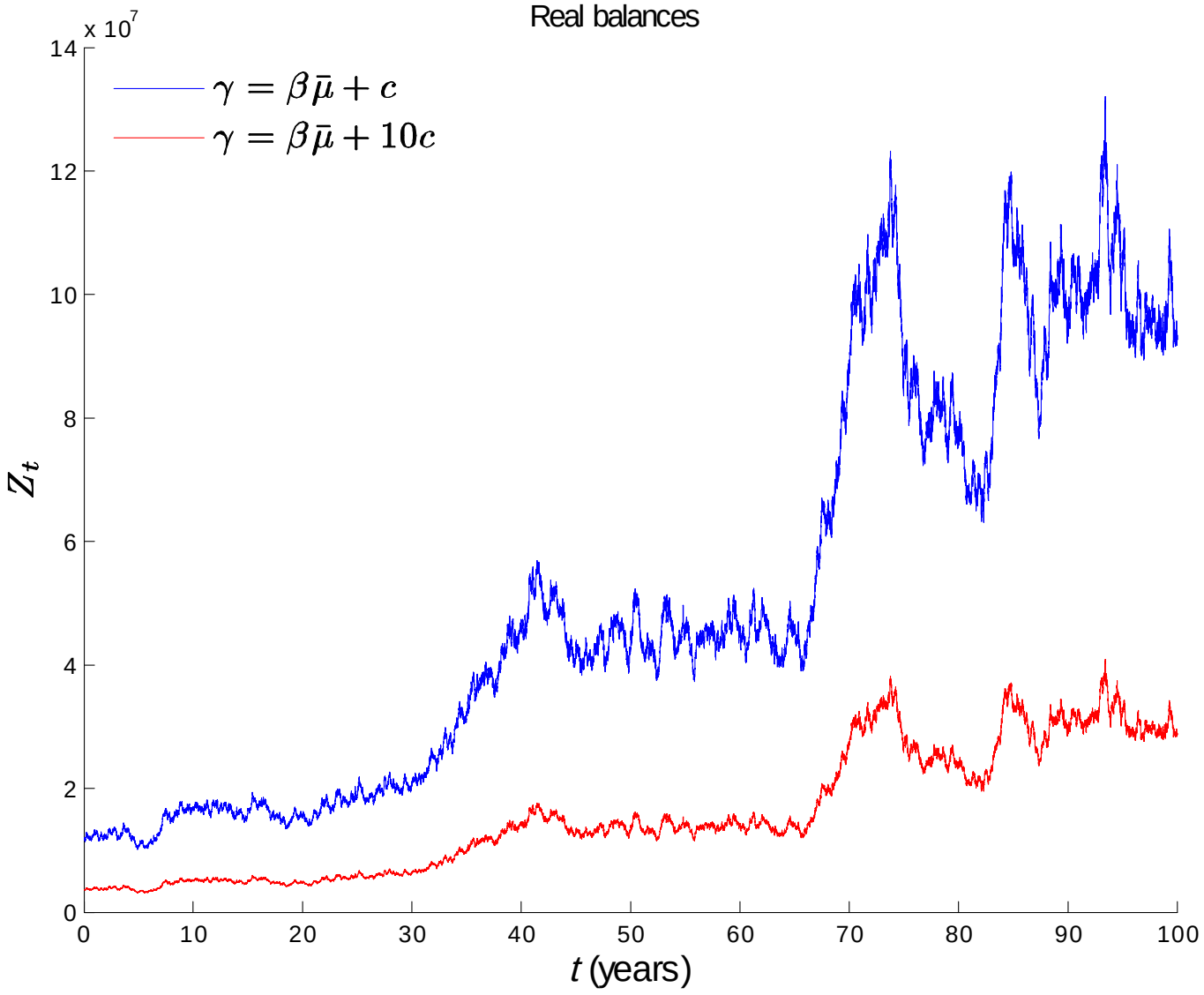
(ii) $\partial Z / \partial\gamma < 0$ *and* $\partial\phi_t^m / \partial\gamma < 0$

(iii) $\partial\bar{\phi}^s / \partial\gamma < 0$

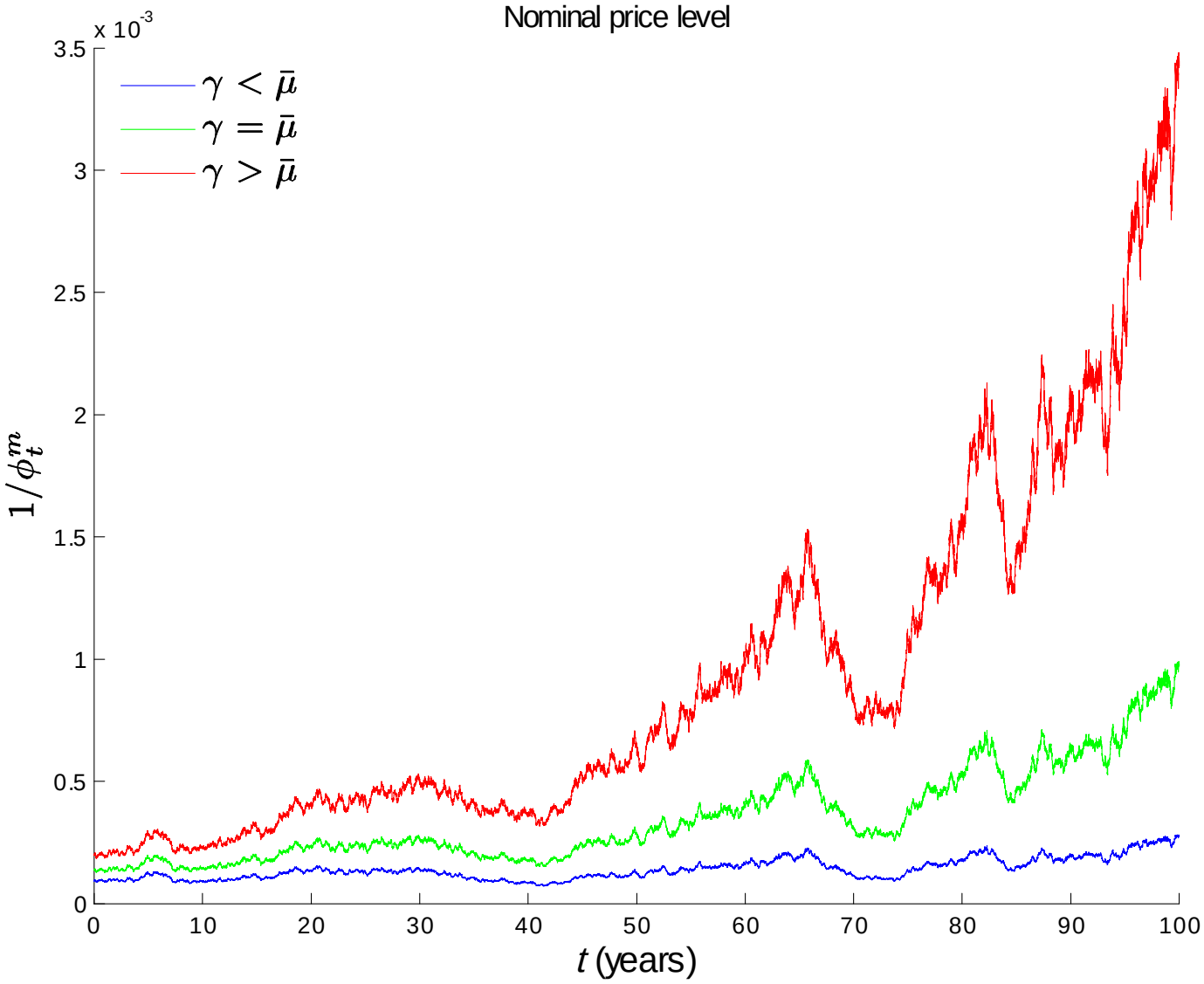
Asset prices and inflation: equity



Asset prices and inflation: real balances



Asset prices and inflation: price of money



Asset prices and OTC frictions (delays and market power)

Proposition (Pure-dealer market)

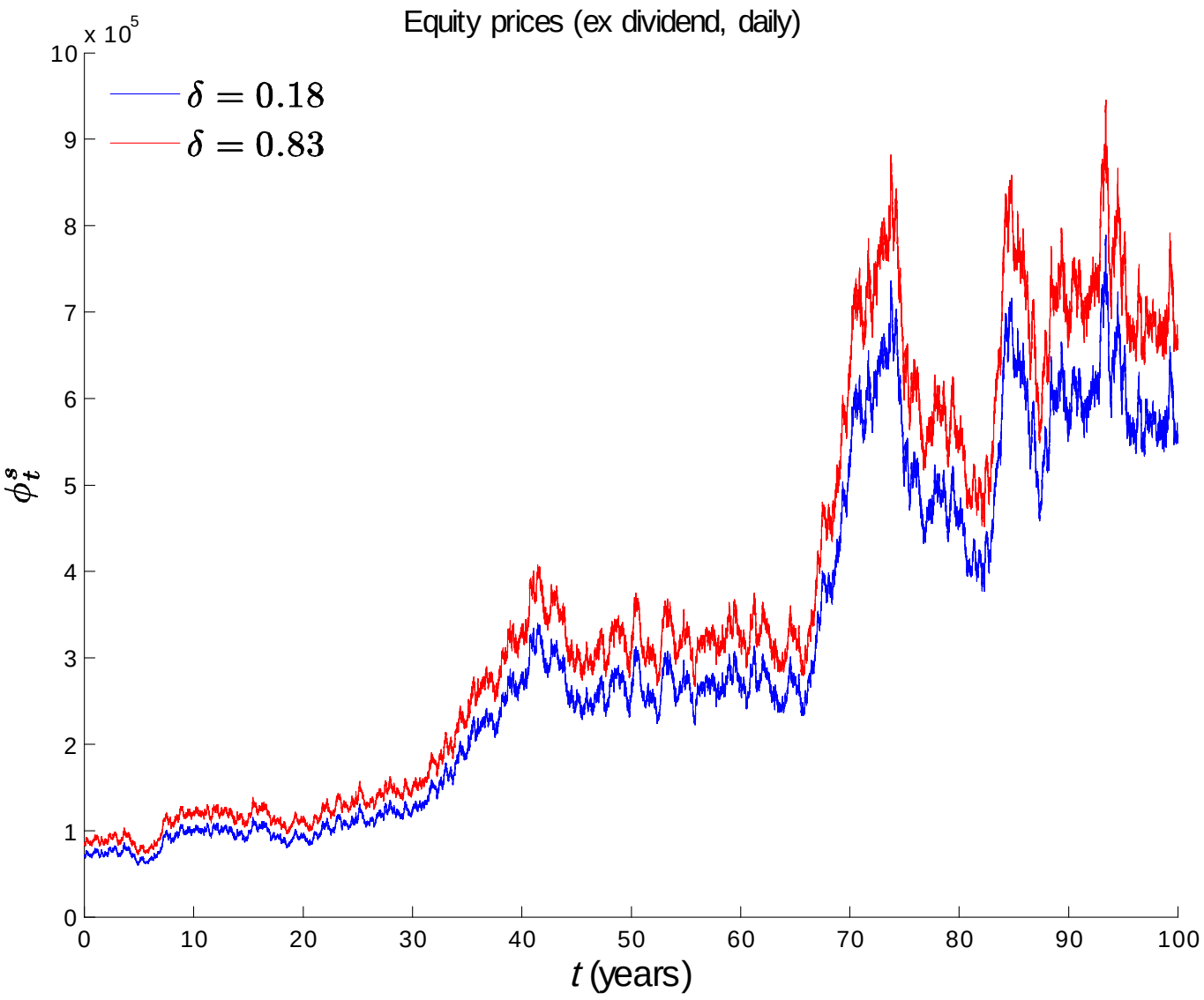
In the stationary monetary equilibrium with $\alpha = 0$:

(i) $\partial \phi^s / \partial (\delta\theta) > 0$

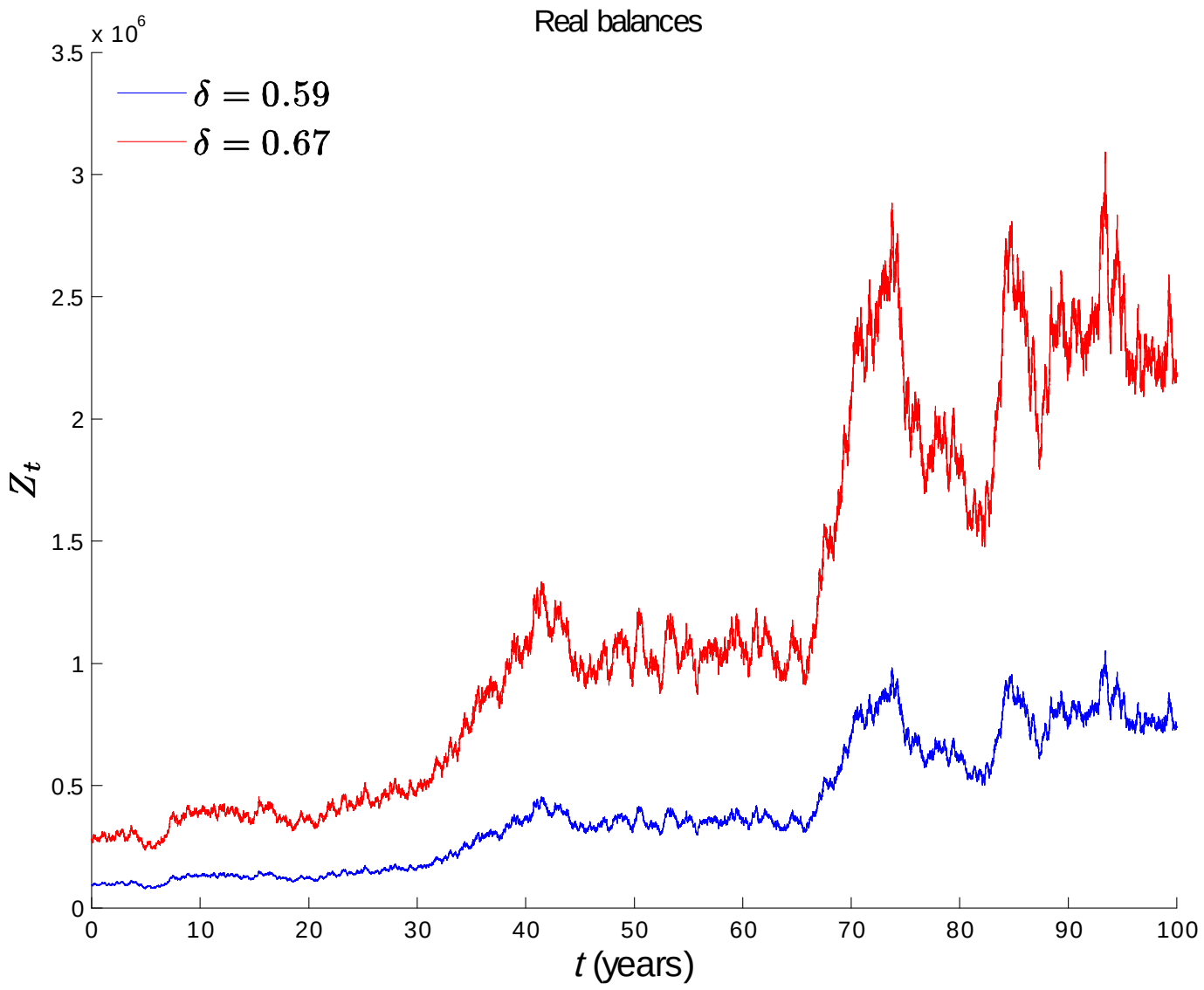
(ii) $\partial \bar{\phi}^s / \partial (\delta\theta) > 0$

(iii) $\partial Z / \partial \delta > 0$ *and* $\partial \phi_t^m / \partial \delta > 0$, *for* $\gamma \in (\hat{\gamma}, \bar{\gamma})$

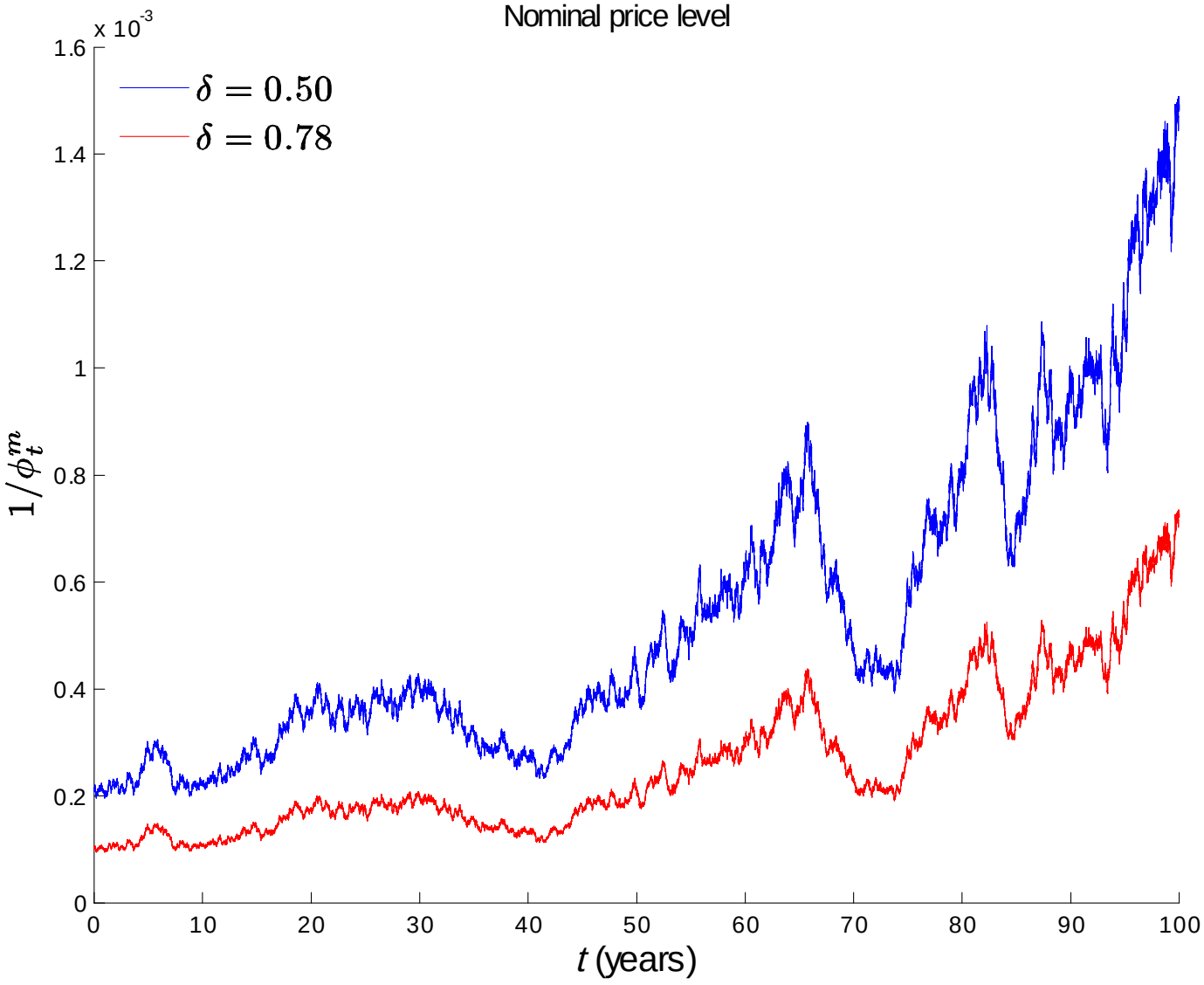
Asset prices and OTC frictions: equity



Asset prices and OTC frictions: real balances



Asset prices and OTC frictions: price of money



Measures of financial liquidity

- Liquidity provision by dealers
- Trade volume
- Bid-ask spreads

Liquidity provision by dealers

Proposition (Pure-dealer market)

In the stationary monetary equilibrium with $\alpha = 0$:

(i) A_D^s is (weakly) decreasing in the inflation rate

(ii) For any γ close to β, A_D^s is nonmonotonic in $\delta\theta$:

- $A_D^s = 0$ *for* $\delta\theta \approx 0$ *and* $\delta\theta \approx 1$
- $A_D^s > 0$ *for intermediate values of* $\delta\theta$

Volume

Proposition (Pure-dealer market)

In the stationary monetary equilibrium with $\alpha = 0$:

(i) trade volume is decreasing in γ

(ii) trade volume is increasing in δ and θ

Bid-ask spreads

A dealer with bargaining power who executes a trade:

- Pays bid price $p_t^o(\varepsilon) \equiv \left(\frac{\varepsilon y + \phi_t^s}{\varepsilon_t^* y + \phi_t^s}\right) p_t < p_t$ to investor who sells
- Charges ask price $p_t^o(\varepsilon) > p_t$ to investor who buys

In each of these transactions, the dealer earns a real spread $\mathcal{S}$:

$$\mathcal{S} = \frac{|p_t^o(\varepsilon) - p_t|}{p_t} = \frac{|\varepsilon - \varepsilon^*| y}{\varepsilon^* y + \phi^s}$$

Speculation (e.g., Harrison and Kreps, 1978)

Define the speculative premium as

$$\mathcal{P} = \phi^s - \frac{\beta\pi}{1-\beta\pi}\bar{\varepsilon}y$$

Speculative premium

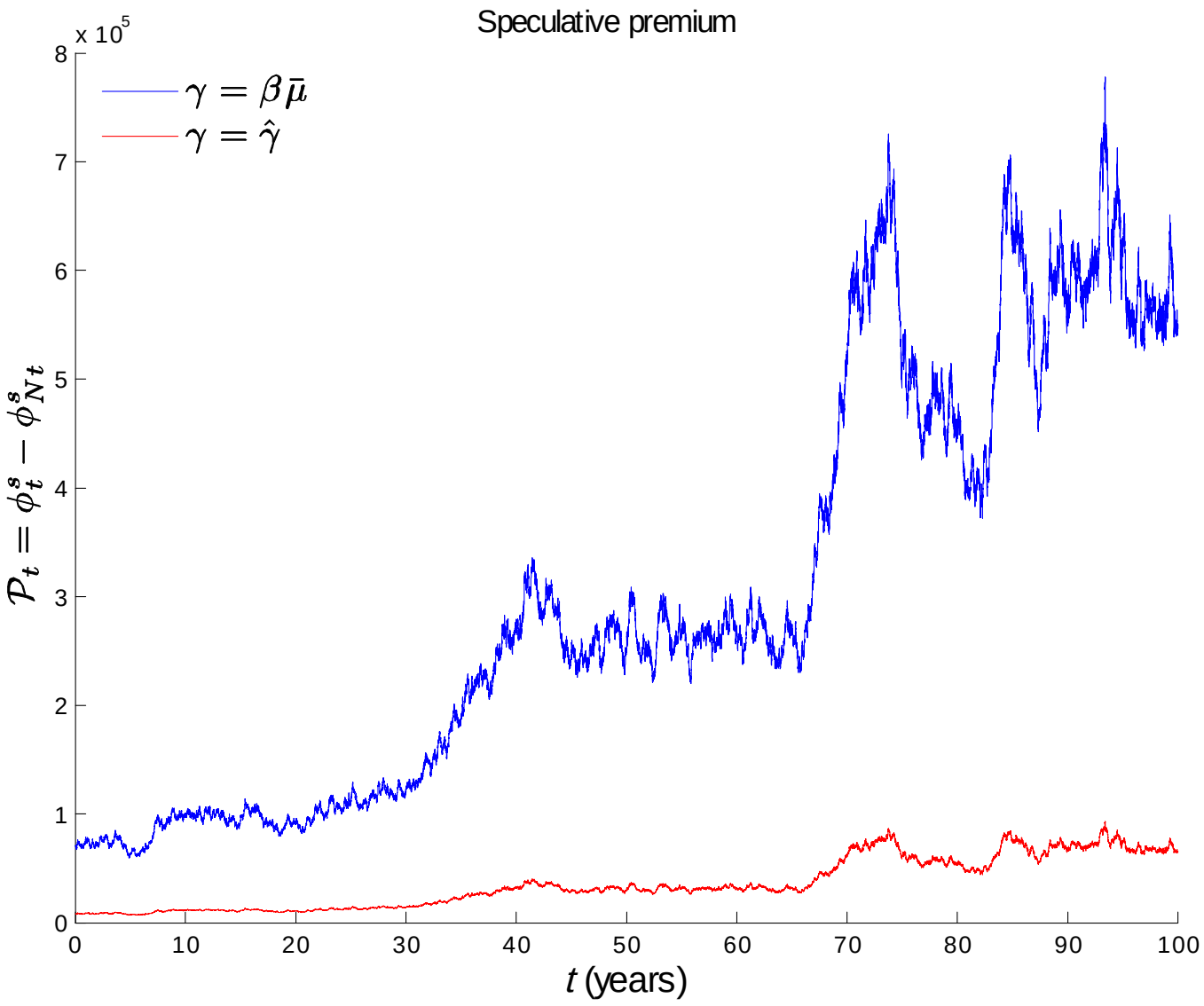
Proposition (Pure-dealer market)

In the stationary monetary equilibrium with $\alpha = 0$:

$$\mathcal{P} = \begin{cases} \frac{\beta\pi}{1-\beta\pi}\left(\varepsilon^* - \bar{\varepsilon}\right) y & \text{if } \beta < \gamma \leq \hat{\gamma} \\ \frac{\beta\pi}{1-\beta\pi}\delta\theta y \int_{\varepsilon_L}^{\varepsilon^*} G\left(\varepsilon\right) d\varepsilon & \text{if } \hat{\gamma} < \gamma < \bar{\gamma} \end{cases}$$

(i) $\partial\mathcal{P}/\partial\gamma < 0$, *(ii)* $\partial\mathcal{P}/\partial\left(\delta\theta\right) > 0$

Speculative premium



Endogenous trading delays: dealer entry

- $\delta(v)$: probability investor contacts a dealer
- $\kappa(v) \equiv \delta(v)/v$: probability dealer contacts an investor
- $\kappa'(v) < 0 < \delta'(v)$
- Free entry: to participate in OTCM of $t+1$ dealer must pay $k > 0$ general goods in the CM of t

Social planner with free entry of dealers

Proposition (Free entry, pure-dealer market)

The efficient allocation is characterized by three properties:

(i) Only dealers carry equity overnight.

(ii) Among those investors who have a trading opportunity with a dealer, only those with the highest preference type hold equity shares at the end of the OTC round of trade.

(iii) $\beta\delta'(v_t)(\varepsilon_H - \bar{\varepsilon})\, y\,(1-\pi)\, A^s - k \leq 0$, "$=$" *if* $v_t > 0$.

Free-entry equilibrium

Equilibrium conditions as before, plus the free-entry condition

$$\Phi_{t+1} - k \leq 0, \text{ with } \text{“}=\text{”} \text{ if } v_{t+1} > 0$$

where

$$\Phi_{t+1} = \beta\kappa\left(v_{t+1}\right)(1-\theta)\left\{G\left(\varepsilon^*_{t+1}\right)\mathcal{S}^b_{t+1} + \left[1 - G\left(\varepsilon^*_{t+1}\right)\right]\mathcal{S}^a_{t+1}\right\}\bar{\phi}_{t+1}$$

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$$\mathcal{S}^b_{t+1} \equiv \int_{\varepsilon_L}^{\varepsilon^*_{t+1}} \left[p_{t+1} - p^o_{t+1}(\varepsilon)\right] A^l_{t+1} \frac{dG(\varepsilon)}{G\left(\varepsilon^*_{t+1}\right)}$$

$$\mathcal{S}^a_{t+1} \equiv \int_{\varepsilon^*_{t+1}}^{\varepsilon_H} \left[p^o_{t+1}(\varepsilon) - p_{t+1}\right] \frac{A^m_{It+1}}{p^o_{t+1}(\varepsilon)} \frac{dG(\varepsilon)}{1 - G\left(\varepsilon^*_{t+1}\right)}$$

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$$\mathcal{S}^a_{t+1} \equiv \int_{\varepsilon^*_{t+1}}^{\varepsilon_H} \left[p^o_{t+1}(\varepsilon) - p_{t+1}\right] \frac{A^m_{It+1}}{p^o_{t+1}(\varepsilon)} \frac{dG(\varepsilon)}{1 - G\left(\varepsilon^*_{t+1}\right)}$$

$$\bar{\phi}_{t+1} \equiv \max\left(\phi^m_{t+1}, \phi^s_{t+1}/p_{t+1}\right)$$

Efficiency of monetary equilibrium

Proposition

Assume $k < \beta (1-\theta)(\varepsilon_H - \bar{\varepsilon}) y (1-\pi) A^s$. *The allocation implemented by the stationary monetary equilibrium converges to the symmetric efficient allocation as* $\gamma \to \beta$, *provided the bargaining power of dealers satisfies* $1-\theta = 1 - \frac{-\kappa'(v)v}{\kappa(v)}$.

Stochastic dividend growth and "sunspots"

$$\sigma_{ij} \equiv \Pr\left(S_{t+1} = S_j | S_t = S_i\right) \ (S_i \text{ is a sunspot})$$

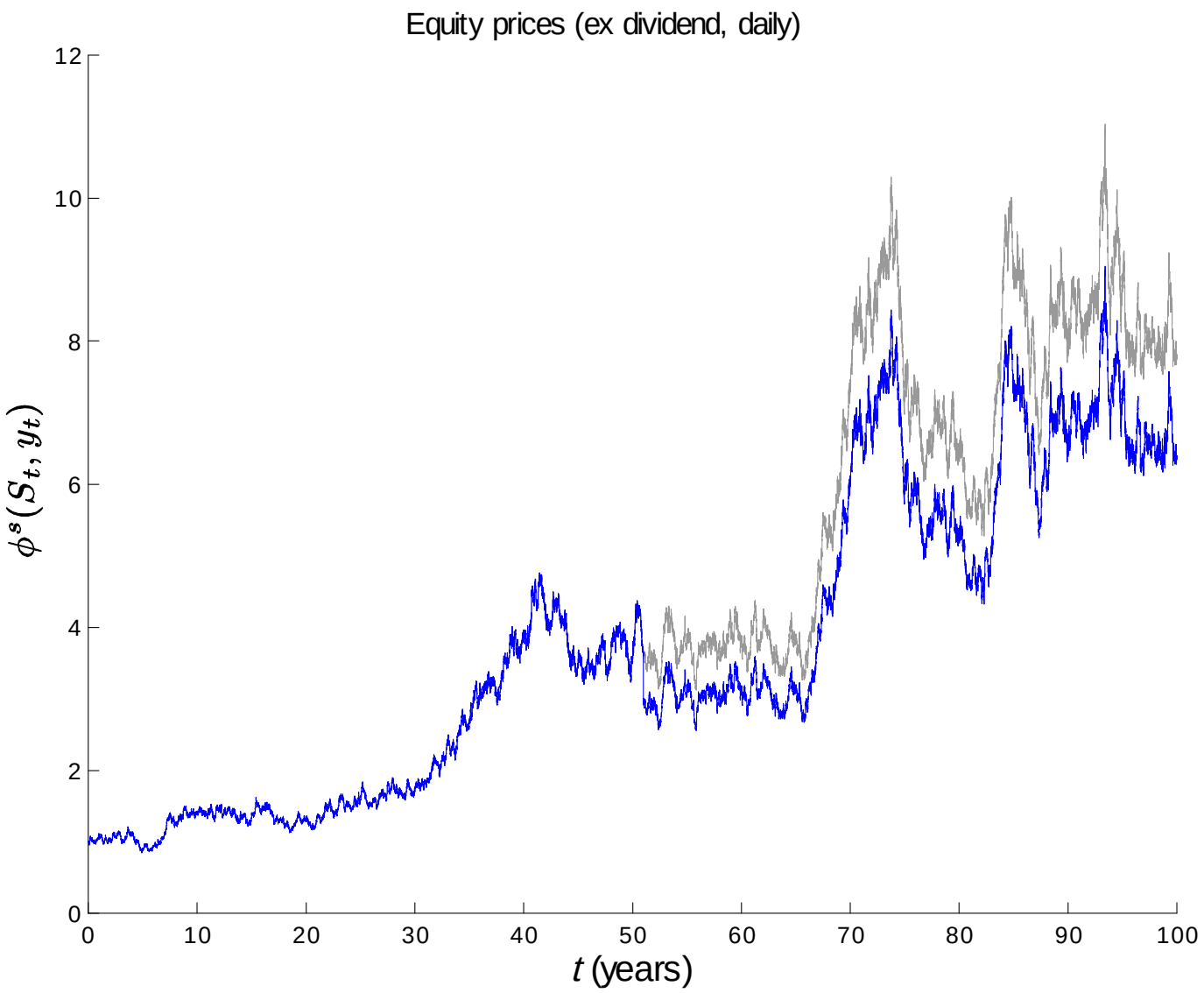
$$\tilde{Z}_i = \frac{\beta\bar{\mu}}{\gamma}\sum_j \sigma_{ij}\left[1 + \delta\theta\int_{\varepsilon_j^*}^{\varepsilon_H} \frac{\varepsilon-\varepsilon_j^*}{\varepsilon_j^*+\tilde{\phi}_j^s} dG(\varepsilon)\right]\tilde{Z}_j$$

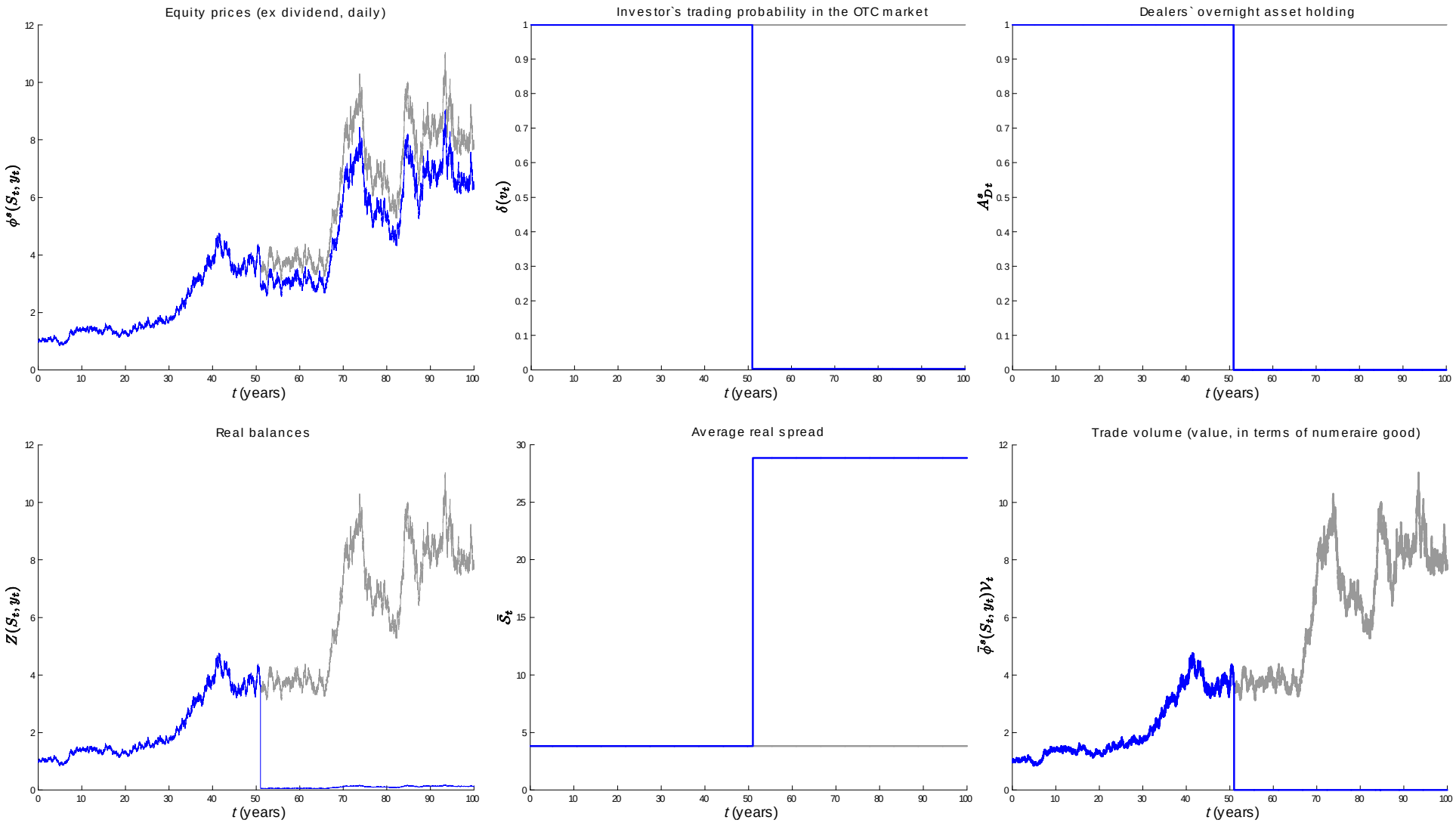
$$\tilde{\phi}_i^s = \beta\bar{\mu}\pi\sum_j \sigma_{ij}\left[\tilde{\phi}_j^s + \max\left(\varepsilon_j^*, \bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{\varepsilon_j^*}(\varepsilon_j^* - \varepsilon)dG(\varepsilon)\right)\right]$$

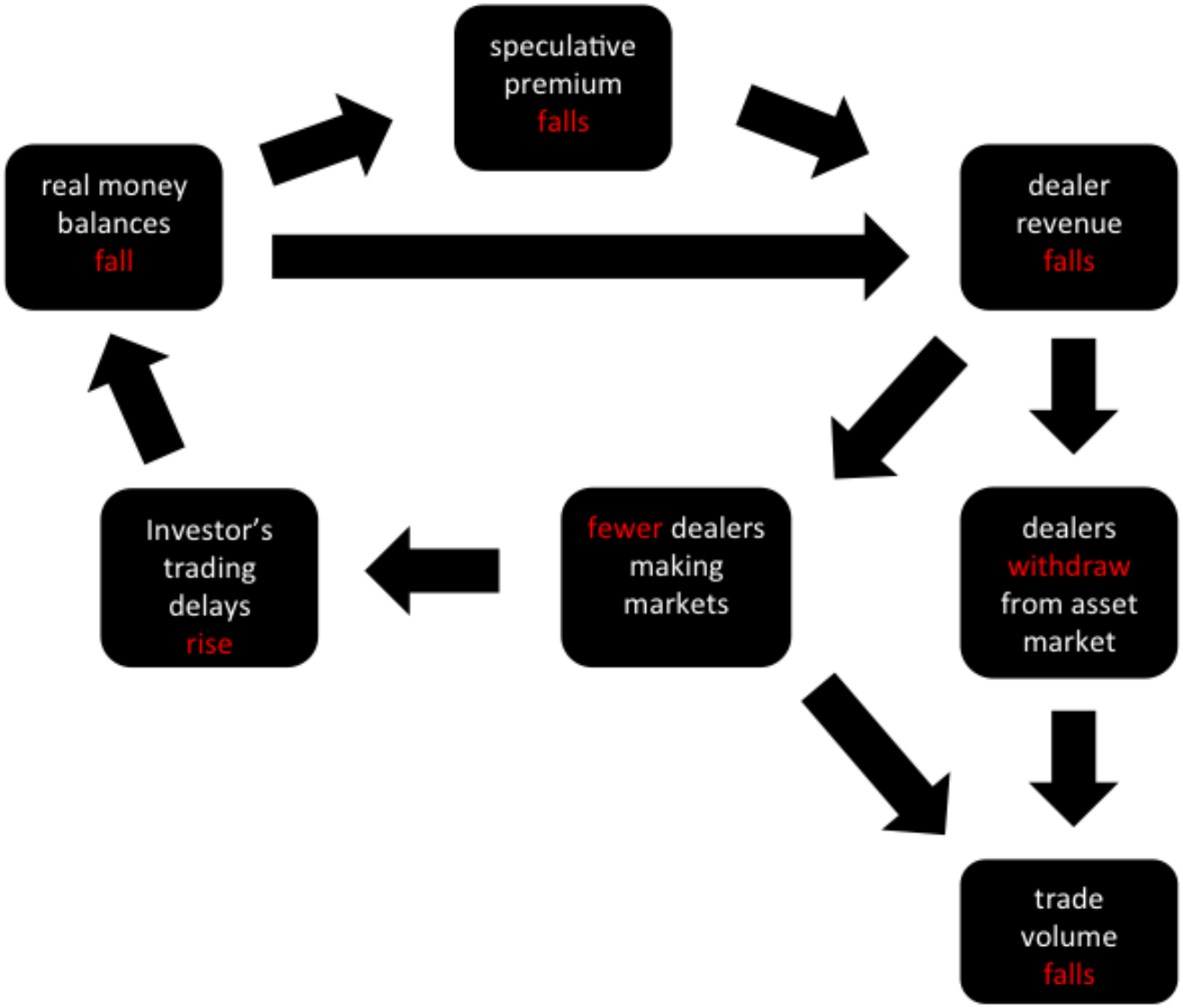
$$k = (1-\theta)\beta\bar{\mu}\frac{\delta(v_j)}{v_j}\left[A_{lj}^s\int_{\varepsilon_L}^{\varepsilon_j^*}(\varepsilon_j^* - \varepsilon)dG(\varepsilon) + \tilde{Z}_j\int_{\varepsilon_j^*}^{\varepsilon_H}\frac{\varepsilon-\varepsilon_j^*}{\varepsilon+\tilde{\phi}_j^s}dG(\varepsilon)\right]$$

$$A_{lj}^s = A^s \text{ if } \varepsilon_j^* < \bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{\varepsilon_j^*}(\varepsilon_j^* - \varepsilon)dG(\varepsilon) \ (= 0 \text{ otherwise})$$

$$\tilde{Z}_j = \frac{A_{Dj}^s + \delta(v_j)G(\varepsilon_j^*)A_{lj}^s}{\delta(v_j)\theta\left[1-G(\varepsilon_j^*)\right]\frac{1}{\varepsilon_j^*+\tilde{\phi}_j^s} + \delta(v_j)(1-\theta)\int_{\varepsilon_j^*}^{\varepsilon_H}\frac{1}{\varepsilon+\tilde{\phi}_j^s}dG(\varepsilon)}$$


Equity prices (ex dividend, daily)
$\phi^s(S_t, y_t)$
t (years)
0
2
4
6
8
10
12
0
10
20
30
40
50
60
70
80
90
100


Equity prices (ex dividend, daily)
$\phi^s(S_t, y_t)$
t (years)
Investor`s trading probability in the OTC market
$\delta(v_t)$
t (years)
Dealers` overnight asset holding
$A^s_{\mathcal{D}t}$
t (years)
Real balances
$Z(S_t, y_t)$
t (years)
Average real spread
$\bar{\mathcal{S}}_t$
t (years)
Trade volume (value, in terms of numeraire good)
$\bar{\phi}^s(S_t, y_t)\mathcal{V}_t$
t (years)


speculative premium falls
real money balances fall
dealer revenue falls
Investor's trading delays rise
fewer dealers making markets
dealers withdraw from asset market
trade volume falls

Summary

- A model of monetary exchange in OTC markets
- Liquidity and asset prices in OTC markets
  - Inflation:
    - distorts the asset allocation across investors
    - reduces trade volume
    - reduces dealers' incentives to provide liquidity
    - increases ask-spreads
  - Asset prices contain a *speculative premium* that:
    - decreases with inflation
    - decreases with OTC frictions (trading delays, power of dealers)

Summary

Dynamic stochastic equilibria with episodes that resemble crises:

- *speculative premium "bursts"*
  - sudden, sharp decline in asset price
- *liquidity "dries up"*
  - sudden, sharp decline in marketmaking and trade volume
  - sudden, sharp increase in trading delays and spreads per share

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Working on...

- More on dynamics
- Concave utility
- Private information
  - Unobservable idiosyncratic preference shocks
  - Lemons

tack så mycket.

Dealer problem in OTCM

$$V_t^D\left(\mathbf{a}_{td}\right) = \kappa\theta \int \hat{W}_t^D\left[\overline{a}_{td}^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon), \overline{a}_{td}^s\left(\mathbf{a}_{ti}, \mathbf{a}_{dt}, \varepsilon\right)\right] dH_t\left(\mathbf{a}_{ti}, \varepsilon\right)$$

$$+\kappa\left(1-\theta\right) \int \hat{W}_t^D\left[\overline{a}_{td^*}^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon), \overline{a}_{td^*}^s\left(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon\right)\right] dH_t\left(\mathbf{a}_{ti}, \varepsilon\right)$$

$$+\left(1-\kappa\right) \hat{W}_t^D\left(\mathbf{a}_{td}\right)$$

Investor problem in OTCM

$$
\begin{aligned}
V_t^I\left(\mathbf{a}_{ti},\varepsilon_i\right) \;=\; & \delta\int\theta\{\varepsilon_i y\overline{a}_{ti*}^s\left(\mathbf{a}_{ti},\mathbf{a}_{td},\varepsilon_i\right)+ \\
& W_t^I\left[\overline{a}_{ti*}^m\left(\mathbf{a}_{ti},\mathbf{a}_{td},\varepsilon_i\right),\overline{a}_{ti*}^s\left(\mathbf{a}_{ti},\mathbf{a}_{td},\varepsilon_i\right)\right]\}dF_t^D\left(\mathbf{a}_{td}\right) \\
& +\delta\int\left(1-\theta\right)\{\varepsilon_i y\overline{a}_{ti}^s\left(\mathbf{a}_{ti},\mathbf{a}_{td},\varepsilon_i\right)+ \\
& W_t^I\left[\overline{a}_{ti}^m\left(\mathbf{a}_{ti},\mathbf{a}_{td},\varepsilon_i\right),\overline{a}_{ti}^s\left(\mathbf{a}_{ti},\mathbf{a}_{td},\varepsilon_i\right)\right]\}dF_t^D\left(\mathbf{a}_{td}\right) \\
& +\alpha\int\tilde{\eta}\left(\varepsilon_i,\varepsilon_j\right)\{\varepsilon_i y\underline{a}_{ti*}^s\left(\mathbf{a}_{ti},\mathbf{a}_{tj},\varepsilon_i,\varepsilon_j\right)+ \\
& W_t^I\left[\underline{a}_{ti*}^m\left(\mathbf{a}_{ti},\mathbf{a}_{tj},\varepsilon_i,\varepsilon_j\right),\underline{a}_{ti*}^s\left(\mathbf{a}_{ti},\mathbf{a}_{tj},\varepsilon_i,\varepsilon_j\right)\right]\}dH_t\left(\mathbf{a}_{tj},\varepsilon_j\right) \\
& +\alpha\int\left[1-\tilde{\eta}\left(\varepsilon_i,\varepsilon_j\right)\right]\{\varepsilon_i y\underline{a}_{ti}^s\left(\mathbf{a}_{ti},\mathbf{a}_{tj},\varepsilon_i,\varepsilon_j\right)+ \\
& W_t^I\left[\underline{a}_{ti}^m\left(\mathbf{a}_{ti},\mathbf{a}_{tj},\varepsilon_i,\varepsilon_j\right),\underline{a}_{ti}^s\left(\mathbf{a}_{ti},\mathbf{a}_{tj},\varepsilon_i,\varepsilon_j\right)\right]\}dH_t\left(\mathbf{a}_{tj},\varepsilon_j\right) \\
& +\left(1-\alpha-\delta\right)\left[\varepsilon_i y a_{ti}^s+W_t^I\left(\mathbf{a}_{ti}\right)\right]
\end{aligned}
$$

Sunspots example

$\beta = (0.9598)^{1/365}$	$\bar{\mu} = E\left(\frac{y_{t+1}}{y_t}\right) = 1 + \frac{0.04}{365}$
$\varepsilon \sim U[0.01, 20]$	$\Sigma = SD\left(\frac{y_{t+1}-y_t}{y_t}\right) = \frac{0.12}{\sqrt{365}}$
$\delta(v) = 1 - e^{-v}$	$\pi = (0.95)^{1/365}$
$k = 0.2$	$\theta = 0.6$
$y_{t+1} = \bar{\mu} e^{x_{t+1}} y_t$	$\gamma = 1$
$x_{t+1} \sim \mathcal{N}\left(-\Sigma^2/2, \Sigma^2\right)$	$\sigma_{00} = (0.9964)^{1/365}$; $\sigma_{11} \approx 1$

$\delta(v_0)$	$\delta(v_1)$	ϕ_0^s/ϕ_1^s	Z_0/Z_1	$\varepsilon_0^*/\varepsilon_1^*$
0.9995	0.003	1.22	60	12